SIMON FRASER UNIVERSITY
Department of Economics
ECONOMICS 435

Fall 1992 Midterm Examination M. Kamstra

1) a) Suppose

$$Y = \beta_1 + \beta_2 X + \varepsilon \quad E[\,\varepsilon\,] = 0\,,$$

$$X = \alpha_1 + \alpha_2 Z + \eta \quad E[\,\eta\,] = 0\,,$$

where Z, η and ε are independent of each other.

Give $E[Y]$, $E[Y \mid X]$, $E[Y \mid Z]$.

2 points

b) Suppose that η and ε were not independent of each other. Does this pose a problem for us if we are using least squares to estimate

i) $E[Y \mid X]$?

ii) $E[Y \mid Z]$?

Explain. (Hint: Are we violating any assumptions that we make when we perform least squares?)

4 points

2) Short Answer: Explain briefly each of the following, and give an example if appropriate.

a) Non-linear model
b) BLUE
c) Post-sample stability test (Chow's short test)
d) Type I and Type II errors
e) Maximum Likelihood estimator
f) Equality of the F-test and the square of the t-test.

12 points

3) A study was conducted to assess the influence of various factors on the start of new firms in the electronics components industry in New York. With twenty years of data the following model was estimated:

$$\underset{}{y_t} = -59.31 + \underset{(1.156)}{4.983x_{t,1}} + \underset{(.210)}{2.1983x_{t,2}} + \underset{(2.063)}{3.816x_{t,3}} - \underset{(.330)}{.310x_{t,4}} - \underset{(3.055)}{.886x_{t,5}} + \underset{(1.568)}{3.21x_{t,6}} - \underset{(.354)}{.085x_{t,7}}$$

$R^2 = .766$, DW-statistic $= .1$

where

y = New business starts in the industry
x_1 = Population, in millions
x_2 = Industry size
x_3 = Measure of economic quality of life
x_4 = Measure of political quality of life
x_5 = Measure of environmental quality of life
x_6 = Measure of health and educational quality of life

x_7 = Measure of social quality of life
The figures in parentheses beneath the coefficient estimates are their estimated standard errors.

Suppose we wished to test the hypothesis that the coefficients on x_3, x_4, x_5 and x_7 were jointly 0.

a) Suggest an incorrect approach that a beginning statistics student might take to test this joint hypothesis.

b) Write out carefully a test of this hypothesis making use of only quantities estimated under the unrestricted alternative. Substitute actual values wherever possible.

10 points

4) What assumptions are needed to yield unbiasness of the least squares estimator? Point out where these assumptons come into play in showing unbiasness.

5 points

5) Write out the hypothesis to be tested by the Chow test, in the form $R\beta = c$ with all terms written out explicitly, i.e. what is R, β and c?

3 points

6) Give an example of an estimator that has smaller variance than the least squares estimator, even when all the assumptions of the classical linear regression model are satisfied. Discuss a criterion that is alternative to the least squares criterion for picking an estimator that could justify the estimator you proposed. List advantages and disadvantages of using such a criterion to pick an estimator.

5 points

7) List the properties of least squares estimators and Maximum Likelihood estimators, and compare and contrast.

3 points

Formulae

Given the Guass-Markov assumptions and the PRF

$$y = X\beta + \varepsilon$$

we have the following from a least squares regression on n observations producing regression residuals e .

$$\hat{se}(e_{n+1}) = \, = \hat{\sigma}\left(X'_{n+1}\ (X'X)^{-1}\ \ X_{n+1} + 1 \right)^{½}$$

Given normality of the innovations ε it follows directly that

$$\frac{e_{n+1}}{\hat{se}(e_{n+1})} \approx t_{n-k}$$

$$b = (X'X)^{-1}X'y$$

$$t = \frac{b_i - \beta_{oi}}{\hat{se}(\ b_i)}$$

$$\hat{se}(\ b_i) = \ s_X^2\ X_{ii}^{-1}$$

$$X_{ii}^{-1} : i^{th}\ \ diagonal\ \ element\ \ of\ \ (X'X)^{-1}$$

$$F = (R\ b - c)'(R\ v\hat{a}r(\ b)\ R')^{-1}(R\ b - c)/q$$

$$= \frac{(e'_* e_* - e'e)/q}{(e'e)/(n-q)}$$