Assignment #2

1) Get on the computer of your choice and run a regression of the following dependent variable on a constant and the following independent variable, producing the usual descriptive statistics (LS estimates std errors, mean, variance, etc.)

$$Y = \quad 12 \;\; 11 \;\; 8 \;\; 7 \;\; 13 \;\; 12 \;\; 17 \;\; 11 \;\; 9 \;\; 14 \;\; 21$$

$$X = 1 \;\; 11 \;\; 31 \;\; 12 \;\; 17 \;\; 11 \;\; 9 \;\; 19 \;\; 14 \;\; 12 \;\; 16$$

2) i) $Y_t = \beta_1 + \beta_2 X_{2,t} + \epsilon$
ii) $Y_t = \beta_1 + \beta_2 X_{2,t}^2 + \epsilon$
iii) $Y_t = \beta_1 + \beta_2 X_{2,t} + \epsilon$ and $X_{2,t} = \alpha Z_t + \eta$
Where ϵ_t and η_t are innovations (unanticipated), and $E(\epsilon_t) = 0 = E(\eta_t)$. For each of i) ii) and iii), give
a) $E(Y_t)$.
b) $E(Y_t|Z_t)$.
c) $E(Y_t|X_{2,t})$.

3) If the population regression model is

$$Y_t = \beta_1 + \beta_2 X_{2,t} + \epsilon_t$$

and the OLS estimates are $\hat{\beta}_1$ and $\hat{\beta}_2$ based on 'n' observations $(Y_1, X_{2,1}) \cdots (Y_n, X_{2,n})$, evaluate

$$\sum_{t=1}^{n} (\hat{\beta}_1 + \hat{\beta}_2 X_{2,t})(Y_t - \hat{\beta}_1 - \hat{\beta}_2 X_{2,t}).$$

Explain your answer.

4) The OLS estimators of the population regression model

$$Y_t = \beta_1 + \beta_2 X_{2,t} + \epsilon_t$$

are

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}; \qquad \hat{\beta}_2 = \frac{\sum_{t=1}^{n} (X_t - \bar{X})(Y_t - \bar{Y})}{\sum_{t=1}^{n} (X_t - \bar{X})^2}$$

a) Suppose $n = 2$, $Y_1 = 3, X_1 = -1$, $Y_2 = 5, X_2 = 1$. Do you need the formulas as above to solve for $\hat{\beta}_1$ and $\hat{\beta}_2$?
b) Suppose $n = 3$ and $(Y_1, X_1) = (17, 11)$ $(Y_2, X_2) = (14, 21)$ $(Y_3, X_3) = (15, 19)$. Solve for $\hat{\beta}_1$ and $\hat{\beta}_2$. Show that $\sum_{t=1}^{3} \hat{\epsilon}_t = 0$ where $\hat{\epsilon}_t = (Y_t - \hat{\beta}_1 - \hat{\beta}_2 X_{2,t})$. What is $\sum_{t=1}^{3} \hat{Y}_t \hat{\epsilon}_t$?