STAT 870

Problems: Assignment 3

1. Customers arrive at a facility according to a Poisson Process with rate λ. There is a waiting time cost of c per unit time for each customer waiting. At fixed times T, $2T$, $3T$, and so on the customers are "processed" (so that the customer's waiting ends). The cost of dispatch is K.

   (a) What is the expected cost of the first cycle from time 0 to time T counting both dispatch cost and customer waiting cost.

   (b) What value of T minimizes the expected cost per unit time?

2. Assume that points are scattered in the plane according to a Poisson counting process with rate λ. Around each point we draw a circle with random radius R_i independently of the location in the plane of the centre. Assume that R has density f and finite second moment τ^2.

   (a) If $C(r)$ is the number of circles which cover the origin in the plane and have centres located at a distance less than r from the origin show that C is an inhomogeneous Poisson process with intensity $\lambda_r = 2\pi\lambda r \int_r^\infty f(u)\,du$.

   (b) Show that $C(\infty)$, which is the number of circles covering the origin, has a Poisson distribution with parameter $\lambda\pi\tau^2$.

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