Assignment #1

1) Given matrices

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \quad and \quad B = \begin{bmatrix} -4 & 2 & 5 \\ 0 & 1 & 1 \end{bmatrix}$$

verify that AB and BA are quite different matrices but tr(AB) equals tr(BA).

2) Given

$$X' = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 3 & 2 \end{bmatrix}$$

compute $P = X(X'X)^{-1}X'$ and $M = I - X(X'X)^{-1}X'$ and MP. Show that both M and P are symmetric idempotent, and determine their rank (hint: for an idempotent matrix, rank equals trace).

3) Given $Y' = [1 \quad 1 \quad 3 \quad 2]$ and X is as in question 2),
a) calculate the least squares estimates of $\hat{\beta}$.
b) test the hypothesis that $\hat{\beta}_2 = 0$
c) find the sum of squared residuals, the R^2 and $\bar{R}^2$ for the regression.
I want to see the work done by hand - feel free to check it on the computer.

4) Consider the regression $y = X\beta + \epsilon$ where y is n by 1, and X is n by 3.
a) State assumptions necessary to ensure that the OLS estimates of β_2 and β_3 are unbiased (do not include unnecessary assumptions).
b) Suppose you had estimated β, $\hat{\beta}$. How would you estimate $\gamma = \beta_2 + 2\beta_3$ without re-running the regression?
c) Prove your estimator of γ is unbiased, stating clearly any assumptions you need to make, such as those made in part a).