Heterogeneous Aging in Spin Glasses

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References:

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Motivation

• Glassiness is an intrinsically non-equilibrium phenomenon – on any experimental time-scale, the state of a glassy system is one of continual relaxation

• There are many examples of glassiness: structural glasses, polymer glasses, supercooled liquids, spin glasses, vortices in High T_c superconductors, electron glass, . . .

• Theoretical problem: co-operative relaxation of large numbers of microscopic degrees of freedom in the presence of disorder

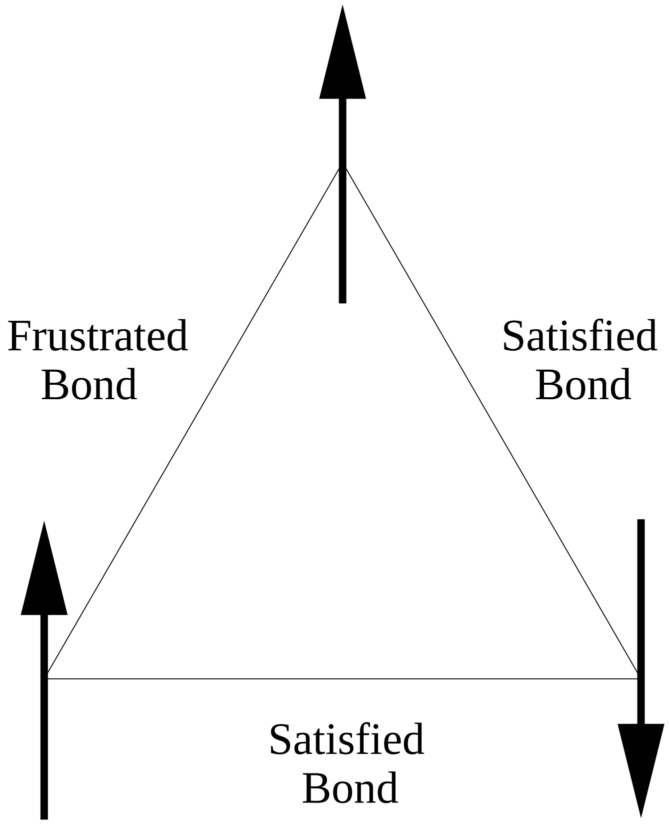
• Spin glass models – simple models that can shed light on glassiness in more complicated systems

• Spatial structure of aging: dynamic heterogeneities appear to play an important role in slow dynamics and need to be understood theoretically

Introduction to spin glasses

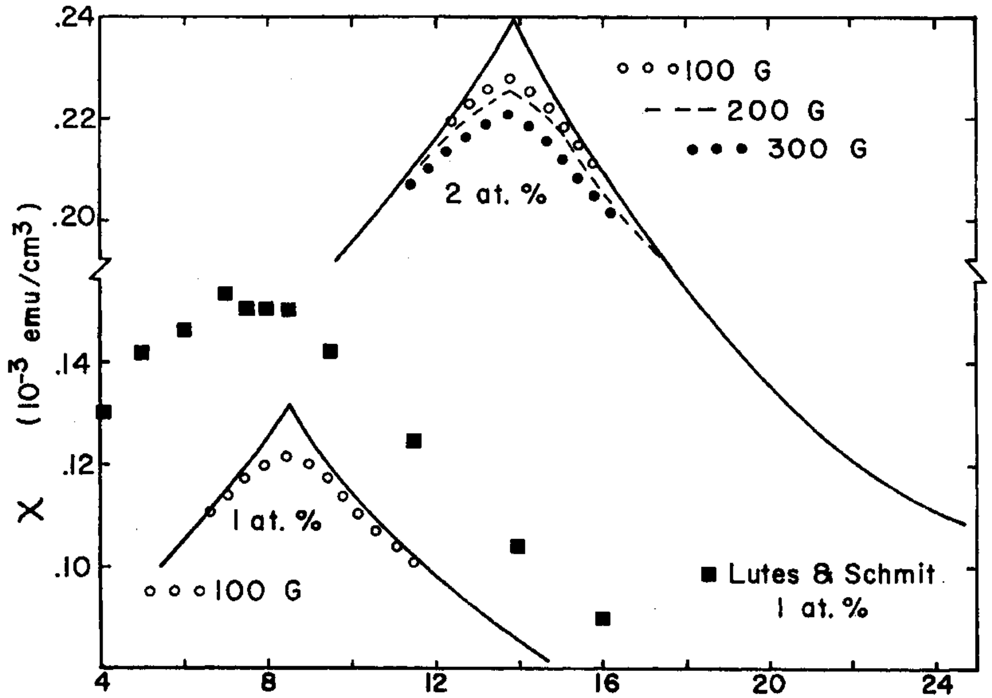
Spin glasses are materials with disordered, frustrated exchange interactions between local magnetic moments (spins)

- Frustration : Ising spins with antiferromagnetic exchange on the triangular lattice



Spin Freezing

- Below a temperature T_f, the dynamics of the spins do not equilibrate, due to the combination of frustration and quenched disorder
- Signature is a cusp in the magnetic susceptibility



Magnetic susceptibility against temperature for Cu:Mn
(Canella and Mydosh, 1972)

Spin Glass models

Edwards-Anderson (EA) Model

- Hamiltonian:

$$\mathcal{H}_{EA} = \sum_{ij} J_{ij} S_i S_j \tag{1}$$

- Distribution of short-ranged exchange interactions J_{ij} may be Gaussian or $\pm J$
- Edwards-Anderson order parameter:

$$q_{EA} = \lim_{t\to\infty} \lim_{N\to\infty} \overline{\langle S_i(t_0) S_i(t_0 + t) \rangle} \tag{2}$$

Sherrington-Kirkpatrick (SK) Model

- Mean-field version of EA model:
  All spins interact (infinite dimensional model)
- Equilibrium Solution of SK model: Replica Symmetry Breaking (Parisi, 1979)
- Infinite number of ground states – distribution of order parameters $P(q)$

Aging

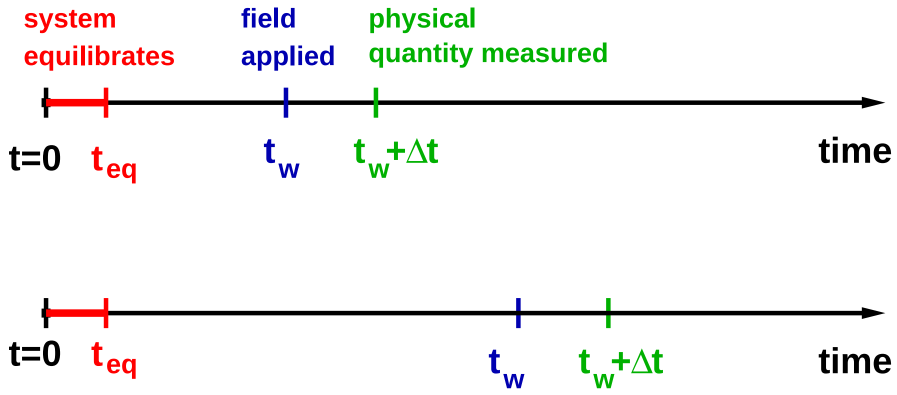
- Frustrated spin interactions lead to very long time-scales for spin relaxation

- Physical properties depend on the time since the system was prepared, the waiting time, t_w , as well as the time between measurements

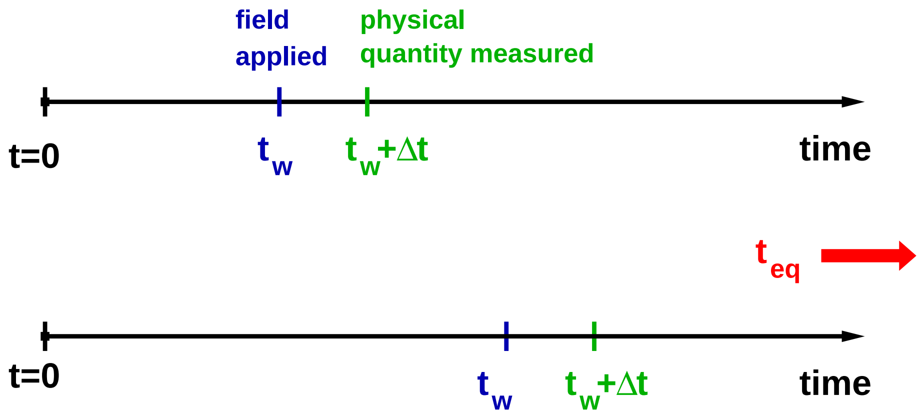
- In equilibrium, equilibration time is much shorter than experimentally interesting time-scales, and hence the time since the system was prepared is irrelevant: time translation invariant (TTI) dynamics

- Out of equilibrium systems – system has memory of its previous state, physical properties depend on the waiting time and the measuring time

Time-translation invariant (TTI) systems (Equilibrium)



Aging systems (Out of equilibrium)



Fluctuation-Dissipation Theorem

- Correlation:

$$C(t_w+\tau,t_w)=\frac{1}{N}\sum_{i=1}^{N}\overline{\langle \mathbf{S}_i(t_w+\tau)\cdot\mathbf{S}_i(t_w)\rangle}, \tag{3}$$

- Response to an infinitesimal field applied at the waiting time, $h(t_w)$,

$$R(t_w+\tau,t_w)=\frac{1}{N}\sum_{i=1}^{N}\frac{\partial\overline{\langle S_i(t_w+\tau)\rangle}}{\partial h(t_w)}\theta(\tau). \tag{4}$$

- In equilibrium, response and correlation are related by the Fluctuation-Dissipation Theorem (FDT):

$$R(\tau)=\frac{1}{T}\frac{\partial}{\partial\tau}C(\tau). \tag{5}$$

Out of equilibrium effective temperature

Dynamical Solution to SK model (Cugliandolo and Kurchan, 1994) :

- Separation of time-scales

$$C(t_w+\tau,t_w) = C_{ST}(\tau) + C_{AG}(t_w+\tau,t_w) \tag{6}$$

- Short time differences: Fluctuation-Dissipation Theorem (FDT) holds

$$R_{ST}(\tau) = \frac{1}{T}\frac{\partial}{\partial\tau}C_{ST}(\tau). \tag{7}$$

- Long time differences: Out of equilibrium fluctuation-dissipation relation (OEFDR)

$$R_{AG}(t_w+\tau,t_w) = \frac{X[C_{AG}]}{T}\frac{\partial}{\partial t_w}C_{AG}(t_w+\tau,t_w), \tag{8}$$

Effective Temperature

$$T_{\rm eff} = \frac{T}{X[C_{AG}]} \tag{9}$$

- $T_{\rm eff}$ governs heat transfer and partial equilibration, similarly to thermodynamic T (Cugliandolo, Kurchan, and Peliti, 1997)

Graphical Representation of the OEFDR

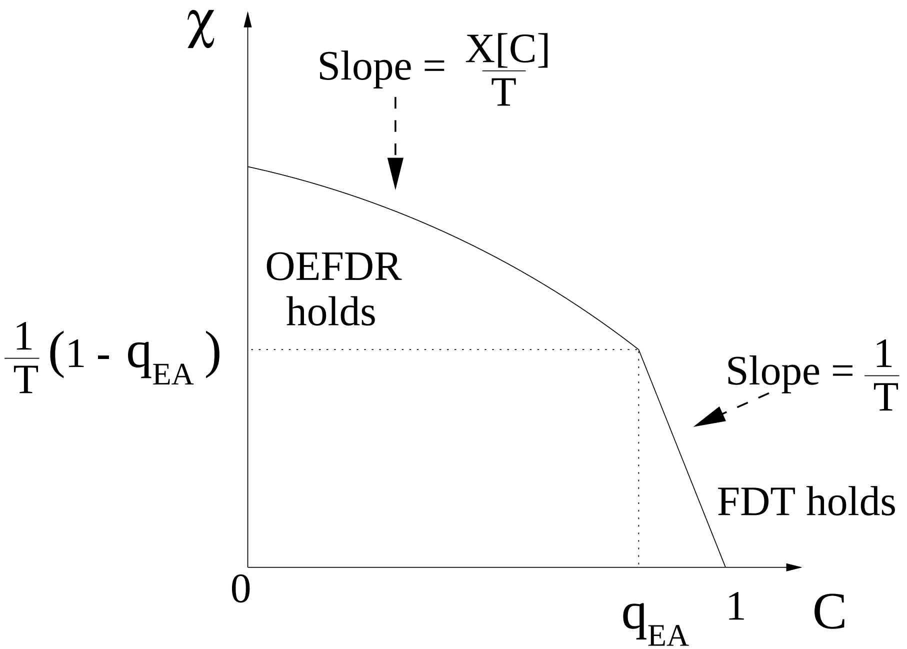
- Short times:

$$\chi(C) = \frac{1}{T}(1 - C) \tag{10}$$

- Long times

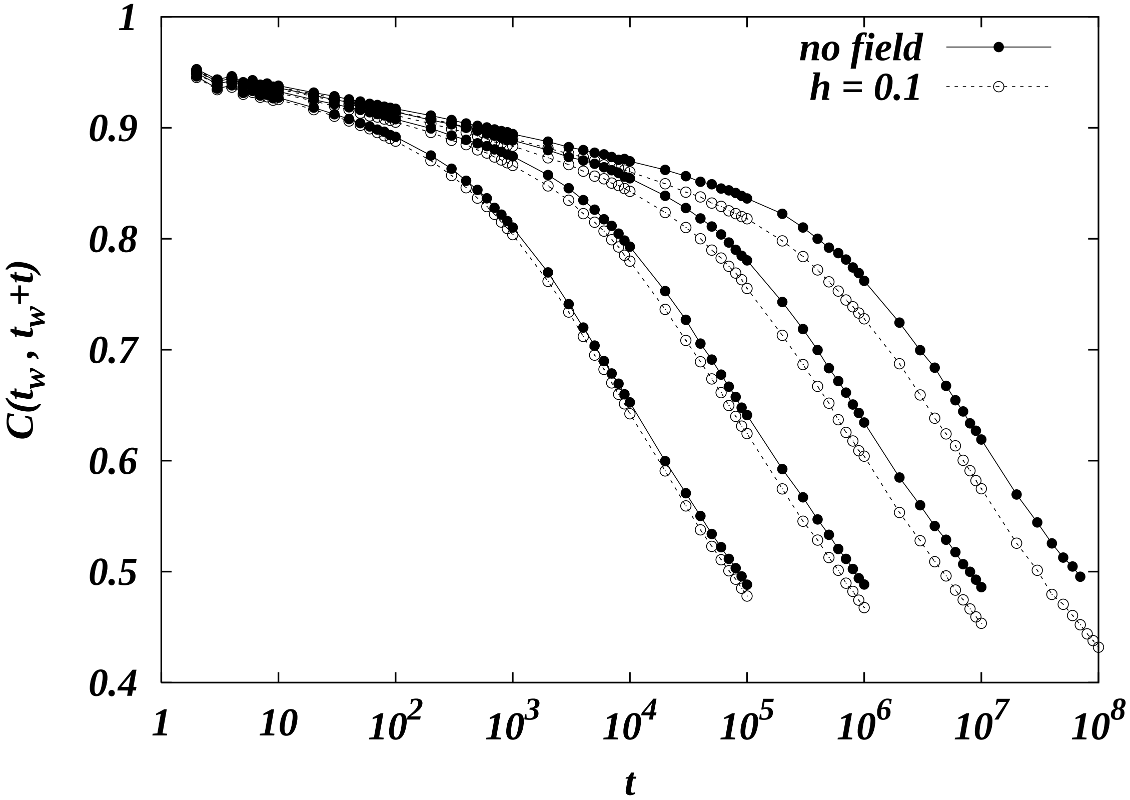
$$\chi(C) = \frac{1}{T_{\text{eff}}}(q_{EA} - C) + \frac{1}{T}(1 - q_{EA}) \tag{11}$$

Integrated response - $\chi(C)$ curve



Examples of aging behaviour

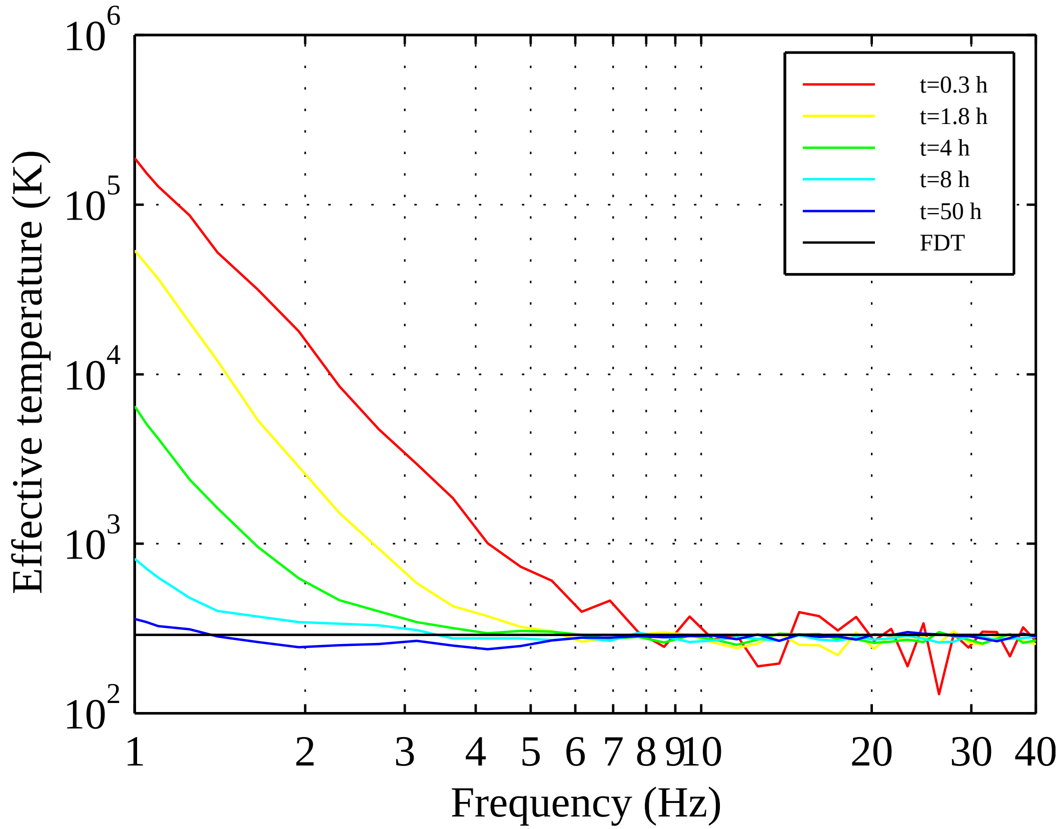
- Spin Glassses: Simulations of correlations in the 3 d EA model



Picco *et al.* (2001)

Examples of aging behaviour

- Effective temperatures in Laponite



Bellon *et al.* (2001)

Reparametrization Invariance

Time Reparametrizations: $t \to h(t)$

- Correlation Functions:

$$G(t_1, t_2) \to \left(\frac{\partial h(t_1)}{\partial t_1}\right)^{\Delta_1} \left(\frac{\partial h(t_2)}{\partial t_2}\right)^{\Delta_2} \tilde{G}(h_1, h_2) \tag{12}$$

- The dynamical equations for the aging of mean field spin glass models are governed by terms which are invariant under time reparametrizations.

- OEFDR is a time reparametrization invariant expression.

$$C(t_1, t_2) \;\to\; \tilde{C}(h_1, h_2), \tag{13}$$

$$R(t_1, t_2) \;\to\; \frac{\partial h(t_2)}{\partial t_2} \tilde{R}(h_1, h_2). \tag{14}$$

- Reparametrization invariance (RI) is a symmetry of the dynamical equations for mean field spin glass models

RI for aging in short-range models

- Mean field models have no fluctuations! ($N \to \infty$ limit)

Short range models :

- To include fluctuations, need to study action , not dynamical equations.

Is the aging action RI ? YES!

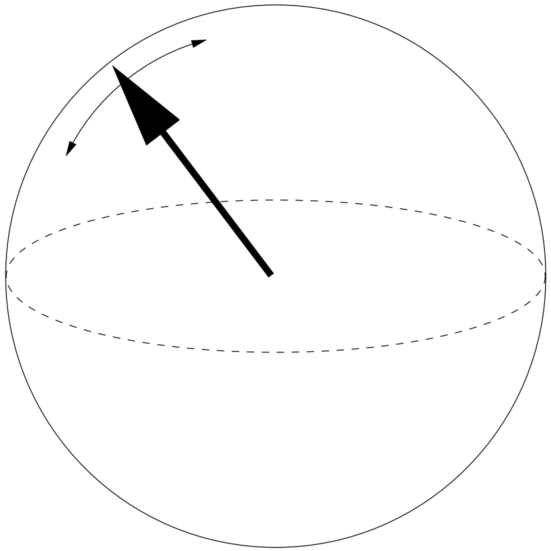
- Assumptions required
  - a) Causality
  - b) A separation of time scales between fast and slow dynamics
- Steps in proof
  - a) Separate fields into fast and slow modes, and perform RG in time
  - b) Naively relevant terms in the action vanish due to causality
  - c) Only marginal terms remain, the long time dynamics are RI
- However, the short time dynamics may act as a symmetry breaking field and choose a particular reparametrization function $h(t)$

Analogy with a ferromagnet

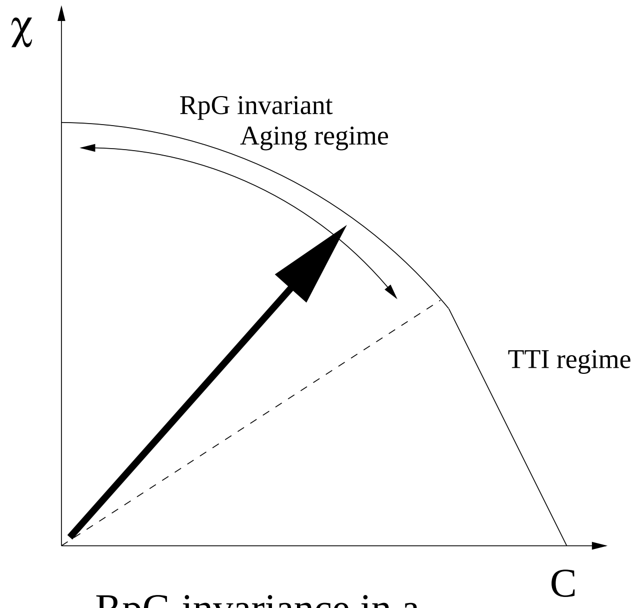
- Ferromagnet: global symmetry under rotations

  Low energy modes: spin waves, preserve magnitude of magnetization
- Spin glass dynamics: global symmetry under RI

  Low energy modes: spatially varying time reparametrizations



Rotation Invariance in a Ferromagnet



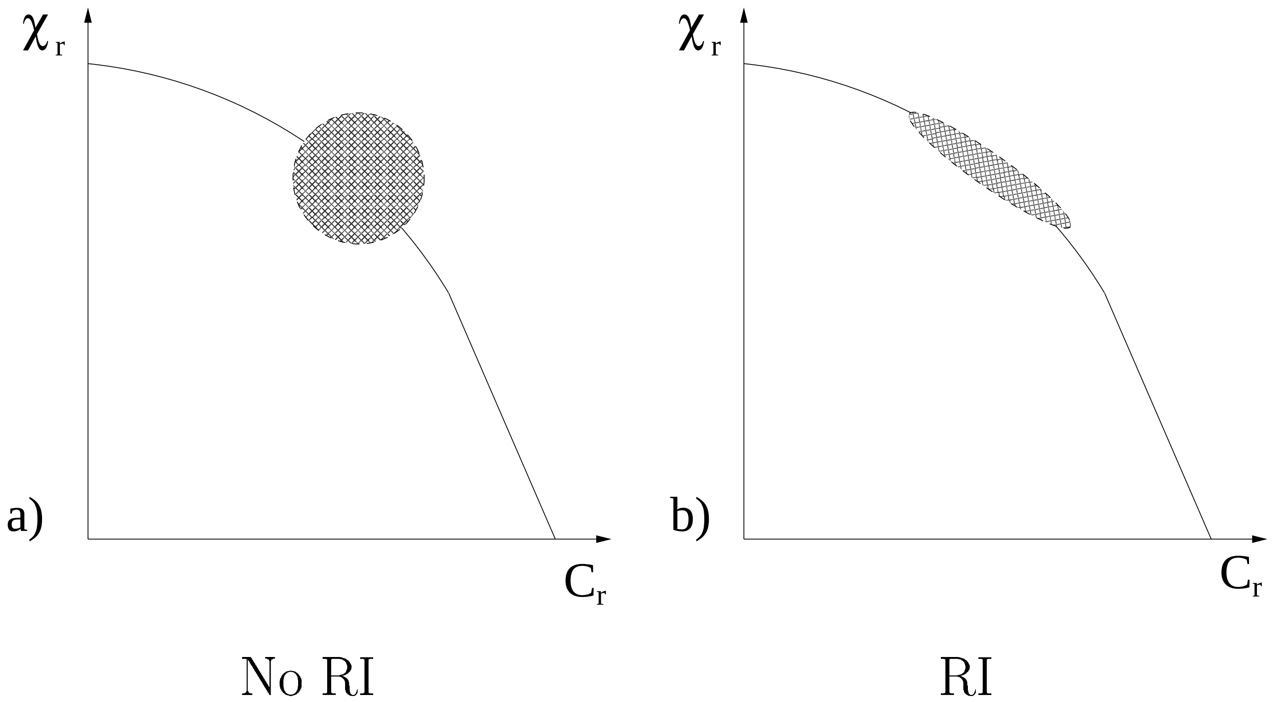
RpG invariance in a Spin Glass

- Low energy fluctuations preserve the invariance, localized fluctuations break the invariance .

Local OEFDR

- The OEFDR holds globally. Is there a local OEFDR ?

Predictions from ferromagnet analogy :

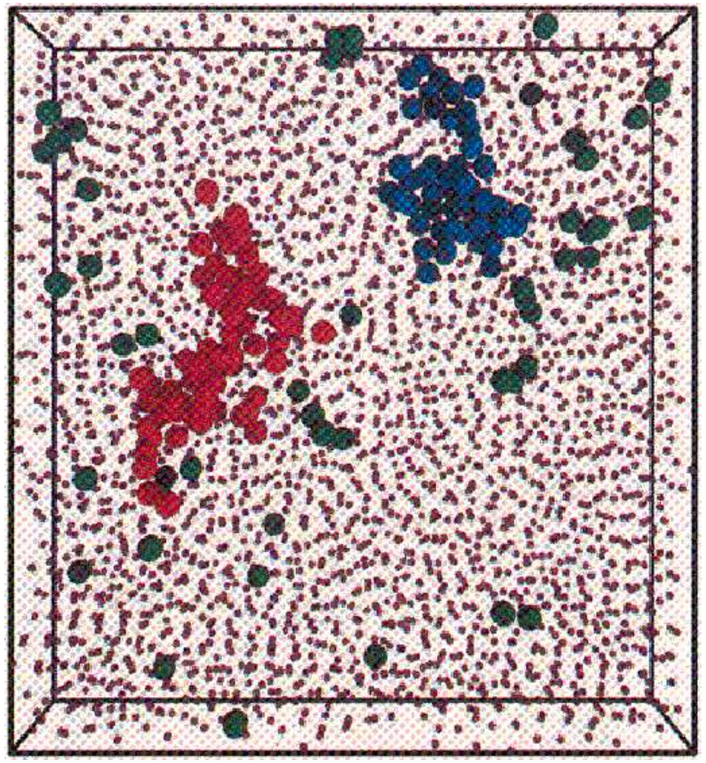


- Reparametrization invariant fluctuations should follow the bulk $\chi(C)$ curve (soft modes), whilst localized fluctuations should lie perpendicular to the bulk $\chi(C)$ curve in the $\chi - C$ plane.

Dynamic heterogeneities

• Local regions can have dynamics quite different from the bulk

Colloidal Glass :



Dark regions have faster dynamics (Weeks *et al.*, 2000)

Local Correlations and Responses

- Edwards-Anderson model studied with Monte Carlo simulations:
  L^3 sites, $L = 32, 64$ Ising spins: $S_i = \pm 1$
  Nearest neighbour couplings: $J_{ij} = \pm 1$, $T = 0.72\, T_c$
- Coarse-grained spin (over $\tau = 1000$ Monte Carlo steps):

$$\bar{s}_i(t) = \frac{1}{\tau} \sum_{t'=t-\tau}^{t'=t-1} s_i(t'). \tag{15}$$

- Coarse-grained local correlation:

$$C_r(t, t_w) = \frac{1}{V} \sum_{i \in V_r} \bar{s}_i(t) \bar{s}_i(t_w) \tag{16}$$

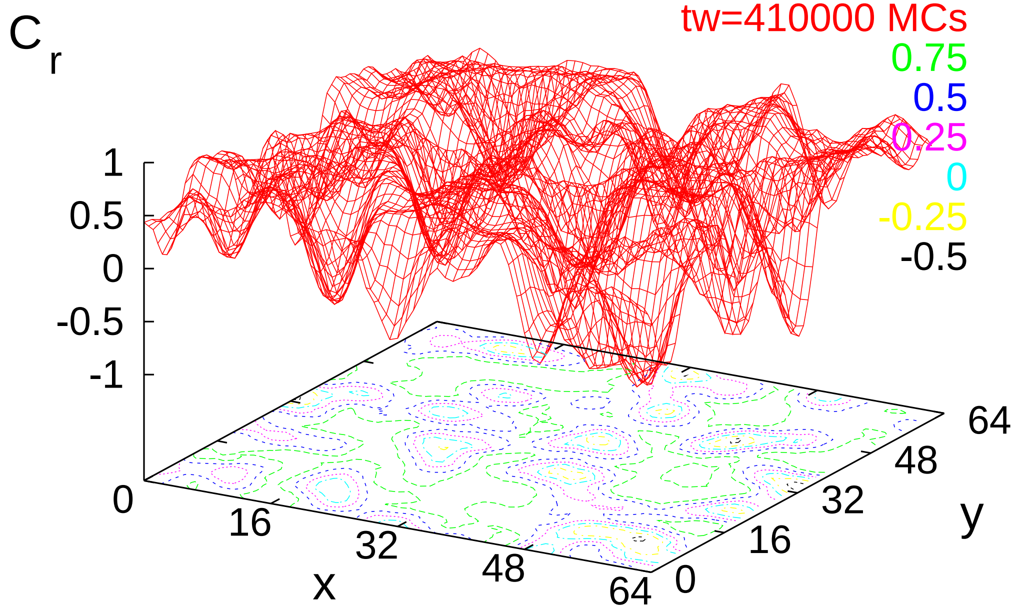
- Coarse-grained local response:

$$\chi_r(t, t_w) = \frac{1}{N_f} \sum_{k=1}^{N_f} \frac{1}{V} \sum_{i \in V_r} \frac{\overline{S}_i(t)|_{h^{(k)}} - \overline{S}_i^0(t)}{h_i^{(k)}} \theta(t - t_w). \tag{17}$$

V is the coarse-graining volume, N_f is the number of field realizations, $h_i^{(k)}$ is the k^{th} realization of the field $\pm h$ at site i.

Local correlations are spatially inhomogeneous

- Two dimensional slice of 3 dimensional simulation



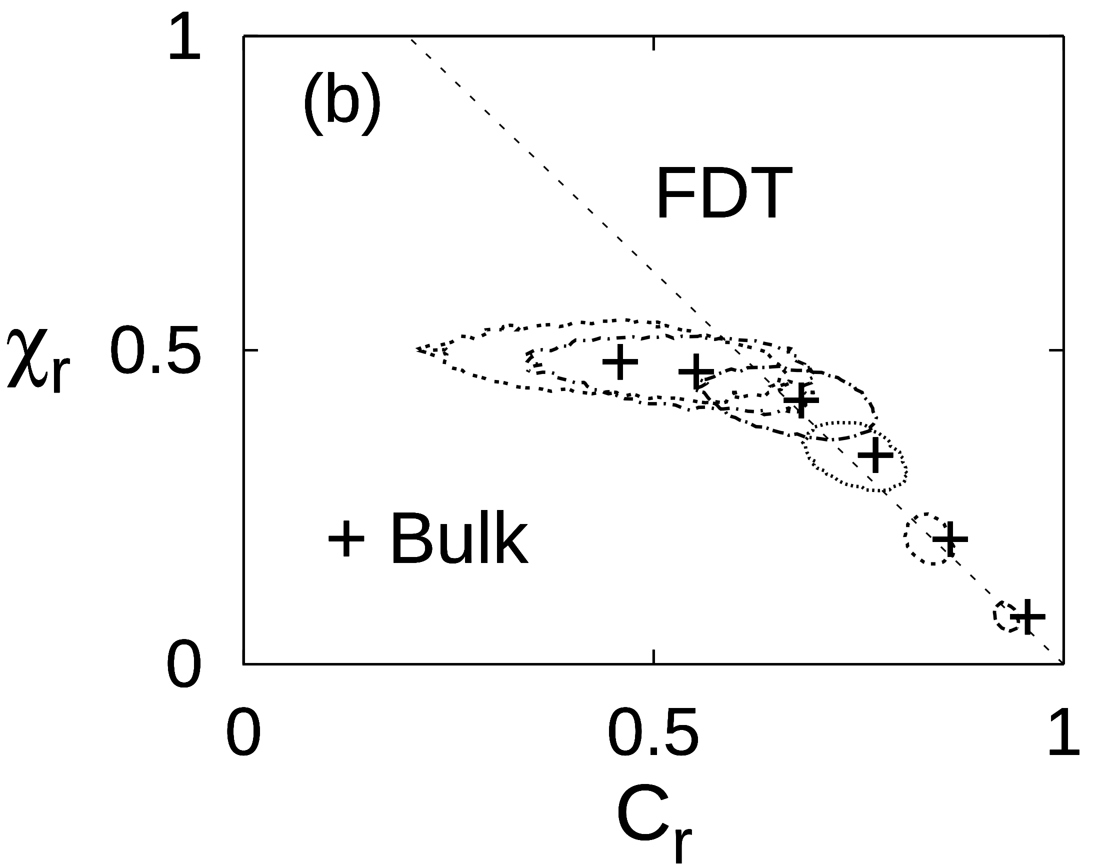
- Parameters:

$L = 64$, $T = 0.72\,T_c$, $V = 3^3$, $t_w = 4.1 \times 10^5$ MCs, $t = 2.8 \times 10^6$ MCs

No clear formation of domain structure at numerically accesible times (unlike a ferromagnet)

Local OEFDR

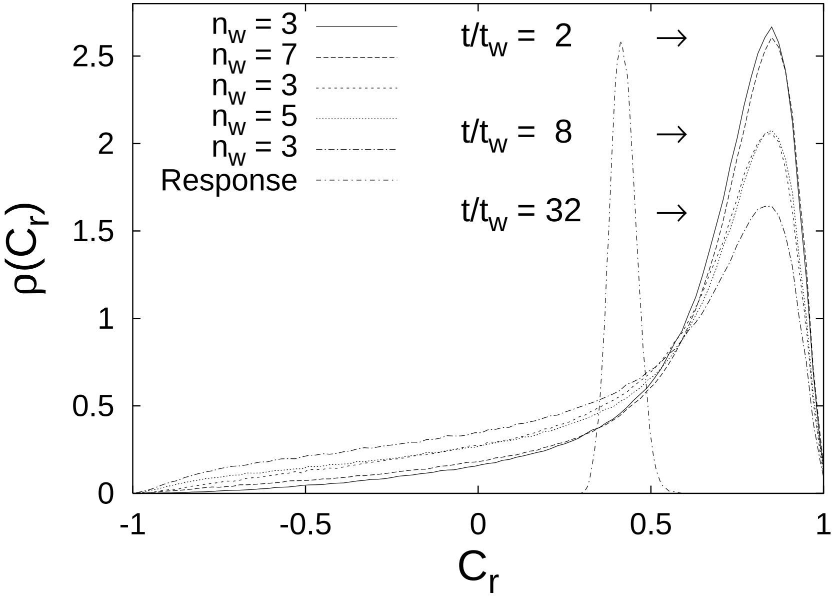
- Relation between local correlations and local susceptibilities



Different regions in spin glasses age at different rates!

Scaling of the PDF for local correlations

- Scaling for fixed t/t_w



The PDF $\rho(C_r(t, t_w))$ is the same for different t and t_w when t/t_w is held fixed.

Explanation in terms of Reparametrization invariance:

- Bulk correlation $C_{SP}(t, t_w) \simeq C(t/t_w)$, i.e. $h(t) \simeq t$ for the 3DEA model
- Implies $C_r(t, t_w) \simeq C_{SP}(h_r(t)/h_r(t_w)) \simeq C((t\epsilon(r, t))/(t_w\epsilon(r, t_w)))$
- Provided the distribution of $\epsilon(r, t)$ depends weakly on time , $\rho(C_r(t, t_w))$ should only depend on t/t_w.

Scaling of the PDF for local correlations

- Write $h(t) = e^{\varphi(t)}$, then $C_r(t, t_w) \simeq C_{SP}(h_r(t)/h_r(t_w)) \simeq C_{SP}\left(e^{\varphi_r(t)-\varphi_r(t_w)}\right)$

- For $h(t) = t$, an effective action for spatially varying time reparametrizations leads to the suggestion:

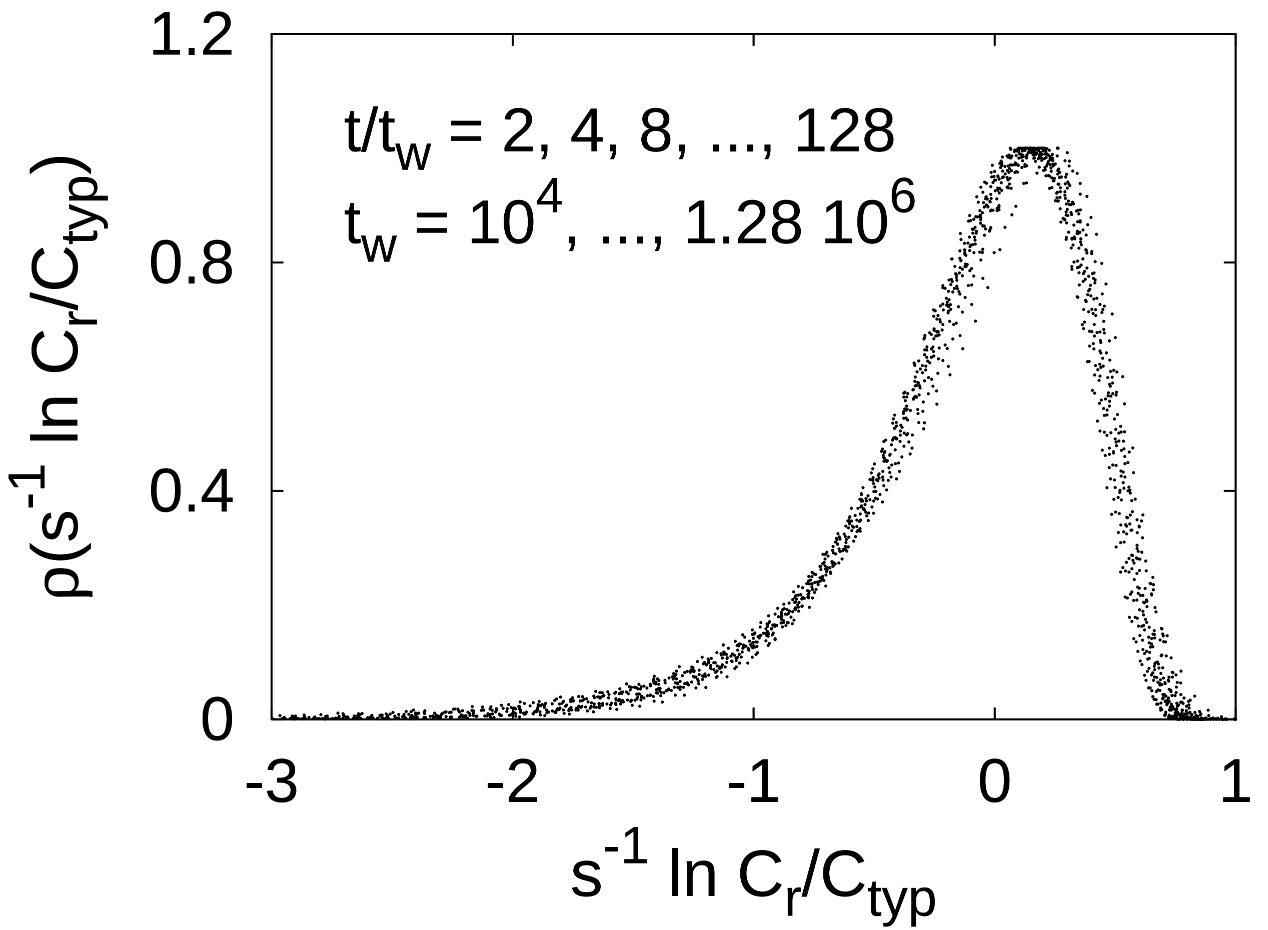
$$
\begin{aligned}
\varphi_r(t) - \varphi_r(t_w) &= \ln(t/t_w) + \delta\varphi_r(t) - \delta\varphi_r(t_w) \\
&= \ln(t/t_w) + (a + b\ln(t/t_w))^\alpha X_r(t, t_w)
\end{aligned}
$$

- a and b are determined by fluctuations, and $X_r(t, t_w)$ is a random variable drawn from a time-independent probability distribution.

- In the simplest case (uncorrelated X), $\alpha = 1/2$.

Scaling for all t, t_w in the aging regime

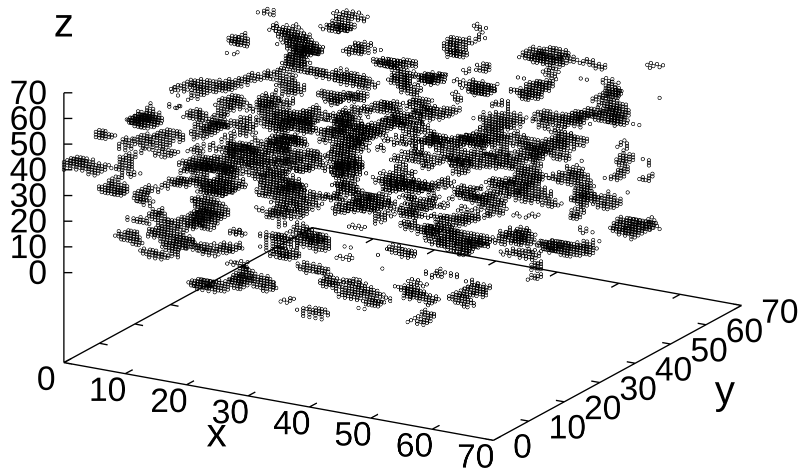
- Using the suggested rescaling and fitting a, b, α:



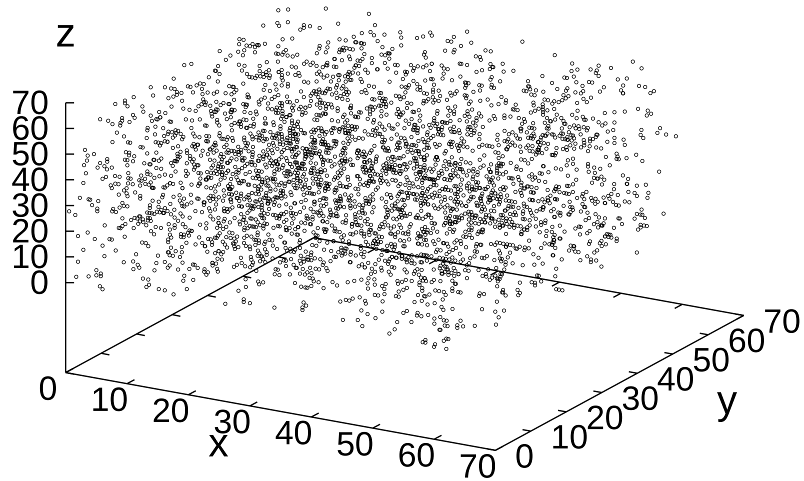
Geometric properties of correlations

- Consider clusters of spins with $C_r \in [\mathcal{C}, \mathcal{C} + d\mathcal{C}]$

Fast regions – localized
$C_r \leq 0$



Slower regions – extended
$0.65 \leq C_r \leq 0.66$



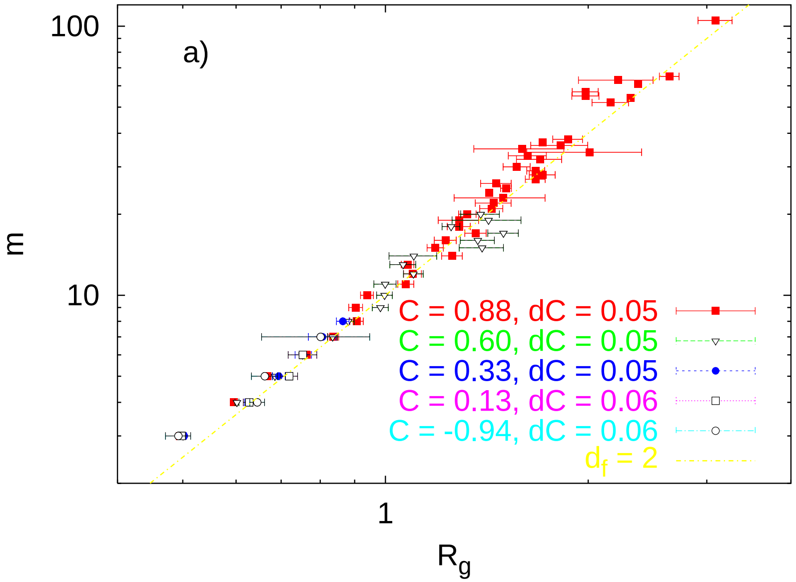
Geometric properties of correlations

- Fractal dimension d_f:

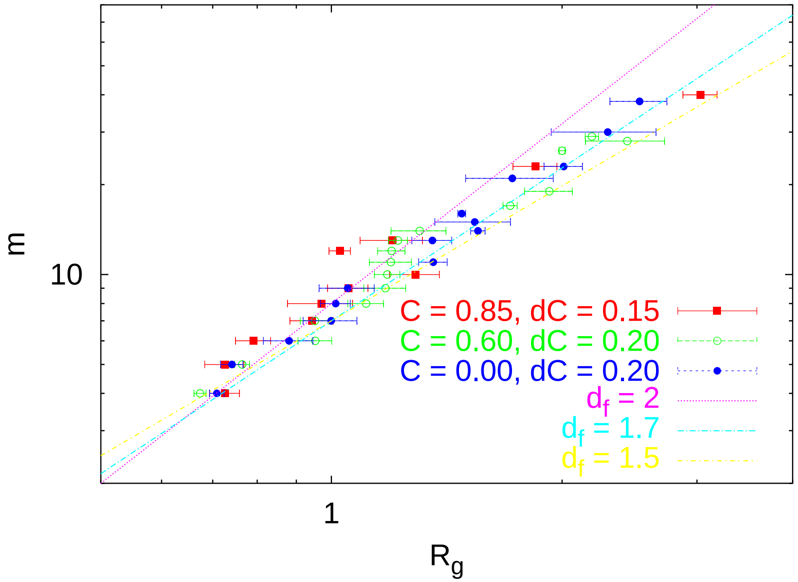
$$m \propto R_g^{d_f} \tag{18}$$

- m is the "mass" (number of spins) of a cluster and R_g is the radius of gyration

3DEA model



2DEA model



- Results are consistent with $d_f \simeq 2$.
- Recent equilibrium calculations also show excitations with $d_f \simeq 2$. (Lamarcq *et al.*, 2002)

Summary

- The action for short range spin glasses is invariant under global time reparametrizations ($t \to h(t)$)

- The low energy modes in short range spin glasses are proposed to be spatially varying time reparametrizations

- Short-range spin glasses have local dynamics that are spatially inhomogeneous

- The PDF for local correlations has uniform behaviour determined by the ratio t/t_w

- The PDF of local correlations obeys a scaling relation for all numerically accessible values of t and t_w

- Time reparametrization invariance in short-range spin glasses has been predicted analytically and is found to be consistent with numerical simulations

Future Directions

- Analysis of length scales as well as time scales

- Do these ideas work for other glassy systems, where local probes are possible experimentally?

- Developing analytic approaches to give a better understanding of dynamic heterogeneities and their role in glassy dynamics