STAT 890: Assignment 1

Instructions: In the following S, d and any S_i, d_i are separable metric spaces.

1. Show that if $f : S_1 \mapsto S_2$ is continuous then
   (a) For each closed set F in S_2 the inverse image $f^{-1}(K)$ is closed in S_1.
   (b) For each compact set K in S_1 the forward image $f(K)$ is compact in S_2.
2. Find a continuous function from $\mathbb{R}$ to $\mathbb{R}$ and an open set O for which $f(O)$ is not open. Do the same for closed.
3. Prove that the function

$$d(x, y) = \sup\{|x(t) - y(t)|; t \in [0, 1]\}$$

   on $\mathcal{C} \times \mathcal{C}$ is a metric.

Due: September 13, 2006.