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# The object detection logic of latent variable technologies

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**Abstract** Endemic to theoretical and applied psychometrics is a failure to appreciate that the logic at root of each and every latent variable technology is object detection logic. The predictable consequence of a discipline's losing sight of an organizing logic, is that superficiality, confusion, and mischaracterization are visited upon discussion. In this paper, I elucidate the detection logic that is the foundational, and unifying, logic, of latent variable technology, and discuss and dissolve a number of the more egregious forms of confusion and mischaracterization that, consequent upon its having been disregarded, have come to infect psychometrics.

**Keywords** Latent variable models · Object detection · Latent structure detection

Uniquely with respect to the quantitative technologies employed within Psychology, the history of the latent variable technologies is the history of the discipline's attempts to provide quantitative solutions to what it has seen as its technical problems, most vexatious and consequential. Spearman's invention of linear factor analysis, the first full-fledged latent variable technology, was an attempt to remedy, in mathematics, what he saw to be a science-undermining definitional murkiness inherent to the concept *intelligence*, in particular, and psychological concepts in general.

The concepts *latent variable* and *manifest variable* were invented as a means of technically characterizing the problem—the existence of which was suggested by analogical comparisons with the physical sciences—of error laden variables. The concept of *local independence* which finds its way into many latent variable technologies was invented as a quantitative paraphrase of the concept of *causal dependency*. The invention of structural equation modelling technologies was a technically dazzling effort with roots in the genetic models of Sewall Wright. It represented an attempt to provide the social scientist with a

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means of testing networks of causal claims, using, as input, easily generated aggregate-level quantities such as first and second order moments.

Accordingly, the history of latent variable technologies is far more than merely a sequence of mathematical and statistical innovations. It is the story of Psychology's most determined and creative efforts to solve scientific problems using mathematics (cf. Schonemann, 1994). Though the scientific problems that latent variable technologies were invented to address might strike one as simply too diverse to share a strong unifying communality, all of these problems, in fact, become salient under commitment to a particular conception of science. This is the empirical realist inspired conception of psychological science that derives from the philosophy of, most notably, Feigl (1953), Hempel (1958), and Sellars (1956), and which was introduced into psychology by methodological table-setters such as L.L. Thurstone, Frederik Lord, Melvin Novick, and Alan Birnbaum, Lee Cronbach and Paul Meehl, Paul Lazarsfeld, and William Rozeboom. This empirical realist inspired conception of science (hereafter, the ERC[1]) can be summarized as follows:

*ERC1* Psychological phenomena are classifiable as either observable or unobservable[2];
*ERC2* Observables are dependent upon unobservables;
*ERC3* The nature of the relational linkage between unobservable and dependent observable is either that of: (a) cause (unobservable) to partially determined consequent (observable) [e.g., Cattell[3]; Rozeboom[4]; Meehl[5]; Mulaik[6]; McDonald[7]]; or (b) conceptual

[1] For an elucidation of the manner in which the ERC is presupposed in various of the methodological orientations indigenous to the social and behavioural sciences, see Maraun et al. (2008), wherein the ERC is referred to as the Augustinian Conception of Reality.

[2] Those within the class of the unobservable-phenomena such as "…a person's anxiety, extraversion, intelligence, or goal orientation" (Strauss 1999, p. 19), "…general intelligence…" (Borsboom 2008, p. 27), and "…*social class*, *public opinion* or *extrovert personality*…" (Everitt 1984, p. 2) referred to, variously, and among others, as *latent constructs* (e.g., Lord and Novick 1968), *latent variables* (e.g., Green 1954; Lazarsfeld 1950), *latent concepts* (Bollen 2002), *underlying abilities* (Lord 1952), *underlying traits* (Lord 1953), *hypothetical variables* (e.g., Green, 1954, p. 725), and *latent traits* (Birnbaum 1968).

[3] "It would seem that in general the variables highly loaded in a factor are likely to be the causes of those which are less loaded, or, at least that the most highly loaded measure-the factor itself-is causal to the variables that are loaded in it" (Cattell 1952, p. 362).

[4] "And what we want to learn is not so much $F_i$-scores in **AM** solution-range most closely aligned with scores in $P$ on causal sources of $Z$ as the non-extensional nature of these causal variables" (Rozeboom 1988, p. 225).

[5] "If a causal conjecture substantively entails the existence of a taxon specifying (on theoretical grounds) its observable indicators, a clear-cut non taxonomic result of taxometric data analysis discorroborates the causal conjecture"(Meehl 1992, p. 152).

[6] "To use Guttman's measure of indeterminacy in factor analysis, we need not assume that the factors *generating the data* [italics added] on the observed variables…" (Mulaik 1976, p. 252). "which set of variables actually are the factors that *generated the data* [italics added]…"; "He then establishes a set of empirical operations that will measure that *causal factor* [italics added]…" Later in the same paper, in considering the idea of factors as constructed variates, he states that "such artificial variables in $\underline{\mathbf{x}}$ could not serve as causal explanations of the variables in $\underline{\mathbf{n}}$" (Mulaik 1976, p. 254).

[7] "If, in the first place, we are willing to regard the factor model as describing an aspect of the real world whereby unobserved processes *give rise* [italics added] to "observable" random variables, it is then a contradiction to suppose that those processes are not unique. It is another contradiction to suppose that they are known… In short, most accounts of the fundamental factor model use the non-mathematical qualifier "unobservable" to describe the common factor scores. It is not yet proven that it is philosophically naive to do this." (McDonald 1972, p. 18).

essence (unobservable) to error-laden exemplar/indicator (observable) [e.g., Thurstone[8]; Guilford[9]; Green[10]; McDonald[11]; McDonald and Mulaik[12]].

*ERC4* Fundamental aims of psychological science are to detect and identify efficacious unobservables; make discoveries as to the natures of the relationships that hold among these unobservables; and come to an understanding as to the natures of the dependencies of observables upon them.

Under the ERC, a need arose for the psychological scientist to possess tools that could take as input, (a) extant knowledge about relationships among observables, and (b) theoretical speculations about the natures of the dependencies of both observables, and unobservables, on unobservables, and return the desired inferences respecting scientifically important unobservables. The tools that were seen as satisfying, uniquely so, this need, were the latent variable technologies; a category which, nowadays, is populated by such as linear factor analysis, latent profile analysis, quadratic factor analysis, and item response theory.

Lord and Novick put it this way: "…the abilities or traits that psychologists wish to study are usually not directly measureable; rather they must be studied indirectly, through measurements of other quantities" (1968, p. 13); Latent variable technologies "…have in part been offered as a means of linking the more precise but limited mathematical models of behaviour with the more broadly conceived psychological theories" (1968, p. 19); "The factor analytic model is one of a number of models that give concrete form to the concepts used in theoretical explanations of human behaviour in terms of *latent traits*. In any theory of latent traits, one supposes that human behaviour can be accounted for, to a substantial degree, by isolating certain consistent and stable human characteristics, or *traits*, and by using a person's values on those traits to predict or explain his performance in relevant situations" (1968, p. 537).

In Lazarsfeld's words, "Empirical observations locate our objects in a manifest property space. But this is not what we are really interested in. We want to know their location in a latent property space. *Our problem is to infer this latent space from the manifest data*"

8 "The factorial methods were developed primarily for the purpose of identifying the principal dimensions or categories of mentality…" (Thurstone 1947, p. 55).

9 "The task of isolating the independent aspects of experience has been a difficult one. Armchair methods dominated by deductive logic rather than by observation led to the faculty psychologies, traditionally unacceptable to modern psychology. Direct observation has likewise failed to arrive at any set of unitary traits which even approach a universal acceptance. Factor analysis or some similar objective process had to be brought into the *search for the unitary traits of personality* [italics added]" (Guilford 1954, p. 470).

10 "To obtain a more precise definition of attitude, we need a mathematical model that relates the responses, or observed variables, to the latent variable" (Green 1954, p. 725).

11 "In using the common factor model, then, with rare exceptions the researcher makes the simple heuristic assumption that two tests are correlated because they in part measure the same trait, rather than because they are determined by a common cause, or linked together in a causal sequence, or related by some other theoretical mechanism. It is a consequence of this heuristic assumption that we interpret a common factor as a characteristic of the examinees that the tests measure in common, and, correlatively, regard the residual (the unique factor) as a characteristic of the examinees that each test measures uniquely" (McDonald 1981, p. 107); "…I am describing the rule of correspondence which I both recommend as the normative rule of correspondence, and conjecture to be the rule as a matter of fact followed in most applications, namely: In an application, the common factor of a set of tests/items corresponds to their common property" (McDonald 1996, p. 670).

12 "..the widely accepted aim of factor analysis, namely, the interpretation of a common factor in terms of the common attribute of the tests that have high loadings on it…"; "what attribute of the individuals the factor variable represents" (McDonald and Mulaik 1979, p. 298).

(1950, p. 490). Latent variable technologies such as linear factor analysis, item response theory, Lazarsfeld's own latent structure analysis, and Meehl's taxometric procedures, were invented expressly, and for no other reason, than to address scientific problems implied by the ERC. These problems are variations on a theme: The researcher must attempt to "…discover in the factorial analysis the nature of the underlying order" (Thurstone 1947, p. 56).

The manner in which latent variable technologies address the scientific problems implied by the ERC is encoded in a dense, informally scripted, story consisting of[13]: (a) a ramifying lexicon comprised of terms such as *model, unobservability*, *true score, causality*, *latent domains*, *measurement error, dimensionality, construct, hypothetical variable, intervening variable, latent variable, factor, underlying functional unity, abstractive property, manifest variable, observable, indicator,* and *latent structure*; (b) elements drawn from the philosophy of science (notably, the interpretations of Feigl, Hempel, and Sellars offered to the discipline by, among others, Meehl, Cronbach, Lazarsfeld, Lord, and Novick); (c) figurative (e.g., the resource extraction metaphor that portrays the indicators as *tapping into* or *picking up on* signals from the latent *core*)[14] and strongly connotative language,[15] the aim of which is to suggest something about the putative relationship between observable and unobservable; (d) analogical identifications with the unobservability problems indigenous to the natural sciences.[16]

Most fundamentally, however, the manner in which latent variable technologies address the scientific problems implied by the ERC is *manifest* in the logic in accordance to which they are built and operated. This logic is an *object detection logic* that was imported from the natural sciences. It is the very logic that underpins the protocols invented by natural scientists for use in the detection of perceptually unobservable material entities; sub-atomic particles, hidden metal objects, stellar objects, viruses, and the like. However, there is evidence, aplenty, that the role of this foundational logic, as the structural communality running through all latent variable technologies, is no longer properly appreciated. And, as is always the case when a discipline loses sight of an organizing logic, this state of affairs has visited upon discussion of issues central to the employment of these technologies, mischaracterization, superficiality, confusion, and incoherence. The aim of the current work is to provide a corrective to this state of affairs by elucidating the detection logic on which rest all extant latent variable technologies.[17] The task undertaken, herein, is wholly elucidatory. No position is taken on the issue of whether the idea of detecting latent

[13] See Maraun (2003) for a detailed analysis of this story, wherein it is called The Central Account.

[14] "The scale was originally designed to tap into a single source of variance…" (Hoyle and Lennox 1991, p. 511); "…in the Big Seven, Extraversion and Neuroticism are called Positive Emotionally and Negative Emotionally, respectively, in recognition of the emotional core of these higher order factors" (Benet and Waller 1995, p. 702).

[15] E.g., "…for the study of individual differences among people, but the individual differences may be regarded as an avenue of approach to the study of the processes which *underlie* [italics added] these differences" (Thurstone 1947, p. 55); "Latent variables can be more or less latent. (i) Firstly, they can be completely latent, unknown, *hidden*, *invisible*, *undercover* [italics added], unmanifested and their scientific purpose is as obscure as the LVs themselves and concepts…" (Lohmoller 1989, p. 81).

[16] E.g., "…"genetic composition" of a herd of cattle cannot be measured directly, but the effects of this composition can be studied through controlled breeding…Each postulates constructs that are not directly measurable and each observes phenomena that manifest these latent constructs but also exhibit fluctuation due to other factors…" (Lord and Novick 1968, p. 14).

[17] I do not view the logic that I, herein, describe as "a way of conceptualizing latent variable technologies", but, rather, as, uncontroversially, *the* logic on which these technologies are founded.

structures is, in the first place, a sensible one.[18] The organization of the paper is as follows: (a) the object detection logic of the natural sciences is reviewed; (b) the dependency of latent variable technologies upon this logic is elucidated; and (c) certain of the more insidious of the consequences of the failure to keep the logic front and centre are identified and briefly discussed.

## 1 The object detection logic of the natural sciences

Detection protocols are invented by natural scientists when decisions must be made about whether or not are present, particular sorts of material entities that, for one reason or another, are perceptually unobservable; sub atomic particles, hidden metal objects, certain stellar bodies, viruses, and the like. A protocol, *D*, for employment in the service of detecting the elements *u* of a class *U* of material entities is comprised of four logically distinct components:

1. *A specification of the class U, the elements of which are the (unobservable) targets of detection*. A detection protocol is created in response to an identified need to make decisions in respect whether particular unobservable entities are present. One specifies the class *U*, the elements *u* of which are the targets of detection of *D*, by laying down a rule that settles the properties that a material entity must possess in order to qualify as an element of *U*. If *U* contains just those material entities, say, *u*-things, that are denoted by a particular concept "*u*", then to specify *U* is equivalent to defining the concept "*u*".
2. *A generating proposition that links the presence of an element u of U to an observable O*. Though a *u* that is an element of *U* is unobservable, it is, nonetheless, a material entity. As such, it will have various sorts of impacts upon other material entities. Some of these impacts may be observable. One must deduce—through a combination of theory, mathematics, and empirical knowledge—a *generating proposition* that links the presence of a *u* at spatio-temporal coordinates (*t*,*s*) to an observable impact, *O*, of *u*'s being present.
   There are three types of generating proposition, each characterized by a particular type of logical linkage between the presence of a *u* at (*t*,*s*) and an *O*:

   *Type I* If $u \in U$ is present at (*t*,*s*), then *O*
   [i.e., *O* is a necessary condition of *u* being present at (*t*,*s*)];
   *Type II* If *O*, then $u \in U$ is present at (*t*,*s*)
   [i.e., *O* is a sufficient condition of *u* being present at (*t*,*s*)];
   *Type III* $u \in U$ is present at (*t*,*s*) if and only if *O*
   [i.e., *O* is a necessary and sufficient condition of *u* being present at (*t*,*s*)].

3. *A tool of detection, T, that yields decisions about whether or not a u is present at a particular (t,s)*. A tool of detection *T* is an implementation of a particular generating proposition; as such, the type of decision-making it supports is determined by the *logical type* of this generating proposition. If *T* is an implementation of a Type I generating proposition, then *T* operates as follows: when *O* is *not* the case, then it can be validly concluded that *u* is *not* present at (*t*,*s*). That is to say, in this case, *T*'s

[18] For an extended discussion of this issue, see Maraun (2003), *Myths and Confusions: Psychometrics and the Latent Variable Model.*

employment is as a *modus tollens* tool of disconfirmation. If it is an implementation of a generating proposition of Type II, then $T$ is a confirmatory tool and operates as follows: when $O$ is the case, then it can be validly concluded that $u$ is present at ($t$,$s$). And, finally, if $T$ is an implementation of a Type III proposition, then it can be employed in both a confirmatory and disconfirmatory fashion.

4. *Side-conditions that must be satisfied in order that tool of detection T functions properly*. A side-condition is a state of affairs (feature of reality) that must hold in order that $T$'s generating proposition be true. Importantly, side-conditions are not, then, properties of the targets of detection $u \in U$, but, rather, have to do with the correct operation of tool of detection $T$.

*Example* The detection of metal objects: Consider the components of the protocol, $D$, constructed for employment in the service of detecting metal objects, and in which is featured the pulse induction metal detector. *Targets of detection*. The targets of detection are metal objects; i.e., elements of the class $U$ of material entities denoted by the concept *metal object*. Thus, in this case, a specification of the targets of detection is effected by providing a definition of the concept *metal object* (to wit, an object made of at least one element, the atoms of which readily lose electrons). *Generating proposition*. The generating proposition is, in this case, of type III: an electro-magnetic impulse of a particular duration was transmitted in the vicinity of a metal object *if and only if* (in consequence of the phenomenon of self-induction) a primary and secondary electro-magnetic impulse occurs in the object (Kanchev 2005). *Tool of detection*. The pulse induction metal detector is an implementation of this type III generating proposition. It transmits an electro-magnetic impulse, and if it registers (does not register) a consequent fading impulse, the decision is made that there exists (does not exist), in the vicinity of the metal detector, a metal object (Kanchev 2005). *Side conditions*. The proper operation of the pulse induction metal detector is ensured through satisfaction of certain side-conditions; notably, that it is not operated in the vicinity of televisions, radios, cell phones, and other entities that produce radio waves (Kanchev 2005).

*Example* The detection of alpha particles: Consider, as a second example, the detection of the sub-atomic particle known as the alpha particle. In this case, the targets of detection are alpha particles. The concept *alpha particle* is defined as follows:

**Definition** (*Alpha particle*) A positively charged nuclear particle consisting of two protons bound to two neutrons.

The referents of this concept—i.e., positively charged nuclear particles consisting of two protons bound to two neutrons—are the targets of detection. A cloud chamber is a vessel several centimetres or more in diameter, with a glass window on one side and a movable piston on the other. The piston is dropped rapidly to expand the volume of the chamber, which is usually filled with dust-free air saturated with water vapour. Dropping the piston causes the gas to expand rapidly and its temperature to fall, thus rendering the air supersaturated with water vapour. The excess vapour cannot condense unless ions are present. Charged nuclear or atomic particles produce such ions. In consequence, when a charged nuclear particle passes through the chamber, it will leave behind it a trail of ionized particles upon which the excess water vapour will condense. This trail renders *manifest*, the presence of elements belonging to this particular class of perceptually unobservable entities.

Theory, borne out by experience, provided a means for scientists to deduce the following type I generating proposition: *if* an alpha particle passes through a cloud chamber situated within a magnetic field, *then* the path it will leave behind is both wide (relative to the widths of the paths left by other subatomic particles) and bending towards the negative pole of the magnetic field. This generating proposition is the core of the following *modus tollens* detection protocol: if a path through a cloud chamber is observed, then the scientist can validly conclude that a subatomic particle has passed through; if the observed path is either narrow, or not bending towards the negative pole of the magnetic field, then the particle responsible for having left this path was not an alpha particle. This detection protocol yields valid decisions about the presence of alpha particles only given satisfaction of a very long list of conditions (side-conditions) bearing on the physical environment in which the cloud chamber is situated.

## 2 Detection protocols for latent structures

Let us now elucidate the dependency of latent variable technologies upon the logic that informs the detection protocols of the natural sciences. To begin, note that: (a) the unobservables to which the ERC refers (which, in social research, are the targets of detection) are paraphrased technically as *latent structures*; (b) every extant latent variable technology (linear factor analysis, quadratic factor analysis, latent profile analysis, the various item response theories, etc.) arises out of a specification of a particular class of latent structures, the elements of which have been identified by social scientists as being of sufficient scientific importance, to warrant development of a working detection protocol.

Consider, then, a researcher who wishes to build a detection protocol $D_{LS*}$ that can be employed in the service of detecting the unobservable elements *ls** of a particular class *LS** of latent structures; i.e., can be used to make decisions about whether or not there is an element of *LS** that happens to be a latent structure of a set of observable indicators X. $D_{LS*}$ is comprised, then, of *five* components, the first four of which are isomorphic with the components of the object detection protocols of the natural sciences:

*Component 1* A specification of the class *LS** of latent structures, the elements *ls** of which are the targets of detection.
*Component 2* A generating proposition that links the presence of an element *ls** of *LS** to a particular observable impact of *ls**, the latter, a restriction on the distribution of X.
*Component 3* A tool of detection that is an implementation of the generating proposition.
*Component 4* Side-conditions that must be satisfied in order for the generating proposition to be true.
*Component 5* (because the researcher will not possess knowledge of the values assumed by population parameters) inferential machinery employed to make population-level decisions on the basis of sample information.

Let us consider each of these components, in turn.

### 2.1 Component 1: specification of *LS**

Let $\theta$ stand for a set of *m* latent variables,[19] X stand for an arbitrary set of $p > m$ indicators, $f_\theta$ be the distribution of $\theta$, and $f_{X|\theta}$ be the distribution of X conditional on $\theta$. What is meant by *latent structure* is captured by the following definition:

[19] See Michell (2012), in regards the general issue of the conditions that must be satisfied in order that a variable have quantitative character.

**Definition** (*Latent structure*) A latent property, $t_i$, is a property of either $f_{X|\theta}$ or $f_\theta$. A latent structure, *ls*, is the intersection $\bigcap_{i=1}^{k} t_{ji}$ of *k* latent properties.

The *k* latent properties $t_i$ in terms of which a particular latent structure is specified, are called its *defining characteristics*. One specifies a *class of latent structures LS** by listing a set of *s* defining characteristics $\{t_{1*},\ldots,t_{s*}\}$ that must be possessed by a latent structure, in order that it qualify as an element of *LS**. Thus, if it has been decided that $\{t_{1*},\ldots,t_{s*}\}$ are the defining characteristics of class *LS**, then

$$LS^* \equiv \left\{ ls_j^* \equiv \bigcap_{i=1}^{k} t_{ji} \middle| \{t_{j1},\ldots,t_{jk}\} \supseteq \{t_{1*},\ldots,t_{s*}\} \right\}. \tag{1}$$

To specify a class of latent structures in this manner is to *identify* (provide a transparent statement of) the latent structures that are the targets of detection of protocol $D_{LS*}$.

*Example 1* Unidimensional, linear, factor structures: A candidate specification of the class of unidimensional, linear, factor structures is

$$LS_{ulf} \equiv \left\{ ls_j \equiv \bigcap_{i=1}^{k} t_{ji} \middle| \{t_{j1},\ldots,t_{jk}\} \supseteq \{t_{1(ulf)}, t_{2(ulf)}, t_{3(ulf)}\} \right\} \tag{2}$$

in which:

$t_{1(ulf)} \rightarrow \theta$ is a single, continuously distributed, variable with a mean of zero and a variance of unity;
$t_{2(ulf)} \rightarrow \mathrm{E}(\mathrm{X}|\theta = \theta^*) = \boldsymbol{\kappa} + \boldsymbol{\Lambda}\theta^*$, wherein $\boldsymbol{\Lambda}$ and $\boldsymbol{\kappa}$ are $p \times 1$ vectors of real numbers;
$t_{3(ulf)} \rightarrow \mathrm{C}(\mathrm{X}|\theta = \theta^*) = \boldsymbol{\Psi}$, the $p \times p$ conditional covariance matrix of X given $\theta$, is diagonal and positive definite.

To state an interest in detecting the presence of those latent structures that go by the name, unidimensional, linear, factor structure, is to state an interest in detecting precisely those latent structures that, by virtue of their defining characteristics, qualify as elements of $LS_{ulf}$.

*Example 2* 2-class latent profile structures: A candidate specification of the class of two-class latent profile structures (for X, continuously distributed) is

$$LS_{2clp} \equiv \left\{ ls_j \equiv \bigcap_{i=1}^{k} t_{ji} \middle| \{t_{j1},\ldots,t_{jk}\} \supseteq \{t_{1(2clp)}, t_{2(2clp)}, t_{3(2clp)}\} \right\}, \tag{3}$$

in which

$t_{1(2clp)} \rightarrow \theta$ is a single, *Bernoulli* (two-point; values $\theta_1$ and $\theta_2$ with probabilities $\pi_1$ and $\pi_2 = (1-\pi_1)$, respectively) distributed variable, with a mean of zero and a variance of unity;
$t_{2(2clp)} \rightarrow \mathrm{E}(\mathrm{X}|\theta = \theta_j) = \boldsymbol{\kappa} + \boldsymbol{\Lambda}\theta_j, j = \{1,2\}$, wherein $\boldsymbol{\Lambda}$ and $\boldsymbol{\kappa}$ are $p \times 1$ vectors of real numbers;
$t_{3(2clp)} \rightarrow \mathrm{C}(\mathrm{X}|\theta = \theta_j) = \boldsymbol{\Psi}_j$, $j = \{1,2\}$, the $p \times p$ conditional covariance matrix of $\mathbf{X}$ conditional on $\theta = \theta_j$, is diagonal and positive definite.

Note that there are *only* two points of distinction in the specifications of $LS_{2clp}$ and $LS_{ulf}$. First, in $LS_{ulf}$, $\theta$ has a continuous distribution, and, in $LS_{2clp}$, a Bernoulli distribution. Second, in $LS_{ulf}$, $C(X|\theta)$ is homoscedastic (constant over the values of $\theta$), and in $LS_{2clp}$, heteroscedastic. From a detection logic perspective, one asks; "what are the implications of the two targets of detection—the unidimensional, linear, factor structure, on the one hand, and the two-class latent profile structure, on the other—differing in only these respects, for the construction of detection protocols for each?" The answer will, of course, be found in a mathematical analysis of the restrictions imposed on the distribution of an arbitrary set of indicators, by each of $\{t_{1(2clp)}, t_{2(2clp)}, t_{3(2clp)}\}$ and $\{t_{1(ulf)}, t_{2(ulf)}, t_{3(ulf)}\}$.

## 2.2 Component 2: generating proposition

Let $f_X$ be the joint density (discrete mass function) of X, and let $\gamma$ be a subset of the $m$ parameters of $f_X$, $\gamma$ ranging over a subspace $\Omega$ of $\Re^m$; i.e., $\gamma \in \Omega \subseteq \Re^m$.

**Definition** A generating proposition $GP_{LS*}$ for $LS^* \equiv \left\{ ls_j^* \equiv \bigcap_{i=1}^{k} t_{ji} \middle| \{t_{j1}, \ldots, t_{jk}\} \supseteq \{t_{1*}, \ldots, t_{s*}\} \right\}$ is a Type I, II, or III proposition in which the antecedent is $\{t_{1*},\ldots,t_{s*}\}$ and the consequent is $\{\gamma \in \Omega_{LS^*} \subset \Omega\}$.

The generating proposition of $D_{LS*}$ expresses a linkage between the state of affairs of $ls^* \in LS^*$ being a latent structure of $X$, and a *restriction* on the values that $\gamma$ can assume; in particular, that $\gamma$ is *restricted* to a subspace $\Omega_{LS^*}$ of $\Omega$. In other words, $GP_{LS*}$ expresses one particular way in which an unobservable latent structure that is an element of $LS^*$, *manifests itself*: it has impact $\gamma \in \Omega_{LS^*} \subset \Omega$ upon the joint distribution of the indicator variables. The generating propositions of latent structure detection protocols are analytically deduced. For example, to derive a type I {type II; type III} generating proposition for $LS^*$ requires that it be proven in mathematics that $\gamma \in \Omega_{LS^*} \subset \Omega$ is a necessary {sufficient; necessary and sufficient} condition of the defining characteristics $\{t_{1*},\ldots,t_{s*}\}$ of $LS^*$.

*Example 3* The generating proposition $GP_{ulf}$ of unidimensional, linear, factor analysis: Let $\gamma$ contain the $\frac{1}{2}p(p+1)$ nonredundant elements of $\Sigma$ arranged in some particular lexicographic order; accordingly, that $\gamma \in \Omega \subseteq \Re^{\frac{1}{2}p(p+1)}$. Let $\tilde{\gamma}$ contain the elements of $\tilde{\Sigma} = \mathbf{\Lambda}\mathbf{\Lambda}' + \mathbf{\Psi}$, arranged in the same lexicographic order, and wherein $\mathbf{\Lambda}$ is a $p \times 1$ vector of real numbers, and $\mathbf{\Psi}$, a $p \times p$ diagonal, positive definite, matrix. Finally, let $\Omega_{ulf}$ be the subspace of $\Omega$ traced out by $\tilde{\gamma}$ as $\mathbf{\Lambda}$ and $\mathbf{\Psi}$ range over their admissible values. Then, as is well known (cf. Wansbeek and Meijer 2000, p. 150; Mardia et al. 1979, p. 257), the type III generating proposition at root of all extant detection protocols for $LS_{ulf}$ is $GP_{ulf}$: an element $ls^*$ of $LS_{ulf}$ is a latent structure of X *iff* $\gamma \in \Omega_{ulf}$.

*Example 4* The generating proposition $GP_{2pir}$ *of* 2-parameter, logistic, item response analysis: Let X be a vector of $p$ dichotomous [0,1] random variables and, in consequence, $f_X$ be a set of $2^p$ probabilities, one for each of the $2^p$ outcomes of X. Let $\gamma$ contain these $2^p$ probabilities, arranged in some lexicographic order, so that $\gamma \in \Omega \subset \Re^{2^p}$, with $\Omega$ the $p$-dimensional unit simplex (cf. Holland 1990). A candidate specification of the 2-parameter, logistic, item response structure is

$$LS_{2pir} \equiv \left\{ ls_j \equiv \bigcap_{i=1}^{k} t_{ji} \middle| \left\{t_{j1}, \ldots, t_{jk}\right\} \supseteq \left\{t_{1(2pir)}, t_{2(2pir)}, t_{3(2pir)}\right\} \right\},$$

in which

$$t_{1(2pir)}.\theta \sim \mathrm{N}(0, 1);$$

$$t_{2(2pir)}.\ \mathrm{P}(X_\mathrm{j} = 1|\theta = \theta^*) = \frac{\exp(\mathrm{a_j}\theta^* - \mathrm{b_j})}{(1 + \exp(a_j\theta^* - b_j))};$$

$$t_{3(2pir)}.\ \mathrm{P}(\mathbf{X} = \mathrm{X}^*|\theta = \theta^*) = \prod_{j=1}^{p} \mathrm{P}(X_\mathrm{j} = 1|\theta = \theta^*)^{\mathrm{X}_\mathrm{j}^*}(1 - \mathrm{P}(X_\mathrm{j} = 1|\theta = \theta^*))^{1-\mathrm{X}_\mathrm{j}^*}.$$

With $\phi(\theta) = \frac{1}{(2\pi)^{1/2}}\exp(-1/2 * \theta^2)$, let $\tilde{\gamma}$ contain the elements

$$\int_{-\infty}^{\infty} \left[\frac{\exp(\mathrm{a_j}\theta - \mathrm{b_j})}{(1 + \exp(a_j\theta - b_j))}\right]^{X_j^*} \left[1 - \frac{\exp(\mathrm{a_j}\theta - \mathrm{b_j})}{(1 + \exp(a_j\theta - b_j))}\right]^{1-\mathrm{X}_\mathrm{j}^*} \phi(\theta)\mathrm{d}\theta$$

arranged in the same lexicographic order as $\boldsymbol{\gamma}$, and let $\Omega_{2pir}$ be the subspace of $\Omega$ traced out by $\tilde{\gamma}$ as the set of item parameters $\{a_1, a_2,\ldots, a_p, b_1, b_2,\ldots, b_p\}$ ranges over its admissible values. Then, the type I generating proposition at root of extant detection protocols for $LS_{2pir}$ is $GP_{2pir}$: *if* an element $ls$* of $LS_{2pir}$ is a latent structure of X, then $\boldsymbol{\gamma} \in \Omega_{2pir}$.

## 2.3 Component 3: tool of detection

Each and every latent variable technology features a tool of detection that is an implementation of an analytically deduced generating proposition. Let $T_{LS*}$ be a tool of detection for $LS$* that is an implementation of a deduced generating proposition $G_{LS*}$. The type of decision-making possible with $T_{LS*}$ is, then, determined by the type of proposition that $GP_{LS*}$ is.

*Example 6* Tool of detection for $LS_{ulf}$: The only tool of detection for $LS_{ulf}$ of which I am aware—and, certainly, the only tool available in extant statistical packages—is the type III tool $T_{ulf}$, that is an implementation of the generating proposition $GP_{ulf}$. It is employed in the following manner. An inferential decision is made in respect the hypothesis pair [$H_0$: $\boldsymbol{\gamma} \in \Omega_{ulf}$, $H_1$: $\boldsymbol{\gamma} \notin \Omega_{ulf}$].[20] If a decision is made in favour of $H_0$, then it is decided that there exists an element $ls$* of $LS_{ulf}$ that is a latent structure of X; else, it is decided that there exists no element $ls$* of $LS_{ulf}$ that is a latent structure of X.

[20] Or, in the event that rmsea-style thinking is adopted, [$H_0$: $D(\boldsymbol{\gamma},\Omega_{ulf}) \le c$, $H_1$: $D(\boldsymbol{\gamma},\Omega_{ulf}) > c$], wherein $D$ is a distance measure, and $c$ is a positive number.

## 2.4 Component 4: side conditions

The fourth component of a latent structure detection protocol is a set of *side conditions*.

**Definition** Let it be the case that $GP_{LS*}$ is true only if $f_{\mathrm{X}}$ it happens to have particular property $b$. Then $b$ is a side condition attendant to the employment of any tool of detection that is an implementation of $GP_{LS*}$.

Side conditions are potential properties of $f_{\mathrm{X}}$ that must obtain empirically (in a population $P$ under study) in order that the generating proposition on which is based a tool of detection, be true; equivalently, in order that the detection protocol which features the generating proposition, operate properly in $P$. A detection protocol need not feature any side conditions. For example, because no side conditions need hold in order that $GP_{ulf}$ be true ($GP_{ulf}$ is unconditionally true), there are no side conditions associated with any detection protocol that features a tool of detection that is an implementation of $GP_{ulf}$.

## 2.5 Component 5: inferential machinery

Finally, $D_{LS*}$ will, of necessity, have an inferential component. This is because, when it comes time to employ tool of detection $T_{LS*}$ (an implementation of generating proposition $GP_{LS*}$) to make a decision as to whether or not (in a particular population $P$) some particular set of indicators X* has, as a latent structure, an element $ls$* of $LS$*, the researcher will only have available to him or her, a sample drawn from $P$. More particularly, on the basis of this sample, the researcher will have to make a decision as to whether or not restriction $\gamma \in \Omega_{LS^*} \subset \Omega$ (the consequent of $GP_{LS*}$) holds in $P$. The necessity of making an inferential decision as to whether or not $\gamma \in \Omega_{LS^*} \subset \Omega$ holds in $P$, visits sampling error upon the employment of a detection protocol in the service of an attempt to detect elements $ls$* of $LS$*.

*Example 7* A complete detection protocol $D_{ulf}$ for $LS_{\mathrm{ulf}}$: Target of detection: $LS_{ulf} \equiv \left\{ ls_j \equiv \bigcap_{i=1}^{k} t_{ji} \middle| \left\{t_{j1}, \ldots, t_{jk}\right\} \supseteq \left\{t_{1(ulf)}, t_{2(ulf)}, t_{3(ulf)}\right\} \right\}$. Generating proposition ($GP_{ulf}$) → an element $ls$* of $LS_{ulf}$ is a latent structure of X iff $\gamma \in \Omega_{ulf}$. Tool of detection: type III tool $T_{ulf}$ (an implementation of $GP_{ulf}$). Side conditions: $GP_{ulf}$ is always true, so there are no side-conditions attendant to the employment of $T_{ulf}$. Inferential component: (one possibility) under multivariate normality of X,[21] maximum likelihood estimation of the $2p$ parameters contained in $\mathbf{\Lambda}$ and $\mathbf{\Psi}$, yielding estimators $\hat{\Lambda}$ and $\hat{\Psi}$ that are optimal in many well known senses. An inferential decision as to which is the case, in a population $P$, $\mathrm{H}_0$: $\gamma \in \Omega_{ulf}$ or $\mathrm{H}_1$: $\gamma \notin \Omega_{ulf}$, is made with the aid of the maximum likelihood loss function $\ln\left|\hat{\hat{\Sigma}}S^{-1}\right| + tr(\hat{\hat{\Sigma}}^{-1}S) - p$ (in which $S$ is the maximum likelihood estimator of $\mathbf{\Sigma}$ under $\mathrm{H}_1$).

[21] Note: the multivariate normality of X is *not* a side condition (i.e., it is not required in order that $GP_{ulf}$ be true), but, rather, a statistical assumption attendant to a particular application of maximum likelihood machinery.

## 3 Consequences of the failure to appreciate the logic

I have now elucidated the detection logic at root of all latent variable technologies. It is patent, from even a cursory examination of the literature, that, by theoretician's and applied researchers, alike, neither the logic, nor the fact of its pre-eminence in providing coherent account of latent variable technologies, is properly appreciated. To disregard an organizing logic is a perilous act; one that can be expected to engender all manner of errors in comprehension and application. I contend that the failure to keep firmly in view the detection logic at root of latent variable technologies has led to wide-ranging confusion, misportrayal, and misidentification. In the remainder of the paper, I examine a number of the more pernicious of such.

### 3.1 Misportrayal of latent variable technologies as having to do with models

Within both applied and theoretical psychometrics, it is a commonplace—and, to my knowledge, unchallenged—supposition that latent variable technologies involve models; and that their employments in research are modelling exercises. But this supposition is mere fallacy; mischaracterization engendered by the joint influence of a loose and infelicitous employment of the concept *model* and the failure to properly appreciate the detection logic that underpins these technologies. Black opined that "Scientists often speak of using models but seldom pause to consider the presuppositions and implications of their practice" (1962, p. 219; cf. Freedman 1985). In my view, this has never been truer, than it is, nowadays, in the social sciences, wherein "models" are invented, dime-a-dozen, and user-friendly computer programs have, it seems, turned the most humble of academics into modellers. There are manifold senses of the term *model*, and, correlatively, different types of models, including, but by no means limited to, the scale model (an instance of which is the architectural model), the analogical model (e.g., electricity understood on the model of hydrodynamics), and the mathematical model (examples of which are Kepler's laws of planetary motions), of which the stochastic model (e.g., any of Mendel's genetic models) is a special case. Though the various types of models can be profitably differentiated along manifold dimensions of comparison, they share a unifying functionality; each is created for the purpose of *representing*. $T$ represents (is a model of) some particular state of affairs $A$, if it was constructed in accordance with antecedently specified rules of correspondence that establish linkages between constituent parts of $A$, say, $\{a_1,a_2,\ldots,a_p\}$, and constituent parts of $T$, say, $\{t_1,t_2,\ldots,t_p\}$.

Now, on the other hand, the laying down of antecedently specified rules of correspondence is neither part of the construction, nor the employment in research, of a latent variable technology. What one encounters, instead, is an entirely different sort of an entity; namely, the analytically deduced generating proposition. Conversely, in true modelling exercises, it is this *latter* entity that is nowhere to be found. Consider, as an example, unidimensional, linear, factor analytic technology. Frequently, the expression $\tilde{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi} - \mathbf{\Lambda}$ a $p \times 1$ vector of real numbers, $\mathbf{\Psi}$, diagonal and positive definite-that is, arguably, the characteristic mark of this technology, is either directly referred to as, or is tacitly portrayed as being, a model. It is not. For it was not invented to represent anything. In particular, it did not come into being by virtue of an act of laying down rules that establish correspondence relations between, say, the elements of covariance matrix $\Sigma$ (of X, in a particular population $P$) and the elements of $\mathbf{\Lambda}$ and $\mathbf{\Psi}$. And if its genesis could be shown to lie in such a representational act, how, then, would be explained the role of the analytically deduced generating proposition $GP_{ulf}$, the consequent of which just happens to be $\tilde{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi}$?

The expression $\tilde{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi} - \mathbf{\Lambda}$ a $p \times 1$ vector of real numbers, $\mathbf{\Psi}$ diagonal and positive definite-is the consequent of deduced biconditional $GP_{ulf}$. The antecedent of $GP_{ulf}$ is $\{t_{1(ulf)}, t_{2(ulf)}, t_{3(ulf)}\}$; i.e., the unidimensional, linear, factor structure. What this means is that $\tilde{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi} - \mathbf{\Lambda}$ a $p \times 1$ vector of real numbers, $\mathbf{\Psi}$ diagonal and positive definite-is a claim of *restriction* on $f_X$ that obtains in a population $P$—i.e., $\Sigma_P = \tilde{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi}$—*if and only if* an element of the class $LS_{ulf}$ happens to be a latent structure of X. The logical character of $\tilde{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi}$ is, then, akin to that of the coloration of litmus paper in tests of pH: *it is an observable indication of the existence, in P, of a particular unobservable state of affairs*. And whereas the sole relevance of the restriction $\tilde{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi}$ rests on its being an observable indication of the particular unobservable state of affairs wherein X has, as a latent structure, an element of $LS_{ulf}$, a model has manifold practical uses that, generally speaking, rest on its capacity to serve as proxy for that which it was created to represent.

It is a fruitful exercise to remind oneself that there are countless decompositions of Σ—the Cholesky, LDU, Jordan, Schur, and eigen, for starters—that are guaranteed to hold in *P*. In contrast, just as with blue litmus paper's disposition to turn red in response to the alkalinity of a solution, the decomposition $\tilde{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi}$—in no way guaranteed to hold—obtains in response to the event of a set of variables X having, as a latent structure, an element of the class $LS_{ulf}$. One *builds* a model to represent. In contrast, one *observes* whether $\Sigma_P = \tilde{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi}$ holds, and, on the basis of this observation, makes a decision as to whether or not an unobservable state of affairs is extant.

Nor is it the case that the set of characteristics $\{t_{1(ulf)}, t_{2(ulf)}, t_{3(ulf)}\}$ is a model. For there is nothing that this particular set of latent properties *represents*. Rather, this set of latent properties *is* a particular thing; namely, the unidimensional, linear factor structure. The unidimensional, linear, factor structure—that thing specified as $\{t_{1(ulf)}, t_{2(ulf)}, t_{3(ulf)}\}$—is an unobservable. Consequently, if it happens to be underlying a particular X, its presence must be detected. A decision that the unidimensional, linear, factor structure has been detected is made when it is observed that $f_X$ is in a state of restriction that this unobservable is known to induce. Finally, though the purpose of certain data analytic technologies, e.g., principal component analysis, is, indeed, to create representations of *data* (high dimensional data structures are not directly inspectable, and so can only be visualized through the taking of low dimensional projections), this is *not* the purpose of linear factor analysis (nor, for that matter, any other latent variable technology). For latent variable technologies are not constructed in accordance with the logic of low dimensional approximation; but, rather, in accordance with detection logic. And, in contradistinction to the logic of dimensional approximation, it is detection logic under which generating propositions—each of which expresses a linkage between the presence of an efficacious unobservable and one of its manifest effects—must be deduced.

### 3.2 Misportrayal of latent variable technologies as general, open-ended, exploratory procedures

Latent variable technologies such as linear factor analysis, latent class analysis, structural equation modelling, and non-linear factor analysis, are frequently portrayed in a manner that ascribes to them the character of open-ended, exploratory, data analytic tools; tools, the purpose of which is to provide the researcher with a means by which to investigate the association structures of sets of variables. In fact, it is hard to imagine anything *less* open-ended exploratory, than a latent variable technology. For each is founded on a generating proposition which expresses a linkage between the (possible) state of nature wherein an element *ls** of a *particular* class of latent structures *LS** happens to be a latent structure of a set of variables, and a *particular* restriction $\gamma \in \Omega_{LS^*} \subset \Omega$ on the joint distribution of the variables. To employ, in research, a latent variable technology, is, most essentially, to test whether holds in some population *P*, restriction $\gamma \in \Omega_{LS^*} \subset \Omega$ on $f_X$. This restriction is the consequent of the analytically deduced generating proposition on which the technology is founded. Because the antecedent of the proposition is a specification $\{t_{1*},\ldots,t_{s*}\}$ of particular class of latent structures *LS**, the test undertaken is, then, a test of a particular existential hypothesis (cf. Feigl 1953; Rozeboom 1984). It is a test of whether a particular unobservable thing—an element of the class *LS**—*exists*; equivalently, of whether an element of *LS** happens to underlie, in a particular population *P*, some specific X. There is nothing open-ended, or exploratory, about it.

But, in fact, the detection logic on which is erected a particular latent variable technology, invests the technology with an even stronger sense of *non*-open endedness; a sense captured by what might be called the *specificity principle* of latent structure detection. This principle can be expressed as follows: if an analytical linkage (of Type I, Type II, or Type III) does not exist between the defining characteristics $\{t_{1*},\ldots,t_{s*}\}$ of class of latent structures *LS**, and restriction $\gamma \in \Omega^+ \subset \Omega$ on $f_X$, then a test of whether restriction $\gamma \in \Omega^+ \subset \Omega$ holds can have no implications for the issue of whether an element of *LS** underlies X.

The flip side of this coin has, of course, to do with dimensionality. There have been invented countless distinct senses of dimensionality, among them, the classical Euclidean, the Riemannian, the sense appearing in fractal geometry due to Hausdorff, and the sense arising in non-metric multidimensional scaling. Additionally, a distinct sense of dimensionality is defined under each and every distinct latent structure. A decision that *LS**—under which is defined what it means that a set of variables is $\rho$-dimensional in the $\gamma$-sense of dimensionality—underlies a particular set of variables $X^*$, is a decision that these variables are $\rho$-dimensional in the $\gamma$-sense of dimensionality. Accordingly, the specificity principle underlines the facts that: (a) a set of variables does not have *a* dimensionality in a population *P*, but, rather, a multitude of dimensionalities (these, frequently, numerically disparate), one for each of the senses of dimensionality that is applicable to the variables[22]; (b) the employment of a particular latent variable technology can never render an unconditional decision about dimensionality. The decision rendered must always be conditioned on the sense of dimensionality defined by the latent structure that constitutes the antecedent of the technology's generating proposition. Thus, a decision that $LS_{ulf}$ is a latent structure of particular set of variables $X^*$ is not a decision that $X^*$ is unidimensional, but, rather, that it is unidimensional precisely *in the sense of {t1(ulf), t2(ulf), t3(ulf)}.*

## 3.3 Endemic failure to specify target of detection (and to select relevant tool of detection)

When a particular latent variable technology is called for in a research application, the problem at hand is the determination of whether a specific type of latent structure happens

[22] e.g., a set of *p* quasi-continuous variables that happen to be undimensional in the sense of quadratic factor analysis, is 2-dimensional in the sense of linear factor analysis [and, for that matter, *p* dimensional in the classical euclidean sense (the sense of principal component analysis)].

to be underlying a set of observables. An immediate consequence of the misportrayal of latent variable technologies as open-ended data analytic tools, is that researchers do not set up their detection problems with the careful particularity that ensures the fruitful employment of these technologies. It is, in fact, commonplace that they neither bother to *specify* a particular latent structure, the detection of which is called for by the science they are undertaking; nor, in consequence, ensure selection of a *relevant* tool of detection. That is to say, there exists rampant violating of the specificity principle of latent structure detection. And the consequence is the visiting of looseness, ambiguity, and irrelevancy upon the empirical results yielded in the employment of latent variable technologies.

The way the employment of latent variable technologies *should* look, is as follows: (a) antecedent to their employment, *by virtue of the particulars of the empirical science being undertaken*, a need is identified to render detections of the elements *ls** of a class *LS** of (unobservable) latent structures. Because classes of latent structures are specified through a listing of their defining characteristics, such a need cannot *be* identified—hence, coherently expressed—unless the defining characteristics of *LS** have been listed. Consequently, as part of the identification of the need, the set of defining characteristics of *LS**, say, $\{t_{1*},\ldots,t_{s*}\}$, has been duly noted; (b) the need to render detections in respect the targets of detection that are the elements of *LS** necessitates construction of a detection protocol $D_{LS*}$. Accordingly, a generating proposition is invented—or found extant in the literature-, that takes as its antecedent, $\{t_{1*},\ldots,t_{s*}\}$; c) when a decision is required as to whether an *ls** $\in$ *LS** is present (i.e., underlying a particular set of variables), it is effected through an inferential testing of whether restriction $\gamma \in \Omega_{LS^*} \subset \Omega$ on $f_X$—the consequent of the generating proposition—obtains. The type of decision made, on the basis of $D_{LS*}$ (*modus tollens* disconfirmatory, confirmatory, or bidirectional), is in keeping with the logical type of the generating proposition.

As manifest in the following sorts of quotes—"The first goal was to test the underlying structure of memory compensation reports… We expected to observe a coherent measurement structure… (de Frias and Dixon 2005, p. 168); "…we calculated maximum-likelihood confirmatory factor analysis…" (Mottram and Donders 2005, p. 214); "This study applied latent class analysis (LCA) to identify subgroups of female juvenile offenders…" (Odgers et al. 2007)—what one encounters, throughout the literature of the social sciences, is something quite different; employment grounded in a *laissez-faire*, choose and use, button-pushing mentality, that betrays few signs of comprehension as to the highly specific purposes latent variable technologies were created to serve.

### 3.4 Misportrayal of generating propositions and defining characteristics as *assumptions*

Every bit as ubiquitous as modelling-talk, within the literature on latent variable technologies, are references to assumptions. Not infrequently, one comes upon quotes of the following sort:

> "…the well-known factor analysis model assumes that $\mathbf{\Sigma} = \mathbf{\Lambda\Lambda}' + \mathbf{\Psi}$…" (Shapiro 1985, p. 84)
> "Similarly, assume that the elements of $\boldsymbol{\varepsilon}$ are normally and independently distributed with mean zero and variances $\mathrm{var}(\varepsilon_i) = \Psi_i$…" (Morrison 1967, p. 305).
> "The elements of **f** are called *common* factors and the elements of **u** *specific* or *unique* factors. We shall suppose $E(\mathbf{f}) = \mathbf{0}$, $V(\mathbf{f}) = \mathbf{I}$, $E(\mathbf{u}) = \mathbf{0}$, $C(u_i,u_j) = 0$, $i \neq j$…" (Mardia et al. 1979).

"If one assumed that the common factors were uncorrelated." (Long 1983, p. 24).

However, the *assumptions* referred to in these quotes, are not assumptions at all. The expression $\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Lambda}' + \boldsymbol{\Psi}$ referred to in the first quote, for example, is not an assumption, but, rather, the consequent of the analytically deduced material implication (Type I generating proposition), on which (multidimensional) linear factor analysis is founded. As such, it is a restriction on $f_X$. If, in a particular population *P*, this restriction does not obtain, then, in *P*, X does not have, as a latent structure, a linear factor analytic structure. One neither does, nor does not, *assume* that restriction $\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Lambda}' + \boldsymbol{\Psi}$ holds in a particular population *P* under study; rather, one undertakes the inferential testing of whether it does. In so doing, one undertakes a *modus tollens* test of disconfirmation of the existential hypothesis that an element of the class of linear factor structures, is a latent structure of X.

Nor are the properties referred to in the final three quotes, assumptions. They are, in marked contradistinction, *defining characteristics of particular latent structures*. To state, for example, that "$\boldsymbol{\varepsilon}$ are normally and independently distributed with mean zero and variances $\mathrm{var}(\varepsilon_i) = \Psi_i$" or that "the common factors are uncorrelated" is to *specify* certain of the properties that a latent structure must possess in order that it belong to particular classes of latent structures (i.e., in order that it *be*, say, a unidimensional, quadratic, factor structure). When, for example, in a scientific investigation, the elements of $LS_{ulf}$ are nominated as the targets of detection, the targets of detection are, then, *precisely* those unobservable things that have—among other properties-, *by virtue of the definition of* $LS_{ulf}$, the property that "θ is a continuous random, latent variable for which E(θ) = 0 and V(θ) = 1."

It is no more an assumption of (unidimensional) linear factor analysis that θ is a continuous random variable for which E(θ) = 0 and V(θ) = 1, than it is an assumption, attendant to the correct employment of a metal detector, that a metal object loses electrons. To specify the targets of detection in a legitimate employment of a metal detector—i.e., hidden metal objects-, is to lay down a definition of the concept *metal object* that signifies these particular unobservables. And it is a matter of definition that a metal object is an object that loses electrons. So too is it a matter of definition that each of the elements of $LS_{ulf}$—which are the targets of detection relative to (unidimensional) linear factor analysis—has, as a property, the property that "θ is a continuous random, latent variable for which E(θ) = 0 and V(θ) = 1."

There are two components of a latent structure detection protocol that could, arguably, be said to involve assumptions. Component 5 is an inferential component that comes into play when there is a need to make sample-based decisions as to whether the restriction $\gamma \in \Omega_{LS^*} \subset \Omega$ named in the generating proposition of a given technology, obtains in populations under study. The inferential procedures available to the researcher having to undertake such decision-making—e.g., maximum likelihood-, standardly rest on conditions on population distributions that must be satisfied in order that the procedure perform optimally. The side-conditions of Component 4—i.e., conditions on $f_X$ that must be satisfied in order that the generating proposition of a given detection protocol be logically true—are entities of similar sort, in that their satisfaction is necessary for the optimal performance of a procedure or protocol.

However, though a case could be made, along these lines, for assigning to the side-conditions of component 4, the appellation, *assumption*, I am strongly against doing so. In my view, it is very much more preferable to retain the label *assumption* solely for the designation of statistical assumptions; conditions on $f_X$ that must be satisfied in order that an inferential procedure performs adequately. In so doing, one maintains a sharp focus on the logical distinction between—relatively trivial—inferential issues pertaining to the optimality of statistical procedures, and the central, but patently non-inferential, issues pertaining to the optimality of detection tools in delivering detection-oriented decision-making.

## 3.5 Confusions over what it means for a latent variable technology to perform adequately

It is proper that effort be dedicated to the adjudication of the absolute performance level of a particular latent variable technology; so too, to the performances, relative to one another, of a set of competitor technologies. As one might expect, given the preeminent role of latent variable technologies in social research, there has been no dearth of attempts to render adjudication of this sort. However, for an analysis of the performance of *any* technology to have relevance, its design must arise out of a perspicuous grasp of the purposes for which the technology was designed to serve. Accordingly, the pervasive failure to appreciate that latent variable technologies are *detection* technologies—which amounts to a failure to maintain a clear view of the purpose for which these technologies were meant to serve-, has served to undermine the vast majority of attempted adjudications. I note, herein, two crucial flaws in the adjudicative program, to date.

In the first place, the failure to keep, front and center, the raison d'etre of a latent variable technology as *detection*, has meant that adjudicative efforts undertaken are, in the aggregate, improperly targeted. Virtually all of the attention has been focussed on component (5), the inferential component. Volumes have been published on the estimation and hypothesis testing features inherent to the employments of latent variable technologies. But though not unimportant, component (5) is—of the five components of a latent variable technology—most tangential to the purpose for which these technologies were designed to serve. Latent variable technologies are detection protocols, and it is components (1)–(4) that, most centrally, bear on the issue of a given technology's performing of its role as detector of the elements of a class of latent structures.

Consider a detection protocol $D_{LS*}$, the targets of detection of which are the elements *ls** of a class *LS** of latent structures, and let it be the case that *LS** is specified as $\{t_{1*},\ldots,t_{s*}\}$. Most fundamentally, what it *means* for $D_{LS*}$ to perform adequately, is that the tool of detection featured in $D_{LS*}$ is an implementation of a generating proposition $GP_{LS*}$ that is both: (a) *relevant* (i.e., its antecedent is, in fact, $\{t_{1*},\ldots,t_{s*}\}$); and (b) *logically true* (the truth of $GP_{LS*}$ conferring upon $D_{LS*}$ the property of yielding logically valid decision-making[23] in respect the existence of elements of *LS**). However, little is published within the psychometric literature, these days, on the logical properties of extant, and candidate, generating propositions; or, in other words, analytic work directed at the issues of their relevance and truth. Moreover, very little work is being done on the development of new generating propositions; i.e., the establishment of fruitful pairings of *class of latent structure* and *restriction on* $f_X$.

This has led to a state of affairs wherein: (i) working detection protocols are available for but a relatively small subset of the latent structures that might be of scientific interest to the social researcher. If, as Thurstone (1947, p. 56) insisted, the aim is to "…discover… the nature of the underlying order", then it is clearly not in the best interests of social science, that social scientists be in possession of working detectors for such a minutely

[23] Of a type either disconfirmatory, confirmatory, or both, depending on the logical type of $GP_{LS*}$.

small subset of the latent structures that they may encounter in their empirical researches; (ii) for the majority of the latent structures for which detection protocols are available, the extant detection theory amounts to nothing beyond the derivation of a single—very often Type I—generating proposition.[24] Because sounder decision-making in respect the presence of a particular latent structure may well be achievable through the joint employment of multiple detection protocols, these protocols featuring *different* types of generating propositions (those of type II and III as well as the, seemingly default, type I proposition), this state of affairs is suboptimal.[25]

In the second place, it has meant that attempts to analyze the performances of latent variable technologies are frequently undermined by full-blown irrelevancy. A commonly observed path to irrelevancy can be described as follows: (a) under consideration is a latent variable technology, $D_{LS*}$, founded on a generating proposition which takes as its antecedent, $\{t_{1*},\ldots,t_{s*}\}$. That is to say, $D_{LS*}$ is a detector of elements *ls** of a class *LS** of latent structures, the defining characteristics of which are $\{t_{1*},\ldots,t_{s*}\}$; (b) $D_{LS*}$ is adjudicated in respect its performance in detecting latent structures, the defining characteristics of which are not $\{t_{1*},\ldots,t_{s*}\}$.[26] Many instances of this sort of irrelevancy are found in the 1990 article by Velicer and Jackson, titled *Component Analysis Versus Factor Analysis: Some Issues in Selecting an Appropriate Procedure*, and the commentaries that followed. Velicer and Jackson (1990, p. 1) ask the question, "…should one do a component analysis or a factor analysis?…" and claim that "…The choice is not obvious, because the two broad classes of procedures serve a similar purpose, and share many important mathematical characteristics…"

However, *the only circumstance under which the performances of two distinct detection protocols, say,* $D_{LS1}$ *and* $D_{LS2}$*, can be coherently compared, is the circumstance wherein* $D_{LS1}$ *and* $D_{LS2}$ *have identical targets of detection*. Let $LS_1$ be specified as $LS_1 \equiv \left\{ ls1_j \equiv \bigcap_{i=1}^{k} t_{ji} \middle| \left\{t_{j1},\ldots,t_{jk}\right\} \supseteq \{t_1,\ldots,t_s\} \right\}$ and $LS_2$ be specified as $LS_2 \equiv \left\{ ls2_j \equiv \bigcap_{i=1}^{l} s_{ji} \middle| \left\{s_{j1},\ldots,s_{jl}\right\} \supseteq \{s_1,\ldots,s_r\} \right\}$. Then $D_{LS1}$ and $D_{LS2}$ can be coherently compared only if the sets $\{t_1,\ldots,t_s\}$ and $\{s_1,\ldots,s_r\}$ are identical. If this condition is not satisfied, then comparing $D_{LS1}$ and $D_{LS2}$ is akin to attempting to answer the question, "on the basis of its performance, which should be preferred (which should be selected, which is better), a metal detector or a bubble chamber." The answer, of course, is that "the comparison is pointless because the targets of detection of these tools are not the same; these tools serve different purposes." To attempt to adjudicate the performance of linear factor analysis, relative to that of principal component analysis, is, in fact, a step further in the direction of incoherence. For not only do these procedures not share the same target of detection, but the latter is not a detection protocol of *any* sort (it was not created to serve in the role of detector of the elements of a class of unobservables).

[24] The tool of detection that is an implementation of this single proposition, having been rapidly mythologized to the status of general, open-ended, data analytic "technique."

[25] Paul Meehl has made this point on many occasions. His belief in the necessity of multiple detectors for a given latent structure was the reason that he invented his multiple consistency tests strategy for use in the detection of the taxonomic latent structure (cf. Meehl 1995).

[26] cf. Linear factor analytic technology and the difficulty factor problem (see, especially, McDonald 1967).

## 4 Conclusions

Latent variable technologies such as linear factor analysis, latent profile analysis, quadratic factor analysis, and item response theory are built on the logic of detection theory. Each is tied to a specification of a particular class of latent structures *LS**. Each is founded on a generating proposition that expresses a linkage between the (possible) state of nature wherein an element *ls** of *LS** happens to be a latent structure of a set of variables, and a restriction $\boldsymbol{\gamma} \in \Omega_{LS^*} \subset \Omega$ on the joint distribution of the variables. The restriction is, then, an (observable) manifestation of the presence of an *ls**. An employment of the technology in research involves a test of whether, in a particular population, the restriction is satisfied by the joint distribution $f_{X^*}$ of a particular set of variables X*. The decision yielded in the conducting of this test is the basis for a decision as to whether, in *P*, an *ls** underlies X*.

Latent variable technologies do not involve models. Their employments are not modelling exercises. Just as the fact of red litmus turning blue, when submerged in a solution, is an *indication* of the solution being alkaline[27]—and not a model of the solution—the restriction, say, that $\tilde{\Sigma} = \boldsymbol{\Lambda\Lambda}' + \boldsymbol{\Psi} - \boldsymbol{\Lambda}$ a $p \times 1$ vector of real numbers, $\boldsymbol{\Psi}$ diagonal and positive definite-is not a model of the covariance matrix $\Sigma_P$ of a set of variables distributed in a population *P*, but, rather, a condition that- if obtains—gives indication that a particular type of latent structure underlies these variables.

Nor are latent variable technologies open-ended, exploratory, procedures. Though, of course, a great mass of quantitative detail tends, over time, to build up around any particular technology, the core of each and every latent variable technology—what makes it a *particular* technology—is, once again, its generating proposition. And its generating proposition expresses a linkage between the (possible) state of nature wherein an element *ls** of a class of latent structures *LS** is present (happens to underlie a set of variables), and a restriction $\boldsymbol{\gamma} \in \Omega_{LS^*} \subset \Omega$ on the joint distribution of the variables. To test whether the restriction holds in some population *P* is to test a *particular* existential hypothesis. Thus, when, for example, one conducts an inferential test of the pair $[\mathrm{H}_0 : \boldsymbol{\gamma} \in \Omega_{ulf}, \mathrm{H}_1 : \boldsymbol{\gamma} \notin \Omega_{ulf}]$, one is testing an existential hypothesis apropos the unidimensional, linear, factor structure; viz., whether it is extant, underlying a set of variables. There is nothing at all open-ended about it.

Assumptions sometimes attend the employments of latent variable technologies. This is because, in the employment of a latent variable technology, the researcher must make an inferential test as to whether or not empirically obtains, the restriction on $f_X$ expressed by the consequent of the technology's generating proposition. The optimal performance of many inferential procedures requires satisfaction of *assumptions*; in this case, conditions on $f_X$ (e.g., multivariate normality). However, it is neither an assumption attendant to the employment of linear factor analytic technology, that $\mathrm{E}(\mathbf{X}|\theta = \theta^*) = \boldsymbol{\kappa} + \boldsymbol{\Lambda}\theta^*$, nor, of latent class analysis, that $\theta$ has a discrete distribution. These are definitional properties of the latent structures that are the targets of these detection technologies.

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[27] When the diprotic acid contained in the limus enters an alkaline solution, the hydrogen ions it releases form, with the base, a blue-coloured conjugated base.

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