Econ435 Midterm Exam

1) What do all of these have in common?

$$E[X] \quad E[X^2] \quad E[(X - E[X])^2] \quad E[Y \mid X]$$

1 point

2) $Y = \beta_1 + \beta_2 X + \varepsilon$, $E[\varepsilon] = 0$, and X, Z, and ε are independent of each other. Give $E[Y]$, $E[Y \mid X]$, $E[Y \mid Z]$.

2 points

3) What assumptions are needed to yield unbiasness of the least squares estimator? Point out where these assumptons come into play.

5 points

4) What assumptions are needed to yield the best linear unbiasness property of the least squares estimator?

2 points

5) Given (x,y) pairs (0,0) (0,1) (1,0) (1,1) show a least squares regression of y on x gives a $R^2 = 0$.

4 points

6) Suppose we have the following information for a model $y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$.

$$(X'X)^{-1} = \begin{bmatrix} .1 & .5 & .1 \\ .5 & .2 & .1 \\ .1 & .1 & .5 \end{bmatrix} \quad X'y = \begin{bmatrix} 8 \\ 4 \\ 6 \end{bmatrix} \quad \hat{\sigma}^2 = 10$$

Setup and perform a t-test and an F-test for $\beta_3 = 0$.

5 points

Setup an F-test for the joint hypothesis $\beta_2 + \beta_3 = 1$ and $\beta_3 = 0$. Do not calculate, but write out the test statistic completely.

5 points

7) Give the relative advantages and disadvantages of least squares and maximum likelihood estimation.

4 points

8) Suppose we have the estimated model from question 6), and recieved an extra observation on the X's, $(X_1, X_2, X_3) = (23,11,44)$. Provide a confidence interval for the Y that corresponds to (23,11,44).

5 points

9) Show $M = I - X(X'X)^{-1}X'$ is idempotent and symmetric.

2 points

Formulae

Given the Guass-Markov assumptions and the PRF

$$y = X\beta + \varepsilon$$

we have the following from a least squares regression on n observations producing regression residuals e .

$$\hat{se}(e_{n+1}) = = \hat{\sigma}\left(X'_{n+1}\ (X'X)^{-1}\ X_{n+1} + 1 \right)^{½}$$

Given normality of the innovations ε it follows directly that

$$\frac{e_{n+1}}{\hat{se}(e_{n+1})} \sim t_{n-k}$$

$$\hat{\beta} = (X'X)^{-1}X'\,y$$

$$t = \frac{\hat{\beta}_i - \beta_{oi}}{\hat{se}(\hat{\beta}_i)}$$

$$\hat{se}(\hat{\beta}_i) = \hat{\sigma}^2 X_{ii}^{-1}$$

$$X_{ii}^{-1} : i^{th} \ \ diagonal \ \ element \ \ of \ \ (X'X)^{-1}$$

$$F = (R\,\hat{\beta} - c)'(R\,\hat{var}(\hat{\beta})\,R')^{-1}(R\,\hat{\beta} - c)/q$$

$$= \frac{(e^{*\prime}e^{*} - e'e)/q}{(e'e)/n-q}$$