Conditional probability

Important modeling and computation technique:

Def'n: $P(A|B) = P(AB)/P(B)$ if $P(B) \neq 0$.

Def'n: For discrete rvs X, Y conditional pmf of Y given X is

$$\begin{aligned} f_{Y|X}(y|x) &= P(Y = y|X = x) \\ &= f_{X,Y}(x, y)/f_X(x) \\ &= f_{X,Y}(x, y)/\sum_t f_{X,Y}(x, t) \end{aligned}$$

IDEA: used as both computational tool and modelling tactic.

Specify joint distribution by specifying "marginal" and "conditional".

Modelling:

Assume $X \sim$ Poisson(λ).

Assume $Y|X \sim$Binomial(X, p).

Let $Z = X - Y$. Joint law of Y, Z?

$$
\begin{aligned}
&P(Y = y, Z = z) \\
&\quad = P(Y = y, X - Y = z) \\
&\quad = P(Y = y, X = z + y) \\
&\quad = P(Y = y | X = y + z) P(X = y + z) \\
&\quad = \binom{z+y}{y} p^y (1-p)^z e^{-\lambda} \lambda^{z+y} / (z+y)! \\
&\quad = \exp\{-p\lambda\} \frac{(p\lambda)^y}{y!} \exp\{(1-p)\lambda\} \frac{\{(1-p)\lambda\}^z}{z!}
\end{aligned}
$$

So: Y, Z independent Poissons.