Problem: (Newtonian fluid shear stress)

The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates (see Figure) is given by the equation

\[ u = \frac{3U_m}{2} \left[ 1 - \left( \frac{y}{h} \right)^2 \right] \]

where \( U_m \) is the mean velocity. The fluid has the viscosity of 0.04 \( \text{lb.s/ft}^2 \). If \( U_m = 2 \text{ ft/s} \) and \( h = 0.2 \text{ in} \), determine:

(a) The shearing stress acting on the bottom wall.
(b) The shearing stress acting on a plane parallel to the walls and passing through the centerline (midplane).
Solution

For this type of parallel flow the shearing stress is obtained from

\[ \tau = \mu \frac{du}{dy} \]  \hspace{1cm} (1)

Thus, if the velocity distribution \( u = u(y) \) is known, the shearing stress can be determined at all points by evaluating the velocity gradient, \( du/dy \). For the distribution given

\[ \frac{du}{dy} = -\frac{3Uy}{h^2} \]  \hspace{1cm} (2)

(a) The bottom wall \( y = -h \) so that

\[ \frac{du}{dy} = \frac{3U}{h} \]  \hspace{1cm} (3)

And therefore the shearing stress is

\[ \tau_{\text{bottom wall}} = \mu \left( \frac{3U}{h} \right) \quad \text{in direction of flow} \]

Always use units in your calculations

This stress creates a drag on the wall. Since the velocity distribution is symmetrical, the shearing stress along the upper wall would have the same magnitude and direction.

(b) Along the midplane where \( y = 0 \) it follows from Eq. (2) that

\[ \frac{du}{dy} = 0 \]  \hspace{1cm} (4)
And thus the shearing stress is

\[ \tau_{midplane} = 0 \]

**Note (1):**
In this problem, we calculated the shearing stress acting on the walls. Same shearing stress but in the opposite direction is acting on the fluid. Why?

**Note (2):**
From Eq. (2) we see that the velocity gradient is a linear function of \( y \) i.e. \( \tau = \alpha y \) with \( \alpha = -3U/h^2 \). Hence, the shearing stress (see Eq. (1)) varies linearly with \( y \) and in this particular problem varies from 0 at the center of the channel to 14.4 lb/ft\(^2\) at the walls. For the more general case, the actual variation will depend on the nature of the velocity distribution.