Chapter 1 • Introduction

1.1 A gas at 20°C may be *rarefied* if it contains less than 10^{12} molecules per mm^3. If Avogadro's number is 6.023E23 molecules per mole, what air pressure does this represent?

Solution: The mass of one molecule of air may be computed as

$$m = \frac{\text{Molecular weight}}{\text{Avogadro's number}} = \frac{28.97\ \text{mol}^{-1}}{6.023\text{E}23\ \text{molecules/g}\cdot\text{mol}} = 4.81\text{E–}23\ \text{g}$$

Then the density of air containing 10^{12} molecules per mm^3 is, in SI units,

$$\rho = \left(10^{12}\ \frac{\text{molecules}}{\text{mm}^3}\right)\left(4.81\text{E–}23\ \frac{\text{g}}{\text{molecule}}\right)$$

$$= 4.81\text{E–}11\ \frac{\text{g}}{\text{mm}^3} = 4.81\text{E–}5\ \frac{\text{kg}}{\text{m}^3}$$

Finally, from the perfect gas law, Eq. (1.13), at 20°C = 293 K, we obtain the pressure:

$$p = \rho RT = \left(4.81\text{E–}5\ \frac{\text{kg}}{\text{m}^3}\right)\left(287\ \frac{\text{m}^2}{\text{s}^2\cdot\text{K}}\right)(293\ \text{K}) = \mathbf{4.0\ Pa} \quad \textit{Ans.}$$

1.2 The earth's atmosphere can be modeled as a uniform layer of air of thickness 20 km and average density 0.6 kg/m^3 (see Table A-6). Use these values to estimate the total mass and total number of molecules of air in the entire atmosphere of the earth.

Solution: Let R_e be the earth's radius ≈ 6377 km. Then the total mass of air in the atmosphere is

$$m_t = \int \rho\, d\text{Vol} = \rho_{avg}(\text{Air Vol}) \approx \rho_{avg} 4\pi R_e^2(\text{Air thickness})$$

$$= (0.6\ \text{kg/m}^3)4\pi(6.377\text{E}6\ \text{m})^2(20\text{E}3\ \text{m}) \approx \mathbf{6.1E18\ kg} \quad \textit{Ans.}$$

Dividing by the mass of one molecule ≈ 4.8E−23 g (see Prob. 1.1 above), we obtain the total number of molecules in the earth's atmosphere:

$$N_{\text{molecules}} = \frac{\text{m(atmosphere)}}{\text{m(one molecule)}} = \frac{6.1\text{E}21\ \text{grams}}{4.8\text{E}-23\ \text{gm/molecule}} \approx \mathbf{1.3E44\ molecules} \quad \textit{Ans.}$$

1.3 For the triangular element in Fig. P1.3, show that a tilted free liquid surface, in contact with an atmosphere at pressure pa, must undergo shear stress and hence begin to flow.

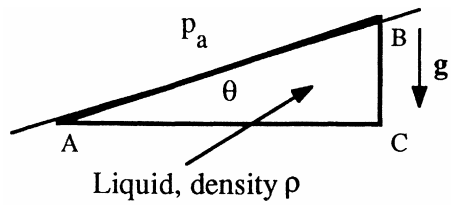
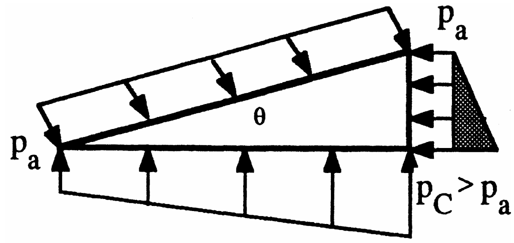


Fig. P1.3

Solution: Assume zero shear. Due to element weight, the pressure along the lower and right sides must vary linearly as shown, to a higher value at point C. Vertical forces are presumably in balance with element weight included. But horizontal forces are out of balance, with the unbalanced force being to the left, due to the shaded excess-pressure triangle on the right side BC. Thus hydrostatic pressures cannot keep the element in balance, and shear and flow result.



P1.4 The Saybolt Universal Viscometer, now obsolete but still sold in scientific catalogs, measures the kinematic viscosity of lubricants [Mott, p. 40]. A container, held at constant temperature, is filled with 60 ml of fluid. Measure the time *t* for the fluid to drain from a small hole or short tube in the bottom. This time unit, called *Saybolt universal seconds*, or SUS, is correlated with kinematic viscosity ν, in centistokes (1 stoke = 1 cm^2/s), by the following curve-fit formula:

$$\nu = 0.215t - \frac{145}{t} \qquad \text{for} \quad 40 < t < 100 \text{ SUS}$$

(*a*) Comment on the dimensionality of this equation. (*b*) Is the formula physically correct? (*c*) Since ν varies strongly with temperature, how does temperature enter into the formula? (*d*) Can we easily convert ν from centistokes to mm^2/s?

Solution: (*a*) The formula is dimensionally *inconsistent*. The right-hand side does not have obvious kinematic viscosity units. The constants 0.215 and 145 must conceal (dimensional) information on temperature, gravity, fluid density, and container shape. (*b*) The formula correctly predicts that the time to drain increases with fluid viscosity. (*c*) The time *t* will reflect changes in ν, and the constants 0.215 and 145 vary slightly (±1%) with temperature [Mott, p. 43]. (*d*) Yes, no conversion necessary; the units of centistoke and mm^2/s are exactly the same..

1.5 A formula for estimating the mean free path of a perfect gas is:

$$\ell = 1.26\frac{\mu}{\rho\sqrt{RT}} = 1.26\frac{\mu}{\text{p}}\sqrt{RT} \quad (1)$$

where the latter form follows from the ideal-gas law, $\rho = \text{p/RT}$. What are the dimensions of the constant "1.26"? Estimate the mean free path of air at 20°C and 7 kPa. Is air *rarefied* at this condition?

Solution: We know the dimensions of every term except "1.26":

$$\{\ell\} = \{\text{L}\} \quad \{\mu\} = \left\{\frac{\text{M}}{\text{LT}}\right\} \quad \{\rho\} = \left\{\frac{\text{M}}{\text{L}^3}\right\} \quad \{\text{R}\} = \left\{\frac{\text{L}^2}{\text{T}^2\Theta}\right\} \quad \{\text{T}\} = \{\Theta\}$$

Therefore the above formula (first form) may be written dimensionally as

$$\{\text{L}\} = \{1.26?\}\frac{\{\text{M/L}\cdot\text{T}\}}{\{\text{M/L}^3\}\sqrt{[\{\text{L}^2/\text{T}^2\cdot\Theta\}\{\Theta\}]}} = \{1.26?\}\{\text{L}\}$$

Since we have {L} on both sides, {1.26} = {unity}, that is, the constant is dimensionless. The formula is therefore dimensionally homogeneous and should hold for any unit system.

For air at 20°C = 293 K and 7000 Pa, the density is $\rho = \text{p/RT}$ = (7000)/[(287)(293)] = 0.0832 kg/m^3. From Table A-2, its viscosity is 1.80E–5 N·s/m^2. Then the formula predicts a mean free path of

$$\ell = 1.26\frac{1.80\text{E–}5}{(0.0832)[(287)(293)]^{1/2}} \approx \mathbf{9.4E–7\ m} \quad \textit{Ans.}$$

This is quite small. We would judge this gas to approximate a continuum if the physical scales in the flow are greater than about 100 ℓ, that is, greater than about 94 μm.

P1.6 From the (correct) theory of Prob. P1.5, estimate the pressure, in pascals, of carbon dioxide at 20°C for which the mean free path is (a) 1 micron; and (*b*) 43.3 nm.

Solution: This is essentially an exercise in getting the units right in a formula. The formula for mean free path of a gas is given in Prob. P1.5:

$$l = 1.26\frac{\mu}{\rho\sqrt{RT}} = 1.26\frac{\mu\sqrt{RT}}{p} \quad \text{from the perfect gas law} \quad p = \rho RT$$

From Table A.4 in the appendix, for CO_2, take R = 189 $m^2/(s^2\text{-}K)$ and μ = 1.48E-5 kg/m-s. The temperature must be in Kelvin, T = 20°C+273 = 293K. (*a*) Introduce these into the formula:

$$l = 1E-6m = 1.26\,\frac{\mu\sqrt{RT}}{p} = 1.26\frac{(1.48E-5kg/m-s)\sqrt{(189m^2/s^2-K)(293K)}}{p}$$

$$\text{Solve for}\quad p = 4388\,N/m^2 \approx \mathbf{4400\,Pa} \qquad Ans.(a)$$

(*b*) This time, with *l* known, solve for the pressure:

$$p = 1.26\frac{\mu\sqrt{RT}}{l} = 1.26\frac{(1.48E-5kg/m-2)\sqrt{(189m^2/s^2-K)(293K)}}{43.3E-9\ m} = \mathbf{101{,}350\ Pa} \quad Ans.(b)$$

Thus, as carefully prepared, 43.3 nm is the mean free path of CO_2 at 20°C and 1 atmosphere.

P1.7 To determine the flow rate of water at 20°C through a hose, a student finds that the hose fills a 55-gallon drum in 2 minutes and 37 seconds. Estimate (*a*) the volume flow rate in m^3/s; and (*b*) the weight flow in N/s.

Solution: This is a straightforward exercise in conversion factors. From Appendix C, or the inside front cover, convert

$$55\,gal = (55gal)(3.7854E-3\,\frac{m^3}{gal}) = 0.2082\,m^3 \;;\quad 2\min 37s = 157\,s$$

$$\text{Volume flow rate} = Q = \frac{0.2082\,m^3}{157\,s} = \mathbf{0.00133}\,\frac{m^3}{s} \quad (\text{or } 21\,\frac{gal}{min}) \qquad Ans.(a)$$

$$\text{Weight flow rate} = \rho g Q = (998\frac{kg}{m^3})(9.81\frac{m}{s^2})(0.00133\,\frac{m^3}{s}) = 13\,\frac{kg\bullet m/s^2}{s} = \mathbf{13}\,\frac{N}{s} \quad Ans.(b)$$

A cubic meter is a lot of water, and a second is very short, so the m^3/s unit is more suitable for, say, a river flow. Engineers who work with moderate flows, like hoses and pipes, are apt to use gallons per minute, which usually yields easy-to-remember whole numbers.

1.8 Suppose that bending stress σ in a beam depends upon bending moment M and beam area moment of inertia I and is proportional to the beam half-thickness y. Suppose also that, for the particular case M = 2900 in·lbf, y = 1.5 in, and I = 0.4 in^4, the predicted stress is 75 MPa. Find the only possible dimensionally homogeneous formula for σ.

Solution: We are given that $\sigma = $ y fcn(M,I) and we are *not* to study up on strength of materials but only to use dimensional reasoning. For homogeneity, the right hand side must have dimensions of stress, that is,

$$\{\sigma\} = \{y\}\{\text{fcn(M,I)}\}, \quad \text{or:} \quad \left\{\frac{M}{LT^2}\right\} = \{L\}\{\text{fcn(M,I)}\}$$

$$\text{or:} \quad \text{the function must have dimensions } \{\text{fcn(M,I)}\} = \left\{\frac{M}{L^2T^2}\right\}$$

Therefore, to achieve dimensional homogeneity, we somehow must combine bending moment, whose dimensions are $\{ML^2T^{-2}\}$, with area moment of inertia, $\{I\} = \{L^4\}$, and end up with $\{ML^{-2}T^{-2}\}$. Well, it is clear that {I} contains neither mass {M} nor time {T} dimensions, but the bending moment contains both mass and time and in exactly the combination we need, $\{MT^{-2}\}$. Thus it must be that σ _is proportional to M also_. Now we have reduced the problem to:

$$\sigma = yM\,\text{fcn(I)}, \quad \text{or} \quad \left\{\frac{M}{LT^2}\right\} = \{L\}\left\{\frac{ML^2}{T^2}\right\}\{\text{fcn(I)}\}, \quad \text{or:} \quad \{\text{fcn(I)}\} = \mathbf{\{L^{-4}\}}$$

We need just enough *I*'s to give dimensions of $\{L^{-4}\}$: we need the formula to be exactly *inverse* in *I*. The correct dimensionally homogeneous beam bending formula is thus:

$$\boldsymbol{\sigma} = \mathbf{C\frac{My}{I}}, \quad \text{where } \{C\} = \{\text{unity}\} \quad \textit{Ans.}$$

The formula admits to an arbitrary dimensionless constant C whose value can only be obtained from known data. Convert stress into English units: σ = (75 MPa)/(6894.8) = 10880 lbf/in². Substitute the given data into the proposed formula:

$$\sigma = 10880\ \frac{\text{lbf}}{\text{in}^2} = C\frac{My}{I} = C\frac{(2900\ \text{lbf·in})(1.5\ \text{in})}{0.4\ \text{in}^4}, \quad \text{or:} \quad \mathbf{C \approx 1.00} \quad \textit{Ans.}$$

The data show that C = 1, or $\boldsymbol{\sigma}$ **= My/I**, our old friend from strength of materials.

P1.9 A fluid is weighed in a laboratory. It is found that 1.5 U. S. gallons of fluid weigh 136.2 ounces. (*a*) What is the fluid's density, in kg/m³? (*b*) What fluid could this be? Assume standard gravity, $g = 9.807$ m/s².

Solution: (a) Convert the volume and weight to SI units, using Appendix C.

$$\upsilon = 1.5\ \text{US gal} \times 0.0037854\ \text{m}^3/\text{gal} = 0.0056781\ \text{m}^3$$

$$W = 136.2\ \text{oz} \times 4.44822\ \text{N}/16\,\text{oz} = 37.865\ \text{N}$$

$$m = \frac{W}{g} = \frac{37.865\,N}{9.807\,m/s^2} = 3.861\ kg$$

$$\text{Finally,} \quad \text{density} \quad \rho = \frac{m}{\upsilon} = \frac{3.861\,kg}{0.0056781\,m^3} = \mathbf{680}\ \frac{kg}{m^3} \quad \textit{Ans.}$$

(*b*) This is a very light liquid. From Appendix Table A.3, it might very well be *gasoline*.

1.10 The Stokes-Oseen formula [10] for drag on a sphere at low velocity V is:

$$F = 3\pi\mu DV + \frac{9\pi}{16}\rho V^2 D^2$$

where D = sphere diameter, μ = viscosity, and ρ = density. Is the formula homogeneous?

Solution: Write this formula in dimensional form, using Table 1-2:

$$\{F\} = \{3\pi\}\{\mu\}\{D\}\{V\} + \left\{\frac{9\pi}{16}\right\}\{\rho\}\{V\}^2\{D\}^2 \;?$$

$$\text{or:}\quad \left\{\frac{ML}{T^2}\right\} = \{1\}\left\{\frac{M}{LT}\right\}\{L\}\left\{\frac{L}{T}\right\} + \{1\}\left\{\frac{M}{L^3}\right\}\left\{\frac{L^2}{T^2}\right\}\{L^2\} \;?$$

where, hoping for homogeneity, we have assumed that all constants (3,π,9,16) are *pure*, i.e., {unity}. Well, yes indeed, all terms have dimensions $\{ML/T^2\}$! Therefore the Stokes-Oseen formula (derived in fact from a theory) is ***dimensionally homogeneous.***

P1.11 In 1851 Sir George Stokes theorized that the drag force *F* on a particle in high-viscosity (low Reynolds number) flow depended only upon viscosity μ, particle velocity *V*, and particle size *D*. Use the concept of dimensional homogeneity to deduce a possible formula for the force.

Solution: The hypothesis is that $F = \text{fcn}(\mu, V, D)$. To satisfy dimensional homogeneity, the function on the right-hand side must have dimensions of force. The respective dimensions are

$$\{F\} = \{MLT^{-2}\}\ ;\ \{\mu\} = \{ML^{-1}T^{-1}\}\ ;\ \{V\} = \{LT^{-1}\}\ ;\ \{D\} = (L)$$

We need to combine (μ, V, D) so that the combination has force dimensions $\{MLT^{-2}\}$. Multiplying μ by *V* gives us $\{MT^{-2}\}$, but we lost $\{L\}$. So multiplying by *D* gives back the $\{L\}$:

$$\{\mu VD\} = \{ML^{-1}T^{-1}\}\{LT^{-1}\}\{L\} = \{MLT^{-2}\} \quad \text{correct force units}$$

$$\text{Thus}\quad F = \text{constant}\ \mu V D \qquad \textit{Ans.}$$

For a spherical particle, Stokes (1851) found the constant to have the theoretical value 3π.

1.12 For low-speed (laminar) flow in a tube of radius r_o, the velocity u takes the form

$$u = B\frac{\Delta p}{\mu}\left(r_o^2 - r^2\right)$$

where μ is viscosity and Δp the pressure drop. What are the dimensions of B?

Solution: Using Table 1-2, write this equation in dimensional form:

$$\{u\} = \{B\}\frac{\{\Delta p\}}{\{\mu\}}\{r^2\}, \quad \text{or:} \quad \left\{\frac{L}{T}\right\} = \{B?\}\frac{\{M/LT^2\}}{\{M/LT\}}\{L^2\} = \{B?\}\left\{\frac{L^2}{T}\right\},$$

$$\text{or:} \quad \mathbf{\{B\} = \{L^{-1}\}} \quad \textit{Ans.}$$

The parameter B must have dimensions of inverse length. In fact, B is not a constant, it hides one of the variables in pipe flow. The proper form of the pipe flow relation is

$$u = C\frac{\Delta p}{L\mu}\left(r_o^2 - r^2\right)$$

where L is the *length of the pipe* and C is a dimensionless constant which has the theoretical laminar-flow value of (1/4)—see Sect. 6.4.

1.13 The efficiency η of a pump is defined as

$$\eta = \frac{Q\Delta p}{\text{Input Power}}$$

where Q is volume flow and Δp the pressure rise produced by the pump. What is η if $\Delta p = 35$ psi, Q = 40 L/s, and the input power is 16 horsepower?

Solution: The student should perhaps verify that $Q\Delta p$ has units of power, so that η is a dimensionless ratio. Then convert everything to consistent units, for example, BG:

$$Q = 40\ \frac{L}{s} = 1.41\ \frac{ft^2}{s}; \quad \Delta p = 35\ \frac{lbf}{in^2} = 5040\ \frac{lbf}{ft^2}; \quad \text{Power} = 16(550) = 8800\ \frac{ft\cdot lbf}{s}$$

$$\eta = \frac{(1.41\ ft^3/s)(5040\ lbf/ft^2)}{8800\ ft\cdot lbf/s} \approx 0.81 \quad \text{or} \quad \mathbf{81\%} \quad \textit{Ans.}$$

Similarly, one could convert to SI units: Q = 0.04 m^3/s, Δp = 241300 Pa, and input power = 16(745.7) = 11930 W, thus h = (0.04)(241300)/(11930) = **0.81**. *Ans.*

1.14 The volume flow Q over a dam is proportional to dam width B and also varies with gravity g and excess water height H upstream, as shown in Fig. P1.14. What is the only possible dimensionally homogeneous relation for this flow rate?

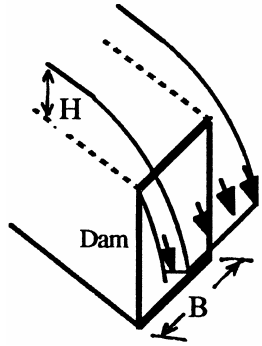


Fig. P1.14

Solution: So far we know that Q = B fcn(H,g). Write this in dimensional form:

$$\{Q\} = \left\{\frac{L^3}{T}\right\} = \{B\}\{f(H,g)\} = \{L\}\{f(H,g)\},$$

$$\text{or:} \quad \{f(H,g)\} = \left\{\frac{L^2}{T}\right\}$$

So the function fcn(H,g) must provide dimensions of $\{L^2/T\}$, but only *g* contains ***time***. Therefore g must enter in the form $g^{1/2}$ to accomplish this. The relation is now

$$Q = Bg^{1/2}\text{fcn}(H), \quad \text{or:} \quad \{L^3/T\} = \{L\}\{L^{1/2}/T\}\{\text{fcn}(H)\}, \quad \text{or:} \quad \{\text{fcn}(H)\} = \{\mathbf{L^{3/2}}\}$$

In order for fcn(H) to provide dimensions of $\{L^{3/2}\}$, the function must be a 3/2 power. Thus the final desired homogeneous relation for dam flow is:

$$\mathbf{Q = C\,B\,g^{1/2}\,H^{3/2}}, \quad \text{where C is a dimensionless constant} \quad \textit{Ans.}$$

P1.15 Mott [50] recommends the following formula for the friction head loss h_f, in ft, for flow through a pipe of length L_o and diameter D (both in ft):

$$h_f = L_o\left(\frac{Q}{0.551AC_hD^{0.63}}\right)^{1.852}$$

where Q is the volume flow rate in ft^3/s, A is the pipe cross-section area in ft^2, and C_h is a dimensionless coefficient whose value is approximately 100. Determine the dimensions of the constant 0.551.

Solution: Write out the dimensions of each of the terms in the formula:

$$\{h_f\}=\{L\}\ ;\ \{L_o\}=\{L\}\ ;\ \{Q\}=\{L^3/T\}\ ;\ \{A\}=\{L^2\}\ ;\ \{C_h\}=\{1\}\ ;\ \{D\}=\{L\}$$

Use these dimensions in the equation to determine {0.551}. Since h_f and L_o have the same dimensions $\{L\}$, it follows that the quantity in parentheses must be dimensionless:

$$\left\{\left(\frac{Q}{0.551AC_hD^{0.63}}\right)\right\} = \{1\} = \left\{\frac{L^3/T}{\{0.551\}\{L^2\}\{1\}\{L\}^{0.63}}\right\} = \left\{\frac{L^{0.37}}{(0.551)T}\right\} = \{1\}$$

$$\text{It follows that} \quad \{0.551\} = \mathbf{\{L^{0.37}/T\}} \qquad \textit{Ans.}$$

The constant has dimensions; therefore *beware*. The formula is valid only for *water* flow at high (*turbulent*) velocities. The density and viscosity of water are hidden in the constant 0.551, and the wall roughness is hidden (approximately) in the numerical value of C_h.

1.16 Test the dimensional homogeneity of the boundary-layer *x*-momentum equation:

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \rho g_x + \frac{\partial \tau}{\partial y}$$

Solution: This equation, like **<u>all</u>** theoretical partial differential equations in mechanics, is dimensionally homogeneous. Test each term in sequence:

$$\left\{\rho u\frac{\partial u}{\partial x}\right\} = \left\{\rho v\frac{\partial u}{\partial y}\right\} = \frac{M}{L^3}\frac{L}{T}\frac{L/T}{L} = \left\{\mathbf{\frac{M}{L^2T^2}}\right\};\quad \left\{\frac{\partial p}{\partial x}\right\} = \frac{M/LT^2}{L} = \left\{\mathbf{\frac{M}{L^2T^2}}\right\}$$

$$\{\rho g_x\} = \frac{M}{L^3}\frac{L}{T^2} = \left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\}; \quad \left\{\frac{\partial\tau}{\partial x}\right\} = \frac{M/LT^2}{L} = \left\{\frac{\mathbf{M}}{\mathbf{L^2T^2}}\right\}$$

All terms have dimension $\{ML^{-2}T^{-2}\}$. This equation may use *any* consistent units.

1.17 Investigate the consistency of the Hazen-Williams formula from hydraulics:

$$Q = 61.9D^{2.63}\left(\frac{\Delta p}{L}\right)^{0.54}$$

What are the dimensions of the constant "61.9"? Can this equation be used with confidence for a variety of liquids and gases?

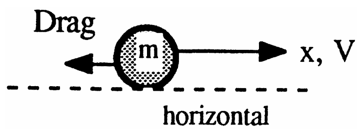
Solution: Write out the dimensions of each side of the equation:

$$\{Q\} = \left\{\frac{L^3}{T}\right\} \stackrel{?}{=} \{61.9\}\{D^{2.63}\}\left\{\frac{\Delta p}{L}\right\}^{0.54} = \{61.9\}\,\{L^{2.63}\}\left\{\frac{M/LT^2}{L}\right\}^{0.54}$$

The constant 61.9 has *fractional* dimensions: $\{61.9\} = \mathbf{\{L^{1.45}T^{0.08}M^{-0.54}\}}$ *Ans.*

Clearly, the formula is extremely inconsistent and cannot be used with confidence for any given fluid or condition or units. Actually, the Hazen-Williams formula, still in common use in the watersupply industry, is valid only for **water** flow in smooth pipes larger than 2-in. diameter and turbulent velocities less than 10 ft/s and (certain) English units. This formula should be held at arm's length and given a vote of "No Confidence."

1.18* ("*" means "difficult"—not just a plug-and-chug, that is) For small particles at low velocities, the first (linear) term in Stokes' drag law, Prob. 1.10, is dominant, hence F = KV, where K is a constant. Suppose a particle of mass m is constrained to move horizontally from the initial position x = 0 with initial velocity V = Vo. Show (a) that its velocity will decrease exponentially with time; and (b) that it will stop after travelling a distance x = mVo/K.



Solution: Set up and solve the differential equation for forces in the *x*-direction:

$$\sum F_x = -\text{Drag} = ma_x, \quad \text{or:} \quad -KV = m\frac{dV}{dt}, \quad \text{integrate} \int_{V_o}^{V} \frac{dV}{V} = -\int_0^t \frac{m}{K}dt$$

$$\text{Solve} \quad \mathbf{V = V_o e^{-mt/K}} \quad \text{and} \quad \mathbf{x = \int_0^t V\,dt = \frac{mV_o}{K}\left(1 - e^{-mt/K}\right)} \quad \textit{Ans.}\text{ (a,b)}$$

Thus, as asked, V drops off exponentially with time, and, as $t \to \infty$, $x = \mathbf{mV_o/K}$.

1.19 *Marangoni convection* arises when a surface has a difference in surface tension along its length. The dimensionless *Marangoni number M* is a combination of thermal diffusivity $\alpha = k/(\rho c_p)$ (where *k* is the thermal conductivity), length scale *L*, viscosity μ, and surface tension difference δY. If *M* is proportional to *L*, find its form.

Solution: List the dimensions: $\{\alpha\} = \{L^2/T\}$, $\{L\} = \{L\}$, $\{\mu\} = \{M/LT\}$, $\{\delta Y\} = \{M/T^2\}$. We divide δY by μ to get rid of mass dimensions, then divide by α to eliminate time:

$$\left\{\frac{\delta Y}{\mu}\right\} = \left\{\frac{M}{T^2}\frac{LT}{M}\right\} = \left\{\frac{L}{T}\right\}, \quad \text{then} \quad \left\{\frac{\delta Y}{\mu}\frac{1}{\alpha}\right\} = \left\{\frac{L}{T}\frac{T}{L^2}\right\} = \left\{\frac{1}{L}\right\}$$

Multiply by L and we obtain the Marangoni number: $\boldsymbol{M = \dfrac{\delta Y L}{\mu\alpha}}$ *Ans.*

1.20C ("C" means computer-oriented, although this one can be done analytically.) A baseball, with m = 145 g, is thrown directly upward from the initial position z = 0 and V_o = 45 m/s. The air drag on the ball is CV^2, where C ≈ 0.0010 $N\cdot s^2/m^2$. Set up a differential equation for the ball motion and solve for the instantaneous velocity V(t) and position z(t). Find the maximum height z_{max} reached by the ball and compare your results with the elementary-physics case of zero air drag.

Solution: For this problem, we include the *weight* of the ball, for upward motion *z*:

$$\sum F_z = -ma_z, \quad \text{or:} \quad -CV^2 - mg = m\frac{dV}{dt}, \quad \text{solve} \int_{V_o}^{V} \frac{dV}{g + CV^2/m} = -\int_0^t dt = -t$$

$$\text{Thus } V = \sqrt{\frac{mg}{C}}\tan\left(\phi - t\sqrt{\frac{Cg}{m}}\right) \quad \text{and} \quad z = \frac{m}{C}\ln\left[\frac{\cos(\phi - t\sqrt{(gC/m)})}{\cos\phi}\right]$$

where $\phi = \tan^{-1}[V_o\sqrt{(C/mg)}]$. This is cumbersome, so one might also expect some students simply to *program* the differential equation, $m(dV/dt) + CV^2 = -mg$, with a numerical method such as Runge-Kutta.

For the given data m = 0.145 kg, V_o = 45 m/s, and C = 0.0010 $N\cdot s^2/m^2$, we compute

$$\phi = 0.8732 \text{ radians}, \quad \sqrt{\frac{mg}{C}} = 37.72\ \frac{m}{s}, \quad \sqrt{\frac{Cg}{m}} = 0.2601\ s^{-1}, \quad \frac{m}{C} = 145 \text{ m}$$

Hence the final analytical formulas are:

$$V\left(\text{in } \frac{m}{s}\right) = 37.72\tan(0.8732 - .2601t)$$

$$\text{and} \quad z(\text{in meters}) = 145\ \ln\left[\frac{\cos(0.8732 - 0.2601t)}{\cos(0.8732)}\right]$$

The velocity equals zero when t = 0.8732/0.2601 ≈ **3.36 s**, whence we evaluate the maximum height of the baseball as z_{max} = 145 ln[sec(0.8734)] ≈ **64.2 meters**. *Ans.*

For zero drag, from elementary physics formulas, $V = V_o - gt$ and $z = V_o t - gt^2/2$, we calculate that

$$t_{max\ height} = \frac{V_o}{g} = \frac{45}{9.81} \approx \mathbf{4.59\ s} \quad \text{and} \quad z_{max} = \frac{V_o^2}{2g} = \frac{(45)^2}{2(9.81)} \approx \mathbf{103.2\ m}$$

Thus drag on the baseball reduces the maximum height by 38%. [For this problem I assumed a baseball of diameter 7.62 cm, with a drag coefficient $C_D \approx 0.36$.]

P1.21 In 1908, Prandtl's student Heinrich Blasius proposed the following formula for the wall shear stress τ_w at a position x in viscous flow at velocity V past a flat surface:

$$\tau_w = 0.332\,\rho^{1/2}\,\mu^{1/2}\,V^{3/2}\,x^{-1/2}$$

Determine the dimensions of the constant 0.332.

Solution: From Table 1.2 we find the dimensions of each term in the equation:

$$\{\tau_w\} = \{ML^{-1}T^{-2}\}\ ;\ \{\rho\} = \{ML^{-3}\}\ ;\ \{\mu\} = \{ML^{-1}T^{-1}\}\ ;\ \{V\} = \{LT^{-1}\}\ ;\ \{x\} = \{L\}$$

Use these dimensions in the equation to determine {0.332}:

$$\{\frac{M}{LT^2}\} = \{0.332\}\,\{\frac{M}{L^3}\}^{1/2}\,\{\frac{M}{LT}\}^{1/2}\,\{\frac{L}{T}\}^{3/2}\,\{L\}^{-1/2}$$

$$\text{Clean up}: \quad \{\frac{M}{LT^2}\} = \{0.332\}\,\{\frac{M}{LT^2}\}\ , \quad \text{or}: \quad \mathbf{\{0.332\} = \{1\}} \quad \textit{Ans.}$$

The constant 0.332 is *dimensionless*. Blasius was one of the first workers to deduce dimensionally consistent viscous-flow formulas without empirical constants.

P1.22 The *Richardson number*, Ri, which correlates the production of turbulence by buoyancy, is a dimensionless combination of the acceleration of gravity g, the fluid temperature T_o, the local temperature gradient $\partial T/\partial z$, and the local velocity gradient $\partial u/\partial z$. Determine an acceptable form for the Richardson number (most workers put $\partial T/\partial z$ in the numerator).

Solution: In the {MLTΘ} system, these variables have the dimensions $\{g\} = \{L/T^2\}$, $\{T_o\} = \{\Theta\}$, $\{\partial T/\partial z\} = \{\Theta/L\}$, and $\{\partial u/\partial z\} = \{T^{-1}\}$. The ratio $g/(\partial u/\partial z)^2$ will cancel time, leaving {L} in the numerator, and the ratio $\{\partial T/\partial z\}/T_o$ will cancel $\{\Theta\}$, leaving $\{L\}$ in the denominator. Multiply them together and we have the standard form of the dimensionless Richardson number:

$$\mathrm{Ri} = \frac{g\left(\dfrac{\partial T}{\partial z}\right)}{T_o\left(\dfrac{\partial u}{\partial z}\right)^2} \qquad \textit{Ans.}$$

P1.23 During World War II, Sir Geoffrey Taylor, a British fluid dynamicist, used dimensional analysis to estimate the energy released by an atomic bomb explosion. He assumed that the energy released, E, was a function of blast wave radius R, air density ρ, and time t. Arrange these variables into a single dimensionless group, which we may term the *blast wave number*.

Solution: These variables have the dimensions $\{E\} = \{ML^2/T^2\}$, $\{R\} = \{L\}$, $\{\rho\} = \{M/L^3\}$, and $\{t\} = \{T\}$. Multiplying E by t^2 eliminates time, then dividing by ρ eliminates mass, leaving $\{L^5\}$ in the numerator. It becomes dimensionless when we divide by R^5. Thus

$$\text{Blast wave number} = \frac{E\,t^2}{\rho R^5}$$

1.24 Consider carbon dioxide at 10 atm and 400°C. Calculate ρ and c_p at this state and then estimate the new pressure when the gas is cooled isentropically to 100°C. Use two methods: (a) an ideal gas; and (b) the Gas Tables or EES.

Solution: From Table A.4, for CO_2, $k \approx 1.30$, and $R \approx 189\ m^2/(s^2\cdot K)$. Convert pressure from p_1 = 10 atm = 1,013,250 Pa, and T_1 = 400°C = 673 K. (a) Then use the ideal gas laws:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{1{,}013{,}250\,Pa}{(189\ m^2/s^2K)(673\,K)} = \mathbf{7.97}\ \frac{kg}{m^3};$$

$$c_p = \frac{kR}{k-1} = \frac{1.3(189)}{1.3-1} = \mathbf{819}\ \frac{J}{kg\cdot K} \qquad \textit{Ans.}\text{ (a)}$$

For an ideal gas cooled isentropically to T_2 = 100°C = 373 K, the formula is

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \frac{p_2}{1013\ kPa} = \left(\frac{373K}{673K}\right)^{1.3/(1.3-1)} = 0.0775, \quad \textbf{\textit{or:}} \quad p_2 = \mathbf{79\ kPa} \quad \textit{Ans.}\text{ (a)}$$

For EES or the Gas Tables, just program the properties for carbon dioxide or look them up:

$$\rho_1 = \mathbf{7.98}\ \text{kg/m}^3; \quad c_p = \mathbf{1119}\ \text{J/(kg·K)}; \quad p_2 = \mathbf{43}\ \text{kPa} \quad \textit{Ans.}\text{ (b)}$$

(NOTE: The large errors in "ideal" c_p and "ideal" final pressure are due to the sharp drop-off in *k* of CO_2 with temperature, as seen in Fig. 1.3 of the text.)

1.25 A tank contains 0.9 m^3 of helium at 200 kPa and 20°C. Estimate the total mass of this gas, in kg, (a) on earth; and (b) on the moon. Also, (c) how much heat transfer, in MJ, is required to expand this gas at constant temperature to a new volume of 1.5 m^3?

Solution: First find the density of helium for this condition, given R = 2077 $m^2/(s^2\cdot K)$ from Table A-4. Change 20°C to 293 K:

$$\rho_{He} = \frac{p}{R_{He}T} = \frac{200000\ \text{N/m}^2}{(2077\ \text{J/kg}\cdot\text{K})(293\ \text{K})} \approx 0.3286\ \text{kg/m}^3$$

Now mass is *mass*, no matter where you are. Therefore, on the moon or wherever,

$$m_{He} = \rho_{He}\upsilon = (0.3286\ \text{kg/m}^3)(0.9\ \text{m}^3) \approx \mathbf{0.296\ kg} \quad \textit{Ans.}\ \text{(a,b)}$$

For part (c), we expand a constant mass isothermally from 0.9 to 1.5 m^3. The first law of thermodynamics gives

$$dQ_{added} - dW_{by\ gas} = dE = mc_v\Delta T = 0 \quad \text{since } T_2 = T_1 \ \text{(isothermal)}$$

Then the heat added equals the work of expansion. Estimate the work done:

$$W_{1\text{-}2} = \int_1^2 p\, d\upsilon = \int_1^2 \frac{m}{\upsilon} RT\, d\upsilon = mRT \int_1^2 \frac{d\upsilon}{\upsilon} = mRT \ln(\upsilon_2/\upsilon_1),$$

$$\text{or:}\quad W_{1\text{-}2} = (0.296\ \text{kg})(2077\ \text{J/kg}\cdot\text{K})(293\ \text{K})\ln(1.5/0.9) = Q_{1\text{-}2} \approx \mathbf{92000\ J} \quad \textit{Ans.}\ \text{(c)}$$

1.26 A tire has a volume of 3.0 ft^3 and a 'gage' pressure of 32 psi at 75°F. If the ambient pressure is sea-level standard, what is the weight of air in the tire?

Solution: Convert the temperature from 75°F to 535°R. Convert the pressure to psf:

$$p = (32\ \text{lbf/in}^2)(144\ \text{in}^2/\text{ft}^2) + 2116\ \text{lbf/ft}^2 = 4608 + 2116 \approx 6724\ \text{lbf/ft}^2$$

From this compute the density of the air in the tire:

$$\rho_{air} = \frac{p}{RT} = \frac{6724\ \text{lbf/ft}^2}{(1717\ \text{ft}\cdot\text{lbf/slug}\cdot{}^\circ\text{R})(535^\circ\text{R})} = 0.00732\ \text{slug/ft}^3$$

Then the total weight of air in the tire is

$$W_{air} = \rho g \upsilon = (0.00732\ \text{slug/ft}^3)(32.2\ \text{ft/s}^2)(3.0\ \text{ft}^3) \approx \mathbf{0.707\ lbf} \quad \textit{Ans.}$$

1.27 Given temperature and specific volume data for steam at 40 psia [Ref. 13]:

T, °F:	400	500	600	700	800
v, ft^3/lbm:	12.624	14.165	15.685	17.195	18.699

Is the ideal gas law reasonable for this data? If so, find a least-squares value for the gas constant R in $m^2/(s^2 \cdot K)$ and compare with Table A-4.

Solution: The units are awkward but we can compute R from the data. At 400°F,

$$\text{"R"}_{400°F} = \frac{pV}{T} = \frac{(40\ \text{lbf/in}^2)(144\ \text{in}^2/\text{ft}^2)(12.624\ \text{ft}^3/\text{lbm})(32.2\ \text{lbm/slug})}{(400+459.6)°R} \approx 2721\ \frac{\text{ft·lbf}}{\text{slug°R}}$$

The metric conversion factor, from the inside cover of the text, is "5.9798": R_{metric} = 2721/5.9798 = 455.1 $m^2/(s^2 \cdot K)$. Not bad! This is only 1.3% less than the ideal-gas approximation for steam in Table A-4: **<u>461</u>** $m^2/(s^2 \cdot K)$. Let's try all the five data points:

T, °F:	400	500	600	700	800
R, $m^2/(s^2 \cdot K)$:	455	457	459	460	460

The total variation in the data is only ±0.6%. Therefore steam *is* nearly an ideal gas in this (high) temperature range and for this (low) pressure. We can take an average value:

$$p = 40\ \text{psia},\ 400°F \le T \le 800°F: \quad \mathbf{R_{steam}} \approx \frac{1}{5}\sum_{i=1}^{5} R_i \approx \mathbf{458\ \frac{J}{kg \cdot K} \pm 0.6\%} \quad \textit{Ans.}$$

With such a small uncertainty, we don't really *need* to perform a least-squares analysis, but if we wanted to, it would go like this: We wish to minimize, for all data, the sum of the squares of the deviations from the perfect-gas law:

$$\text{Minimize } E = \sum_{i=1}^{5}\left(R - \frac{pV_i}{T_i}\right)^2 \quad \text{by differentiating } \frac{\partial E}{\partial R} = 0 = \sum_{i=1}^{5} 2\left(R - \frac{pV_i}{T_i}\right)$$

$$\text{Thus } R_{\text{least-squares}} = \frac{p}{5}\sum_{i=1}^{5}\frac{V_i}{T_i} = \frac{40(144)}{5}\left[\frac{12.624}{860°R} + \cdots + \frac{18.699}{1260°R}\right](32.2)$$

For this example, then, least-squares amounts to summing the (V/T) values and converting the units. The English result shown above gives $R_{least\text{-}squares} \approx 2739$ ft·lbf/slug·°R. Convert this to metric units for our (highly accurate) least-squares estimate:

$$R_{steam} \approx 2739/5.9798 \approx \mathbf{458 \pm 0.6\%\ J/kg{\cdot}K} \quad \textit{Ans.}$$

1.28 Wet air, at 100% relative humidity, is at 40°C and 1 atm. Using Dalton's law of partial pressures, compute the density of this wet air and compare with dry air.

Solution: Change T from 40°C to 313 K. Dalton's law of partial pressures is

$$p_{tot} = 1\text{ atm} = p_{air} + p_{water} = \frac{m_a}{\upsilon}R_aT + \frac{m_w}{\upsilon}R_wT$$

$$\text{or:}\quad m_{tot} = m_a + m_w = \frac{p_a\upsilon}{R_aT} + \frac{p_w\upsilon}{R_wT} \quad \text{for an ideal gas}$$

where, from Table A-4, $R_{air} = 287$ and $R_{water} = 461$ $m^2/(s^2{\cdot}K)$. Meanwhile, from Table A-5, at 40°C, the vapor pressure of saturated (100% humid) water is 7375 Pa, whence the partial pressure of the air is $p_a = 1\text{ atm} - p_w = 101350 - 7375 = 93975$ Pa.

Solving for the mixture density, we obtain

$$\rho = \frac{m_a + m_w}{\upsilon} = \frac{p_a}{R_aT} + \frac{p_w}{R_wT} = \frac{93975}{287(313)} + \frac{7375}{461(313)} = 1.046 + 0.051 \approx \mathbf{1.10\ \frac{kg}{m^3}} \quad \textit{Ans.}$$

By comparison, the density of dry air for the same conditions is

$$\rho_{dry\ air} = \frac{p}{RT} = \frac{101350}{287(313)} = 1.13\ \frac{kg}{m^3}$$

Thus, at 40°C, wet, 100% humidity, air is *lighter* than dry air, by about **2.7%**.

1.29 A tank holds 5 ft^3 of air at 20°C and 120 psi (gage). Estimate the energy in ft-lbf required to compress this air isothermally from one atmosphere (14.7 psia = 2116 psfa).

Solution: Integrate the work of compression, assuming an ideal gas:

$$W_{1\text{-}2} = -\int_1^2 p\,d\upsilon = -\int_1^2 \frac{mRT}{\upsilon}d\upsilon = -mRT\ln\left(\frac{\upsilon_2}{\upsilon_1}\right) = p_2\upsilon_2\ln\left(\frac{p_2}{p_1}\right)$$

where the latter form follows from the ideal gas law for isothermal changes. For the given numerical data, we obtain the quantitative work done:

$$W_{1\text{-}2} = p_2 \upsilon_2 \ln\left(\frac{p_2}{p_1}\right) = \left(134.7 \times 144 \ \frac{\text{lbf}}{\text{ft}^2}\right)(5\ \text{ft}^3)\ \ln\left(\frac{134.7}{14.7}\right) \approx \mathbf{215{,}000\ ft{\cdot}lbf} \quad \textit{Ans.}$$

1.30 Repeat Prob. 1.29 if the tank is filled with compressed *water* rather than air. Why is the result thousands of times less than the result of 215,000 ft·lbf in Prob. 1.29?

Solution: First evaluate the density change of water. At 1 atm, $\rho_o \approx 1.94\ \text{slug/ft}^3$. At 120 psi(gage) = 134.7 psia, the density would rise slightly according to Eq. (1.22):

$$\frac{p}{p_o} = \frac{134.7}{14.7} \approx 3001\left(\frac{\rho}{1.94}\right)^7 - 3000, \quad \text{solve } \rho \approx 1.940753\ \text{slug/ft}^3,$$

$$\text{Hence } m_{water} = \rho\upsilon = (1.940753)(5\ \text{ft}^3) \approx 9.704\ \text{slug}$$

The density change is extremely small. Now the work done, as in Prob. 1.29 above, is

$$W_{1\text{-}2} = -\int_1^2 p\,d\upsilon = \int_1^2 p\,d\left(\frac{m}{\rho}\right) = \int_1^2 p\ \frac{m\,d\rho}{\rho^2} \approx p_{avg} m \frac{\Delta\rho}{\rho_{avg}^2} \quad \text{for a linear pressure rise}$$

$$\text{Hence } W_{1\text{-}2} \approx \left(\frac{14.7 + 134.7}{2} \times 144\ \frac{\text{lbf}}{\text{ft}^2}\right)(9.704\ \text{slug})\left(\frac{0.000753}{1.9404^2}\ \frac{\text{ft}^3}{\text{slug}}\right) \approx \mathbf{21\ ft{\cdot}lbf} \quad \textit{Ans.}$$

[Exact integration of Eq. (1.22) would give the same numerical result.] Compressing water (extremely small $\Delta\rho$) takes *ten thousand times less energy* than compressing air, which is why it is safe to test high-pressure systems with water but dangerous with air.

P1.31 One cubic foot of argon gas at 10°C and 1 atm is compressed isentropically to a new pressure of 601 kPa. (*a*) What will be its new density and temperature? (*b*) If allowed to cool, at this new volume, back to 10°C, what will be the final pressure? Assume constant specific heats.

Solution: This is an exercise in having students recall their thermodynamics. From Table A.4, for argon gas, $R = 208\ m^2/(s^2\text{-}K)$ and $k = 1.67$. Note $T_1 = 283K$. First compute the initial density:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{101350\,N/m^2}{(208\,m^2/s^2\cdot K)(283K)} = 1.72\ kg/m^3$$

For an isentropic process at constant *k*,

$$\frac{p_2}{p_1} = \frac{601{,}000\,Pa}{101{,}350\,Pa} = 5.93 = \left(\frac{\rho_2}{\rho_1}\right)^k = \left(\frac{\rho_2}{1.72}\right)^{1.67},\ \text{Solve}\ \ \rho_2 = \mathbf{5.00\ kg/m^3}\ \ \textit{Ans.}(a)$$

$$\frac{p_2}{p_1} = 5.93 = \left(\frac{T_2}{T_1}\right)^{k/(k-1)} = \left(\frac{T_2}{283K}\right)^{1.67/0.67},\ \text{Solve}\ \ T_2 = \mathbf{578\,K} = 305^\circ C\ \ \textit{Ans.}(a)$$

(*b*) Cooling at constant volume means ρ stays the same and the new temperature is 283K. Thus

$$p_3 = \rho_3 RT_3 = \left(5.00\frac{kg}{m^3}\right)\left(208\frac{m^2}{s^2K}\right)(283K) = 294{,}000\,Pa = \mathbf{294\ kPa}\quad \textit{Ans.}(b)$$

1.32 A blimp is approximated by a prolate spheroid 90 m long and 30 m in diameter. Estimate the weight of 20°C gas within the blimp for (a) helium at 1.1 atm; and (b) air at 1.0 atm. What might the difference between these two values represent (Chap. 2)?

Solution: Find a handbook. The volume of a prolate spheroid is, for our data,

$$\upsilon = \frac{2}{3}\pi LR^2 = \frac{2}{3}\pi(90\text{ m})(15\text{ m})^2 \approx 42412\text{ m}^3$$

Estimate, from the ideal-gas law, the respective densities of helium and air:

$$\text{(a)}\quad \rho_{helium} = \frac{p_{He}}{R_{He}T} = \frac{1.1(101350)}{2077(293)} \approx 0.1832\ \frac{kg}{m^3};$$

$$\text{(b)}\quad \rho_{air} = \frac{p_{air}}{R_{air}T} = \frac{101350}{287(293)} \approx 1.205\ \frac{kg}{m^3}.$$

Then the respective gas weights are

$$W_{He} = \rho_{He}g\upsilon = \left(0.1832\ \frac{kg}{m^3}\right)\left(9.81\ \frac{m}{s^2}\right)(42412\text{ m}^3) \approx \mathbf{76000\ N}\quad \textit{Ans.}\text{ (a)}$$

$$W_{air} = \rho_{air}g\upsilon = (1.205)(9.81)(42412) \approx \mathbf{501000\ N}\quad \textit{Ans.}\text{ (b)}$$

The difference between these two, **425000 N**, is the *buoyancy*, or lifting ability, of the blimp. [See Section 2.8 for the principles of buoyancy.]

1.33 Experimental data for density of mercury versus pressure at 20°C are as follows:

p, atm:	1	500	1000	1500	2000
ρ, kg/m^3:	13545	13573	13600	13625	13653

Fit this data to the empirical state relation for liquids, Eq. (1.19), to find the best values of *B* and *n* for mercury. Then, assuming the data are nearly isentropic, use these values to estimate the speed of sound of mercury at 1 atm and compare with Table 9.1.

Solution: This can be done (laboriously) by the method of least-squares, but we can also do it on a spreadsheet by guessing, say, $n \approx 4,5,6,7,8$ and finding the average *B* for each case. For this data, almost *any* value of $n > 1$ is reasonably accurate. We select:

$$\text{Mercury:} \quad \mathbf{n \approx 7}, \quad \mathbf{B \approx 35000 \pm 2\%} \quad \textit{Ans.}$$

The speed of sound is found by differentiating Eq. (1.19) and then taking the square root:

$$\frac{dp}{d\rho} \approx \frac{p_o}{\rho_o} n(B+1)\left(\frac{\rho}{\rho_o}\right)^{n-1}, \quad \text{hence } a\big|_{\rho=\rho_o} \approx \left[\frac{n(B+1)p_o}{\rho_o}\right]^{1/2}$$

it being assumed here that this equation of state is "isentropic." Evaluating this relation for mercury's values of *B* and *n*, we find the speed of sound at 1 atm:

$$a_{mercury} \approx \left[\frac{(7)(35001)(101350 \text{ N/m}^2)}{13545 \text{ kg/m}^3}\right]^{1/2} \approx \mathbf{1355 \text{ m/s}} \quad \textit{Ans.}$$

This is about 7% less than the value of 1450 m/s listed in Table 9.1 for mercury.

1.34 Consider steam at the following state near the saturation line: $(p_1, T_1) = (1.31$ MPa, 290°C). Calculate and compare, for an ideal gas (Table A.4) and the Steam Tables (or the EES software), (a) the density ρ_1; and (b) the density ρ_2 if the steam expands isentropically to a new pressure of 414 kPa. Discuss your results.

Solution: From Table A.4, for steam, $k \approx 1.33$, and $R \approx 461$ m^2/(s^2·K). Convert T_1 = 563 K. Then,

$$\rho_1 = \frac{p_1}{RT_1} = \frac{1{,}310{,}000\ Pa}{(461\ m^2/s^2K)(563\ K)} = \mathbf{5.05}\ \frac{kg}{m^3} \quad \textit{Ans.}\ \text{(a)}$$

$$\frac{\rho_2}{\rho_1} = \frac{\rho_2}{5.05} = \left(\frac{p_2}{p_1}\right)^{1/k} = \left(\frac{414\ kPa}{1310\ kPa}\right)^{1/1.33} = 0.421, \quad or: \quad \rho_2 = \mathbf{2.12}\ \frac{kg}{m^3} \quad Ans.\ \text{(b)}$$

For EES or the Steam Tables, just program the properties for steam or look it up:

$$\text{EES real steam:} \quad \rho_1 = \mathbf{5.23}\ \text{kg/m}^3 \quad Ans.\ \text{(a)}, \qquad \rho_2 = \mathbf{2.16}\ \text{kg/m}^3 \quad Ans.\ \text{(b)}$$

The ideal-gas error is only about 3%, even though the expansion approached the saturation line.

1.35 In Table A-4, most common gases (air, nitrogen, oxygen, hydrogen, CO, NO) have a specific heat ratio $k = 1.40$. Why do argon and helium have such high values? Why does NH_3 have such a low value? What is the lowest k for any gas that you know?

Solution: In elementary kinetic theory of gases [8], k is related to the number of "degrees of freedom" of the gas: $k \approx 1 + 2/N$, where N is the number of different modes of translation, rotation, and vibration possible for the gas molecule.

Example: Monotomic gas, N = 3 (translation only), thus $k \approx 5/3$

This explains why helium and argon, which are monatomic gases, have $k \approx 1.67$.

Example: Diatomic gas, N = 5 (translation plus 2 rotations), thus $k \approx 7/5$

This explains why air, nitrogen, oxygen, NO, CO and hydrogen have $k \approx 1.40$.

But NH_3 has *four* atoms and therefore more than 5 degrees of freedom, hence k will be less than 1.40 (the theory is not too clear what "N" is for such complex molecules).

The lowest k known to this writer is for *uranium hexafluoride*, $^{238}UF_6$, which is a very complex, heavy molecule with many degrees of freedom. The estimated value of k for this heavy gas is $k \approx \mathbf{1.06}$.

1.36 The *isentropic bulk modulus* of a fluid is defined as $B = \rho(\partial p/\partial \rho)_S$. What are the dimensions of B? Estimate B (in Pa) for (a) N_2O, and (b) water, at 20°C and 1 atm.

Solution: The density units cancel in the definition of B and thus its dimensions are the same as pressure or stress:

$$\{B\} = \{p\} = \{F/L^2\} = \left\{\frac{\mathbf{M}}{\mathbf{LT^2}}\right\} \quad Ans.$$

(a) For an ideal gas, $p = C\rho^k$ for an isentropic process, thus the bulk modulus is:

$$\text{Ideal gas:} \quad B = \rho\frac{d}{d\rho}(C\rho^k) = \rho k C\rho^{k-1} = kC\rho^k = \mathbf{kp}$$

For N_2O, from Table A-4, $k \approx 1.31$, so $B_{N_2O} = 1.31\text{ atm} = \mathbf{1.33E5\ Pa}$ *Ans.* (a)

For water at 20°C, we could just look it up in Table A-3, but we more usefully try to estimate B from the state relation (1-22). Thus, for a liquid, approximately,

$$B \approx \rho\frac{d}{d\rho}[p_o\{(B+1)(\rho/\rho_o)^n - B\}] = n(B+1)p_o(\rho/\rho_o)^n = n(B+1)p_o \quad \text{at 1 atm}$$

For water, $B \approx 3000$ and $n \approx 7$, so our estimate is

$$B_{water} \approx 7(3001)p_o = 21007\text{ atm} \approx \mathbf{2.13E9\ Pa} \quad \textit{Ans. (b)}$$

This is 2.7% less than the value B = 2.19E9 Pa listed in Table A-3.

1.37 A near-ideal gas has M = 44 and c_V = 610 J/(kg·K). At 100°C, what are (a) its specific heat ratio, and (b) its speed of sound?

Solution: The gas constant is $R = \Lambda/M = 8314/44 \approx 189$ J/(kg·K). Then

$$c_v = R/(k-1), \quad \text{or:} \quad k = 1 + R/c_v = 1 + 189/610 \approx \mathbf{1.31} \quad \textit{Ans. (a)} \quad [\text{It is probably } N_2O]$$

With k and R known, the speed of sound at 100ºC = 373 K is estimated by

$$a = \sqrt{kRT} = \sqrt{1.31[189\ m^2/(s^2\cdot K)](373\ K)} \approx \mathbf{304\ m/s} \quad \textit{Ans. (b)}$$

1.38 In Fig. P1.38, if the fluid is glycerin at 20°C and the width between plates is 6 mm, what shear stress (in Pa) is required to move the upper plate at V = 5.5 m/s? What is the flow Reynolds number if "L" is taken to be the distance between plates?

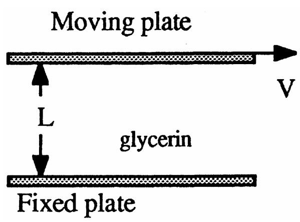


Fig. P1.38

Solution: (a) For glycerin at 20°C, from Table 1.4, $\mu \approx 1.5\ N\cdot s/m^2$. The shear stress is found from Eq. (1) of Ex. 1.8:

$$\tau = \frac{\mu V}{h} = \frac{(1.5\ Pa\cdot s)(5.5\ m/s)}{(0.006\ m)} \approx \mathbf{1380\ Pa} \quad \textit{Ans. (a)}$$

The density of glycerin at 20°C is 1264 kg/m^3. Then the Reynolds number is defined by Eq. (1.24), with L = h, and is found to be decidedly laminar, Re < 1500:

$$Re_L = \frac{\rho VL}{\mu} = \frac{(1264\ kg/m^3)(5.5\ m/s)(0.006\ m)}{1.5\ kg/m\cdot s} \approx \mathbf{28} \quad \textit{Ans. (b)}$$

1.39 Knowing $\mu \approx 1.80E{-}5\ \text{Pa}\cdot\text{s}$ for air at 20°C from Table 1-4, estimate its viscosity at 500°C by (a) the Power-law, (b) the Sutherland law, and (c) the Law of Corresponding States, Fig. 1.5. Compare with the accepted value $\mu(500°C) \approx 3.58E{-}5\ \text{Pa}\cdot\text{s}$.

Solution: First change T from 500°C to 773 K. (a) For the Power-law for air, n ≈ 0.7, and from Eq. (1.30a),

$$\mu = \mu_o (T/T_o)^n \approx (1.80E{-}5)\left(\frac{773}{293}\right)^{0.7} \approx \mathbf{3.55E{-}5\ \frac{kg}{m\cdot s}} \quad \textit{Ans. (a)}$$

This is less than 1% low. (b) For the Sutherland law, for air, S ≈ 110 K, and from Eq. (1.30b),

$$\mu = \mu_o\left[\frac{(T/T_o)^{1.5}(T_o+S)}{(T+S)}\right] \approx (1.80E{-}5)\left[\frac{(773/293)^{1.5}(293+110)}{(773+110)}\right]$$

$$= \mathbf{3.52E{-}5\ \frac{kg}{m\cdot s}} \quad \textit{Ans. (b)}$$

This is only 1.7% low. (c) Finally use Fig. 1.5. Critical values for air from Ref. 3 are:

$$\text{Air:}\quad \mu_c \approx 1.93E{-}5\ \text{Pa·s} \quad T_c \approx 132\ \text{K} \quad (\text{"mixture" estimates})$$

At 773 K, the temperature ratio is $T/T_c = 773/132 \approx 5.9$. From Fig. 1.5, read $\mu/\mu_c \approx 1.8$. Then our critical-point-correlation estimate of air viscosity is only 3% low:

$$\mu \approx 1.8\mu_c = (1.8)(1.93E{-}5) \approx \mathbf{3.5E{-}5\ \frac{kg}{m\cdot s}} \quad \textit{Ans. (c)}$$

1.40 Curve-fit the viscosity data for water in Table A-1 in the form of Andrade's equation,

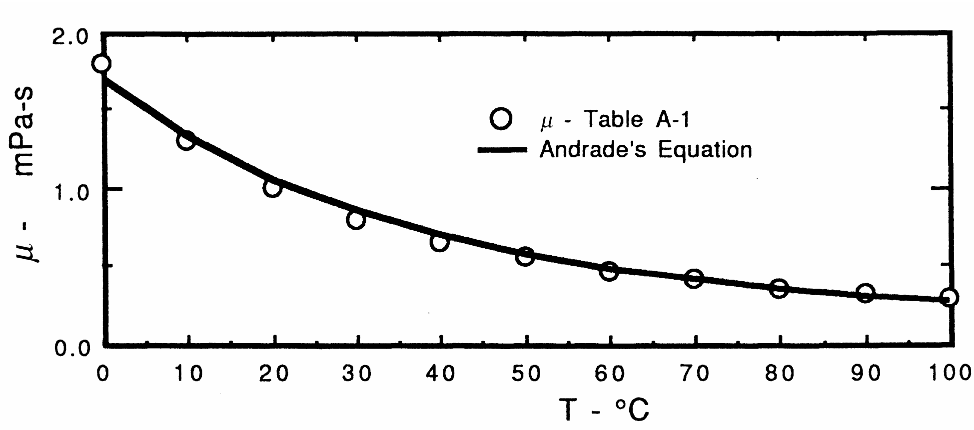
$$\mu \approx A\exp\left(\frac{B}{T}\right) \quad \text{where T is in °K and A and B are curve-fit constants.}$$

Solution: This is an alternative formula to the log-quadratic law of Eq. (1.31). We have eleven data points for water from Table A-1 and can perform a least-squares fit to Andrade's equation:

$$\text{Minimize}\quad E = \sum_{i=1}^{11}[\mu_i - A\exp(B/T_i)]^2, \quad \text{then set}\quad \frac{\partial E}{\partial A} = 0 \quad\text{and}\quad \frac{\partial E}{\partial B} = 0$$

The result of this minimization is: **A ≈ 0.0016 kg/m·s, B ≈ 1903°K.** *Ans.*

The data and the Andrade's curve-fit are plotted. The error is ±7%, so Andrade's equation is not as accurate as the log-quadratic correlation of Eq. (1.31).



1.41 Some experimental values of μ for argon gas at 1 atm are as follows:

T, °K:	300	400	500	600	700	800
μ, kg/m·s:	2.27E–5	2.85E–5	3.37E–5	3.83E–5	4.25E–5	4.64E–5

Fit these values to either (a) a Power-law, or (b) a Sutherland law, Eq. (1.30a,b).

Solution: (a) The Power-law is straightforward: put the values of μ and T into, say, "Cricket Graph," take logarithms, plot them, and make a linear curve-fit. The result is:

$$\text{Power-law fit:}\quad \mu \approx \mathbf{2.29E\text{–}5}\left(\frac{\mathbf{T\,°K}}{\mathbf{300\ K}}\right)^{\mathbf{0.73}} \qquad \textit{Ans.}\text{(a)}$$

Note that the constant "2.29E–5" is slightly higher than the actual viscosity "2.27E–5" at T = 300 K. The accuracy is ±1% and would be poorer if we replaced 2.29E–5 by 2.27E–5.

(b) For the Sutherland law, unless we rewrite the law (1.30b) drastically, we don't have a simple way to perform a linear least-squares correlation. However, it is no trouble to perform the least-squares summation, $E = \Sigma[\mu_i - \mu_o(T_i/300)^{1.5}(300 + S)/(T_i + S)]^2$ and minimize by setting $\partial E/\partial S = 0$. We can try μ_o = 2.27E–5 kg/m·s for starters, and it works fine. The best-fit value of S ≈ 143°K with negligible error. Thus the result is:

$$\text{Sutherland law:}\quad \frac{\mu}{\mathbf{2.27E\text{–}5\ kg/m\cdot s}} \approx \frac{\mathbf{(T/300)^{1.5}(300+143\ K)}}{\mathbf{(T+143\ K)}} \qquad \textit{Ans.}\text{ (b)}$$

We may tabulate the data and the two curve-fits as follows:

T, °K:	300	400	500	600	700	800
$\mu \times$ E5, data:	2.27	2.85	3.37	3.83	4.25	4.64
$\mu \times$ E5, Power-law:	2.29	2.83	3.33	3.80	4.24	4.68
$\mu \times$ E5, Sutherland:	2.27	2.85	3.37	3.83	4.25	4.64

1.42 Some experimental values of μ of helium at 1 atm are as follows:

T, °K:	200	400	600	800	1000	1200
μ, kg/m·s:	1.50E–5	2.43E–5	3.20E–5	3.88E–5	4.50E–5	5.08E–5

Fit these values to either (a) a Power-law, or (b) a Sutherland law, Eq. (1.30a,b).

Solution: (a) The Power-law is straightforward: put the values of μ and T into, say, an Excel graph, take logarithms, plot them, and make a linear curve-fit. The result is:

$$\text{Power-law curve-fit:}\quad \mu_{He} \approx \mathbf{1.505E{-}5}\left(\frac{\mathbf{T\ °K}}{\mathbf{200\ K}}\right)^{\mathbf{0.68}} \qquad \textit{Ans. (a)}$$

The accuracy is less than ±1%. (b) For the Sutherland fit, we can emulate Prob. 1.41 and perform the least-squares summation, $E = \Sigma[\mu_i - \mu_o(T_i/200)^{1.5}(200 + S)/(T_i + S)]^2$ and minimize by setting $\partial E/\partial S = 0$. We can try μ_o = 1.50E–5 kg/m·s and T_o = 200°K for starters, and it works OK. The best-fit value of S ≈ 95.1°K. Thus the result is:

$$\text{Sutherland law:}\quad \frac{\mu_{Helium}}{\mathbf{1.50E{-}5\ kg/m{\cdot}s}} \approx \frac{\mathbf{(T/200)^{1.5}(200+95.1°K)}}{\mathbf{(T+95.1°K)}} \pm \mathbf{4\%} \qquad \textit{Ans. (b)}$$

For the complete range 200–1200°K, the Power-law is a better fit. The Sutherland law improves to ±1% if we drop the data point at 200°K.

P1.43 According to rarefied gas theory [30], the no-slip condition begins to fail in tube flow when the mean free path of the gas is as large as 0.005 times the tube diameter. Consider helium at 20°C (Table A.4) flowing in a tube of diameter 1 cm. Using the theory of Prob. P1.5 (which is correct, not "proposed"), find the helium pressure for which this no-slip failure begins.

Solution: From Table A.4, for helium, take $R = 2077\ m^2/(s^2\text{-}K)$ and μ = 1.97E-5 kg/m-s. The formula for mean free path is given in Prob. P1.5:

$$l \;=\; 1.26\,\frac{\mu}{\rho\sqrt{RT}} \;=\; 1.26\,\frac{\mu\sqrt{RT}}{p} \quad \text{from the perfect gas law}$$

We are given μ and that $T = 20°C = 293$ K and that $l = 0.005D_{tube}$. Substitute into the formula:

$$l \;=\; 0.005(0.01m) \;=\; 1.26\frac{1.97E-5\,kg/m-s}{p\sqrt{(2077m^2/s^2-K)(293K)}}$$

$$\textit{Solve for} \quad p \;=\; 387\,N/m^2 \;=\; \mathbf{387\,Pa} \qquad \textit{Ans.}$$

If p is larger than this, we can neglect *slip* in this tube. Note that l is small, only 50 μm.

1.44 The viscosity of SAE 30 oil may vary considerably, according to industry-agreed specifications [SAE Handbook, Ref. 26]. Comment on the following data and fit the data to Andrade's equation from Prob. 1.41.

T, °C:	0	20	40	60	80	100
μSAE30,kg/m·s:	2.00	0.40	0.11	0.042	0.017	0.0095

Solution: At lower temperatures, 0°C < T < 60°C, these values are up to fifty per cent higher than the curve labelled "SAE 30 Oil" in Fig. A-1 of the Appendix. However, at 100°C, the value 0.0095 is within the range specified by SAE for this oil: $9.3 < \nu < 12.5$ mm^2/s, if its density lies in the range $760 < \rho < 1020$ kg/m^3, which it surely must. Therefore a surprisingly wide difference in viscosity-versus-temperature still makes an oil "*SAE 30.*" To fit Andrade's law, $\mu \approx A\exp(B/T)$, we must make a least-squares fit for the 6 data points above (just as we did in Prob. 1.41):

$$\text{Andrade fit:}\quad \text{With } E = \sum_{i=1}^{6}\left[\mu_i - A\exp\left(\frac{B}{T_i}\right)\right]^2, \quad \text{then set } \frac{\partial E}{\partial A} = 0 \quad \text{and} \quad \frac{\partial E}{\partial B} = 0$$

This formulation produces the following results:

$$\text{Least-squares of } \mu \text{ versus T:}\quad \boldsymbol{\mu \approx 2.35E\text{–}10\,\frac{kg}{m\cdot s}\,\exp\left(\frac{6245\,K}{T°K}\right)} \qquad \textit{Ans.}\ \textbf{(\#1)}$$

These results (#1) are pretty *terrible*, errors of ±50%, even though they are "least-squares." The reason is that μ varies over three orders of magnitude, so the fit is biased to *higher* μ.

An alternate fit to Andrade's equation would be to plot ln(μ) versus 1/T (°K) on, say, an Excel graph, and then fit the resulting near straight line by least squares. The result is:

$$\text{Least-squares of } \ln(\mu) \text{ versus } \frac{1}{T}: \quad \boldsymbol{\mu \approx 3.31E\text{–}9\ \frac{kg}{m \cdot s}\ \exp\left(\frac{5476\ K}{T°K}\right)} \quad \textit{Ans. (\#2)}$$

The accuracy is somewhat better, but not great, as follows:

T, °C:	0	20	40	60	80	100
μSAE30, kg/m·s:	2.00	0.40	0.11	0.042	0.017	0.0095
Curve-fit #1:	2.00	0.42	0.108	0.033	0.011	0.0044
Curve-fit #2:	1.68	0.43	0.13	0.046	0.018	0.0078

Neither fit is worth writing home about. Andrade's equation is not accurate for SAE 30 oil.

1.45 A block of weight W slides down an inclined plane on a thin film of oil, as in Fig. P1.45 at right. The film contact area is A and its thickness h. Assuming a linear velocity distribution in the film, derive an analytic expression for the terminal velocity V of the block.

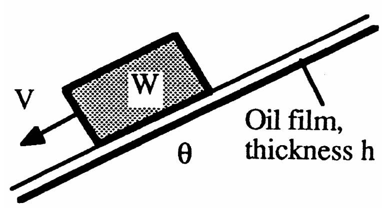


Fig. P1.45

Solution: Let "x" be down the incline, in the direction of V. By "terminal" velocity we mean that there is no acceleration. Assume a linear viscous velocity distribution in the film below the block. Then a force balance in the x direction gives:

$$\sum F_x = W \sin\theta - \tau A = W \sin\theta - \left(\mu \frac{V}{h}\right) A = ma_x = 0,$$

$$\text{or:} \quad V_{terminal} = \boldsymbol{\frac{hW \sin\theta}{\mu A}} \quad \textit{Ans.}$$

P1.46 A simple and popular model for two non-newtonian fluids in Fig. 1.9*a* is the *power-law*:

$$\tau \approx C\left(\frac{du}{dy}\right)^n$$

where C and n are constants fit to the fluid [15]. From Fig. 1.9*a*, deduce the values of the exponent n for which the fluid is (*a*) newtonian; (*b*) dilatant; and (*c*) pseudoplastic. (*d*) Consider the specific model constant $C = 0.4$ N-s^n/m^2, with the fluid being sheared between two parallel plates as in Fig. 1.8. If the shear stress in the fluid is 1200 Pa, find the velocity V of the upper plate for the cases (*d*) $n = 1.0$; (*e*) $n = 1.2$; and (*f*) $n = 0.8$.

Solution: By comparing the behavior of the model law with Fig. 1.9*a*, we see that
(*a*) Newtonian: $n = 1$; (*b*) Dilatant: $n > 1$; (*c*) Pseudoplastic: $n < 1$ *Ans*.(*a,b,c*)
From the discussion of Fig. 1.8, it is clear that the shear stress is *constant* in a fluid sheared between two plates. The velocity profile remains a straight line (if the flow is laminar), and the strain rate $du/dy = V/h$. Thus, for this flow, the model becomes $\tau = C(V/h)^n$. For the three given numerical cases, we calculate:

$$(d)\ n = 1:\ \tau = 1200\frac{N}{m^2} = C(V/h)^n = (0.4\frac{N-s^1}{m^2})(\frac{V}{0.001m})^1,\ \text{solve}\ V = \mathbf{3.0\frac{m}{s}}\quad Ans.(d)$$

$$(e)\ n = 1.2:\ \tau = 1200\frac{N}{m^2} = C(V/h)^n = (0.4\frac{N-s^{1.2}}{m^2})(\frac{V}{0.001m})^{1.2},\ \text{solve}\ V = \mathbf{0.79\frac{m}{s}}\quad Ans.(e)$$

$$(f)\ n = 0.8:\ \tau = 1200\frac{N}{m^2} = C(V/h)^n = (0.4\frac{N-s^{0.8}}{m^2})(\frac{V}{0.001m})^{0.8},\ \text{solve}\ V = \mathbf{22\frac{m}{s}}\quad Ans.(f)$$

A small change in the exponent *n* can sharply change the numerical values.

1.47 A shaft 6.00 cm in diameter and 40 cm long is pulled steadily at V = 0.4 m/s through a sleeve 6.02 cm in diameter. The clearance is filled with oil, $\nu = 0.003\ m^2/s$ and SG = 0.88. Estimate the force required to pull the shaft.

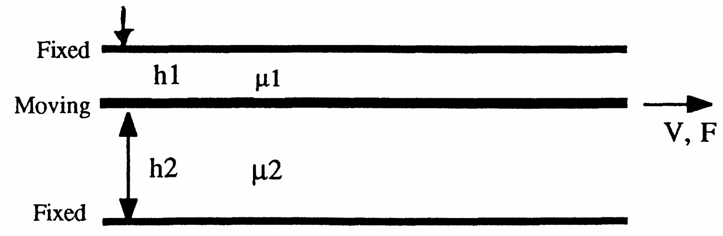
Solution: Assuming a linear velocity distribution in the clearance, the force is balanced by resisting shear stress in the oil:

$$F = \tau A_{wall} = \left(\mu\frac{V}{\Delta R}\right)(\pi D_i L) = \frac{\mu V \pi D_i L}{R_o - R_i}$$

For the given oil, $\mu = \rho\nu = (0.88 \times 998\ kg/m^3)(0.003\ m^2/s) \approx 2.63\ N\cdot s/m$ (or $kg/m\cdot s$). Then we substitute the given numerical values to obtain the force:

$$F = \frac{\mu V \pi D_i L}{R_o - R_i} = \frac{(2.63\ N\cdot s/m^2)(0.4\ m/s)\pi(0.06\ m)(0.4\ m)}{(0.0301 - 0.0300\ m)} \approx \mathbf{795\ N}\quad Ans.$$

1.48 A thin moving plate is separated from two fixed plates by two fluids of unequal viscosity and unequal spacing, as shown below. The contact area is A. Determine (a) the force required, and (b) is there a necessary relation between the two viscosity values?



Solution: (a) Assuming a linear velocity distribution on each side of the plate, we obtain

$$F = \tau_1 A + \tau_2 A = \left(\frac{\mu_1 V}{h_1} + \frac{\mu_2 V}{h_2}\right) A \quad \textit{Ans. (a)}$$

The formula is of course valid only for laminar (nonturbulent) steady viscous flow.
(b) Since the center plate separates the two fluids, they may have separate, unrelated shear stresses, and there is *no necessary relation* between the two viscosities.

1.49 An amazing number of commercial and laboratory devices have been developed to measure fluid viscosity, as described in Ref. 27. Consider a concentric shaft, as in Prob. 1.47, but now fixed axially and rotated inside the sleeve. Let the inner and outer cylinders have radii r_i and r_o, respectively, with total sleeve length L. Let the rotational rate be Ω (rad/s) and the applied torque be M. Using these parameters, derive a theoretical relation for the viscosity μ of the fluid between the cylinders.

Solution: Assuming a linear velocity distribution in the annular clearance, the shear stress is

$$\tau = \mu \frac{\Delta V}{\Delta r} \approx \mu \frac{\Omega r_i}{r_o - r_i}$$

This stress causes a force $dF = \tau\, dA = \tau(r_i\, d\theta)L$ on each element of surface area of the inner shaft. The moment of this force about the shaft axis is $dM = r_i\, dF$. Put all this together:

$$M = \int r_i\, dF = \int_0^{2\pi} r_i \mu \frac{\Omega r_i}{r_o - r_i} r_i\, L\, d\theta = \frac{2\pi\mu\Omega r_i^3 L}{r_o - r_i}$$

Solve for the viscosity: $\mu \approx M(r_o - r_i)/\{2\pi\Omega r_i^3 L\}$ *Ans.*

1.50 A simple viscometer measures the time t for a solid sphere to fall a distance L through a test fluid of density ρ. The fluid viscosity μ is then given by

$$\mu \approx \frac{W_{net} t}{3\pi D L} \quad if \quad t \geq \frac{2\rho D L}{\mu}$$

where D is the sphere diameter and W_{net} is the sphere net weight in the fluid.
(a) Show that both of these formulas are dimensionally homogeneous. (b) Suppose that a 2.5 mm diameter aluminum sphere (density 2700 kg/m^3) falls in an oil of density 875 kg/m^3. If the time to fall 50 cm is 32 s, estimate the oil viscosity and verify that the inequality is valid.

Solution: (a) Test the dimensions of each term in the two equations:

$$\{\mu\} = \left\{\frac{M}{LT}\right\} \quad \text{and} \quad \left\{\frac{W_{net}\, t}{(3\pi)DL}\right\} = \left\{\frac{(ML/T^2)(T)}{(1)(L)(L)}\right\} = \left\{\frac{M}{LT}\right\} \quad \textbf{Yes}, \text{ dimensions OK.}$$

$$\{t\} = \{T\} \quad \text{and} \quad \left\{\frac{2\rho DL}{\mu}\right\} = \left\{\frac{(1)(M/L^3)(L)(L)}{M/LT}\right\} = \{T\} \quad \textbf{Yes}, \text{ dimensions OK.} \quad \textit{Ans. (a)}$$

(b) Evaluate the two equations for the data. We need the net weight of the sphere in the fluid:

$$W_{net} = (\rho_{sphere} - \rho_{fluid}) g (Vol)_{fluid} = (2700 - 875 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(\pi/6)(0.0025 \text{ m})^3$$
$$= 0.000146 \text{ N}$$

$$\text{Then } \mu = \frac{W_{net} t}{3\pi DL} = \frac{(0.000146\ N)(32\ s)}{3\pi(0.0025\ m)(0.5\ m)} = \mathbf{0.40}\ \frac{kg}{m \cdot s} \quad \textit{Ans. (b)}$$

$$\text{Check } t = 32\ s \text{ compared to } \frac{2\rho DL}{\mu} = \frac{2(875\ kg/m^3)(0.0025\ m)(0.5\ m)}{0.40\ kg/m \cdot s}$$
$$= 5.5\ s \quad \text{OK, } t \text{ is greater}$$

P1.51 An approximation for the boundary-layer shape in Figs. 1.6*b* and P1.51 is the formula

$$u(y) \approx U\sin\left(\frac{\pi y}{2\delta}\right), \quad 0 \le y \le \delta$$

where U is the stream velocity far from the wall and δ is the boundary layer thickness, as in Fig. P.151. If the fluid is helium at 20°C and 1 atm, and if U = 10.8 m/s and δ = 3 mm, use the formula to (*a*) estimate the wall shear stress τ_w in Pa; and (*b*) find the position in the boundary layer where τ is one-half of τ_w.

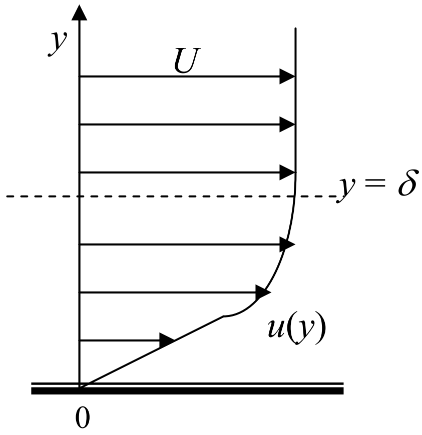


Fig. P1.51

Solution: From Table A.4, for helium, take $R = 2077\ m^2/(s^2\text{-}K)$ and $\mu = 1.97E\text{-}5$ kg/m-s. (*a*) Then the wall shear stress is calculated as
A very small shear stress, but it has a profound effect on the flow pattern.

$$\tau_w = \mu\frac{\partial u}{\partial y}\Big|_{y=0} = \mu\left(U\frac{\pi}{2\delta}\cos\frac{\pi y}{2\delta}\right)_{y=0} = \frac{\pi\,\mu U}{2\delta}$$

$$\text{Numerical values: } \tau_w = \frac{\pi(1.97E-5\,kg/m-s)(10.8\,m/s)}{2(0.003\,m)} = \mathbf{0.11\ Pa} \qquad Ans.(a)$$

(*b*) The variation of shear stress across the boundary layer is simply a cosine wave:

$$\tau(y) = \frac{\pi\mu U}{2\delta}\cos\left(\frac{\pi y}{2\delta}\right) = \tau_w\cos\left(\frac{\pi y}{2\delta}\right) = \frac{\tau_w}{2} \text{ when } \frac{\pi y}{2\delta} = \frac{\pi}{3}, \text{ or: } \mathbf{y} = \frac{\mathbf{2\delta}}{\mathbf{3}} \qquad Ans.(b)$$

1.52 The belt in Fig. P1.52 moves at steady velocity V and skims the top of a tank of oil of viscosity μ. Assuming a linear velocity profile, develop a simple formula for the belt-drive power P required as a function of (h, L, V, B, μ). Neglect air drag. What power P in watts is required if the belt moves at 2.5 m/s over SAE 30W oil at 20°C, with L = 2 m, b = 60 cm, and h = 3 cm?

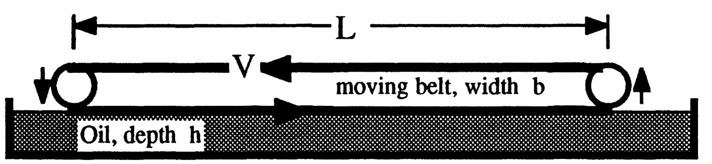


Fig. P1.52

Solution: The power is the viscous resisting force times the belt velocity:

$$P = \tau_{oil} A_{belt} V_{belt} \approx \left(\mu \frac{V}{h}\right)(bL)V = \boldsymbol{\mu V^2 b \frac{L}{h}} \quad \textit{Ans.}$$

(b) For SAE 30W oil, $\mu \approx 0.29$ kg/m·s. Then, for the given belt parameters,

$$P = \mu V^2 bL/h = \left(0.29\ \frac{kg}{m \cdot s}\right)\left(2.5\ \frac{m}{s}\right)^2 (0.6\ m)\frac{2.0\ m}{0.03\ m} \approx 73\ \frac{kg \cdot m^2}{s^3} = \mathbf{73\ W} \quad \textit{Ans.}\ (b)$$

1.53* A solid cone of base r_o and initial angular velocity ω_o is rotating inside a conical seat. Neglect air drag and derive a formula for the cone's angular velocity $\omega(t)$ if there is no applied torque.

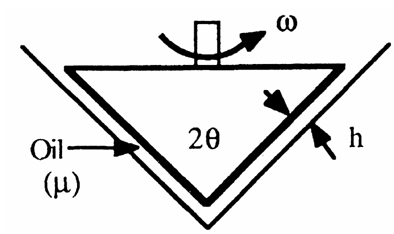


Fig. P1.53

Solution: At any radial position $r < r_o$ on the cone surface and instantaneous rate ω,

$$d(\text{Torque}) = r\tau\, dA_w = r\left(\mu \frac{r\omega}{h}\right)\left(2\pi r \frac{dr}{\sin\theta}\right),$$

$$\text{or:} \quad \text{Torque } M = \int_0^{r_o} \frac{\mu\omega}{h\sin\theta} 2\pi r^3\, dr = \frac{\pi\mu\omega r_o^4}{2h\sin\theta}$$

We may compute the cone's slowing down from the angular momentum relation:

$$M = -I_o \frac{d\omega}{dt}, \quad \text{where } I_o(\text{cone}) = \frac{3}{10} m r_o^2, \quad m = \text{cone mass}$$

Separating the variables, we may integrate:

$$\int_{\omega_o}^{w} \frac{d\omega}{\omega} = -\frac{\pi\mu r_o^4}{2hI_o \sin\theta}\int_0^t dt, \quad \text{or:} \quad \boldsymbol{\omega = \omega_o \exp\left[-\frac{5\pi\mu r_o^2 t}{3mh\sin\theta}\right]} \quad \textit{Ans.}$$

1.54* A disk of radius R rotates at angular velocity Ω inside an oil container of viscosity μ, as in Fig. P1.54. Assuming a linear velocity profile and neglecting shear on the outer disk edges, derive an expression for the viscous torque on the disk.

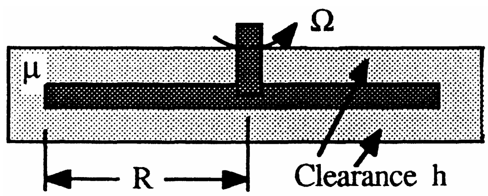


Fig. P1.54

Solution: At any r ≤ R, the viscous shear $\tau \approx \mu\Omega r/h$ on both sides of the disk. Thus,

$$d(\text{torque}) = dM = 2r\tau\, dA_w = 2r\frac{\mu\Omega r}{h}2\pi r\, dr,$$

$$\text{or:} \quad M = 4\pi\frac{\mu\Omega}{h}\int_0^R r^3\, dr = \boldsymbol{\frac{\pi\mu\Omega R^4}{h}} \quad \textit{Ans.}$$

P1.55 A block of weight *W* is being pulled over a table by another weight W_o, as shown in Fig. P1.55. Find an algebraic formula for the steady velocity *U* of the block if it slides on an oil film of thickness *h* and viscosity μ. The block bottom area *A* is in contact with the oil. Neglect the cord weight and the pulley friction.

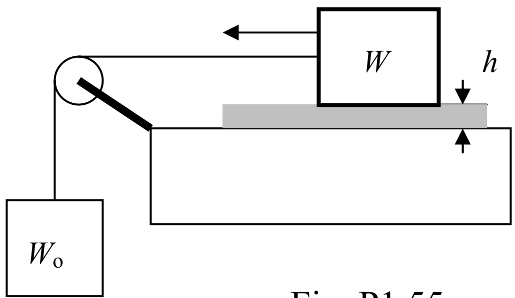


Fig. P1.55

Solution: This problem is a lot easier to *solve* than to set up and sketch. For steady motion, there is no acceleration, and the falling weight balances the viscous resistance of the oil film:

$$\sum F_{x,block} = 0 = \tau A - W_o = (\mu \frac{U}{h}) A - W_o$$

$$\text{Solve for} \quad U = \frac{W_o h}{\mu A} \qquad \textit{Ans.}$$

The block weight *W* has no effect on steady horizontal motion except to smush the oil film.

1.56* For the cone-plate viscometer in Fig. P1.56, the angle is very small, and the gap is filled with test liquid μ. Assuming a linear velocity profile, derive a formula for the viscosity μ in terms of the torque M and cone parameters.

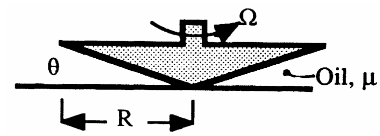


Fig. P1.56

Solution: For any radius r ≤ R, the liquid gap is h = r tanθ. Then

$$\text{d(Torque)} = \text{dM} = \tau\,\text{dA}_w \text{r} = \left(\mu\frac{\Omega \text{r}}{\text{r}\tan\theta}\right)\left(2\pi \text{r}\frac{\text{dr}}{\cos\theta}\right)\text{r}, \quad \text{or}$$

$$\text{M} = \frac{2\pi\Omega\mu}{\sin\theta}\int_0^R \text{r}^2\,\text{dr} = \frac{2\pi\Omega\mu\text{R}^3}{3\sin\theta}, \quad \text{or:} \quad \mu = \frac{\mathbf{3M\sin\theta}}{\mathbf{2\pi\Omega R^3}} \quad \textit{Ans.}$$

P1.57 For the geometry of Prob. P1.55, (*a*) solve the *unsteady* problem *U*(*t*) where the block starts from rest and accelerates toward the final steady velocity U_o of Prob. P1.55. (*b*) As a separate issue, if the table were instead *sloped* at an angle θ up toward the pulley, state the criterion for whether the block moves up or down the table.

Solution: (*a*) Sum forces to the left, with an acceleration to the left:

$$\sum F_{left} = W_o - \frac{\mu A}{h} U = (m\,a)_{block} = \frac{W}{g}\frac{dU}{dt},$$

$$\text{or:} \quad \frac{dU}{dt} + (\frac{\mu g A}{h W}) U = \frac{W_o}{W} g \qquad \text{subject to} \quad U = 0 \quad at \quad t = 0$$

This is a 1st-order linear differential equation, with the solution

$$U = \frac{W_o h}{\mu A}[1 - \exp(-\frac{\mu g A}{h W} t)] \qquad \textit{Ans.}(a)$$

The block begins at rest and exponentially approaches the final velocity ($W_o h/\mu A$) from P1.55.
(*b*) If the block slopes upward at angle θ, the driving force to move the block is $W_o - W\sin\theta$.
The block slides downward if $\mathbf{W\sin\theta > W_o}$. *Ans.*(*b*)

1.58 The laminar-pipe-flow example of Prob. 1.14 leads to a *capillary viscometer* [27], using the formula $\mu = \pi r_o^4 \Delta p/(8LQ)$. Given r_o = 2 mm and L = 25 cm. The data are

Q, m^3/hr:	0.36	0.72	1.08	1.44	1.80
Δp, kPa:	159	318	477	1274	1851

Estimate the fluid viscosity. What is wrong with the last two data points?

Solution: Apply our formula, with consistent units, to the first data point:

$$\Delta p = 159\,\text{kPa}: \quad \mu \approx \frac{\pi r_o^4 \Delta p}{8LQ} = \frac{\pi(0.002\text{ m})^4(159000\text{ N/m}^2)}{8(0.25\text{ m})(0.36/3600\text{ m}^3/\text{s})} \approx 0.040\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

Do the same thing for all five data points:

Δp, kPa:	159	318	477	1274	1851	
μ, N·s/m^2:	**0.040**	**0.040**	**0.040**	*0.080*(?)	*0.093*(?)	*Ans.*

The last two estimates, though measured properly, are *incorrect*. The Reynolds number of the capillary has risen above 2000 and the flow is turbulent, which requires a different formula.

1.59 A solid cylinder of diameter D, length L, density ρ_s falls due to gravity inside a tube of diameter D_o. The clearance, $(D_o - D) \ll D$, is filled with a film of viscous fluid (ρ,μ). Derive a formula for terminal fall velocity and apply to SAE 30 oil at 20°C for a steel cylinder with D = 2 cm, D_o = 2.04 cm, and L = 15 cm. Neglect the effect of any air in the tube.

Solution: The geometry is similar to Prob. 1.47, only vertical instead of horizontal. At terminal velocity, the cylinder weight should equal the viscous drag:

$$a_z = 0: \quad \Sigma F_z = -W + \text{Drag} = -\rho_s g \frac{\pi}{4} D^2 L + \left[\mu \frac{V}{(D_o - D)/2}\right]\pi DL,$$

$$\text{or:} \quad V = \frac{\boldsymbol{\rho_s g D (D_o - D)}}{\boldsymbol{8\mu}} \quad \textit{Ans.}$$

For the particular numerical case given, $\rho_{steel} \approx 7850$ kg/m^3. For SAE 30 oil at 20°C, $\mu \approx 0.29$ kg/m·s from Table 1.4. Then the formula predicts

$$V_{terminal} = \frac{\rho_s g D(D_o - D)}{8\mu} = \frac{(7850\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.02\text{ m})(0.0204 - 0.02\text{ m})}{8(0.29\text{ kg/m}\cdot\text{s})}$$

$$\approx \mathbf{0.265\ m/s} \quad \textit{Ans.}$$

P1.60 Pipelines are cleaned by pushing through them a close-fitting cylinder called a *pig*. The name comes from the squealing noise it makes sliding along. Ref. 52 describes a new non-toxic pig, driven by compressed air, for cleaning cosmetic and beverage pipes. Suppose the pig diameter is 5-15/16 in and its length 26 in. It cleans a 6-in-diameter pipe at a speed of 1.2 m/s. If the clearance is filled with glycerin at 20°C, what pressure difference, in pascals, is needed to drive the pig? Assume a linear velocity profile in the oil and neglect air drag.

Solution: Since the problem calls for *pascals*, convert everything to SI units:
Find the shear stress in the oil, multiply that by the cylinder wall area to get the required force, and divide the force by the area of the cylinder face to find the required pressure difference.

Comment: The Reynolds number of the clearance flow, Re = $\rho VC/\mu$, is approximately 0.8.

$$D_{cyl} = (5\tfrac{15}{16}\,in)(0.0254\tfrac{m}{in}) = 0.1508\,m \quad ; \quad L = (26\,in)(0.0254\tfrac{m}{in}) = 0.6604\,m$$

$$\text{Clearance} = C = (D_{pipe} - D_{cyl})/2 = (6 - 5\tfrac{15}{16}\,in)(0.0254\tfrac{m}{in})/2 = 0.000794\,m$$

$$\text{Table A.3, glycerin: } \mu = 1.49\ kg/m-s \ , \ \rho = 1260\ kg/m^3 \ (\rho \text{ not needed})$$

$$\text{Shear stress } \tau = \mu V/C = (1.49\ kg/m-s)(1.2\,m/s)/0.000794\,m = 2252\,N/m^2$$

$$\text{Shear force } F = \tau A_{wall} = \tau(\pi D_{cyl} L) =$$

$$(2252\,N/m^2)[\pi(0.1508\,m)(0.6604\,m)] = 705\,N$$

$$\text{Finally,} \quad \Delta p = \frac{F}{A_{cyl\,face}} = \frac{705\,N}{(\pi/4)(0.1508\,m)^2} = \mathbf{39500\ Pa} \quad \textit{Ans.}$$

1.61 An air-hockey puck has m = 50 g and D = 9 cm. When placed on a 20°C air table, the blower forms a 0.12-mm-thick air film under the puck. The puck is struck with an initial velocity of 10 m/s. How long will it take the puck to (a) slow down to 1 m/s; (b) stop completely? Also (c) how far will the puck have travelled for case (a)?

Solution: For air at 20°C take $\mu \approx$ 1.8E−5 kg/m·s. Let A be the bottom area of the puck, A = $\pi D^2/4$. Let x be in the direction of travel. Then the only force acting in the x direction is the air drag resisting the motion, assuming a linear velocity distribution in the air:

$$\sum F_x = -\tau A = -\mu \frac{V}{h} A = m\frac{dV}{dt}, \quad \text{where h = air film thickness}$$

Separate the variables and integrate to find the velocity of the decelerating puck:

$$\int_{V_o}^{V} \frac{dV}{V} = -K\int_0^t dt, \quad \text{or} \quad V = V_o e^{-Kt}, \quad \text{where } K = \frac{\mu A}{mh}$$

Integrate again to find the displacement of the puck:

$$x = \int_0^t V\,dt = \frac{V_o}{K}[1 - e^{-Kt}]$$

Apply to the particular case given: air, $\mu \approx 1.8E{-}5$ kg/m·s, m = 50 g, D = 9 cm, h = 0.12 mm, V_o = 10 m/s. First evaluate the time-constant K:

$$K = \frac{\mu A}{mh} = \frac{(1.8E{-}5 \text{ kg/m}\cdot\text{s})[(\pi/4)(0.09 \text{ m})^2]}{(0.050 \text{ kg})(0.00012 \text{ m})} \approx 0.0191 \text{ s}^{-1}$$

(a) When the puck slows down to 1 m/s, we obtain the time:

$$V = 1 \text{ m/s} = V_o e^{-Kt} = (10 \text{ m/s})\,e^{-(0.0191 \text{ s}^{-1})t}, \quad \text{or} \quad t \approx \mathbf{121\ s} \quad \textit{Ans.}\text{ (a)}$$

(b) The puck will stop completely only when $\mathbf{e^{-Kt} = 0}$**, or:** $\mathbf{t = \infty}$ *Ans.* (b)
(c) For part (a), the puck will have travelled, in 121 seconds,

$$x = \frac{V_o}{K}(1 - e^{-Kt}) = \frac{10 \text{ m/s}}{0.0191 \text{ s}^{-1}}[1 - e^{-(0.0191)(121)}] \approx \mathbf{472\ m} \quad \textit{Ans.}\text{ (c)}$$

This may perhaps be a little unrealistic. But the air-hockey puck *does* accelerate slowly!

1.62 The hydrogen bubbles in Fig. 1.13 have D ≈ 0.01 mm. Assume an "air-water" interface at 30°C. What is the excess pressure within the bubble?

Solution: At 30°C the surface tension from Table A-1 is 0.0712 N/m. For a droplet or bubble with one spherical surface, from Eq. (1.32),

$$\Delta p = \frac{2Y}{R} = \frac{2(0.0712 \text{ N/m})}{(5E{-}6 \text{ m})} \approx \mathbf{28500\ Pa} \quad \textit{Ans.}$$

1.63 Derive Eq. (1.37) by making a force balance on the fluid interface in Fig. 1.9c.

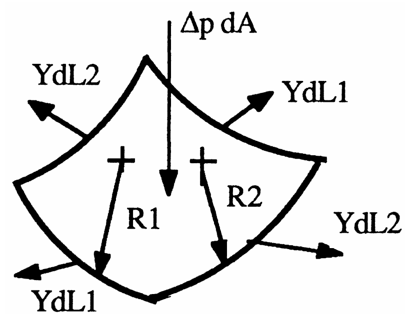


Fig. 1.9c

Solution: The surface tension forces YdL1 and YdL2 have a slight vertical component. Thus summation of forces in the vertical gives the result

$$\sum F_z = 0 = 2YdL_2 \sin(d\theta_1/2) + 2YdL_1 \sin(d\theta_2/2) - \Delta p\, dA$$

But dA = dL1dL2 and sin(dθ/2) ≈ dθ/2, so we may solve for the pressure difference:

$$\Delta p = Y\frac{dL_2 d\theta_1 + dL_1 d\theta_2}{dL_1 dL_2} = Y\left(\frac{d\theta_1}{dL_1} + \frac{d\theta_2}{dL_2}\right) = \mathbf{Y}\left(\frac{\mathbf{1}}{\mathbf{R_1}} + \frac{\mathbf{1}}{\mathbf{R_2}}\right) \quad \textit{Ans.}$$

1.64 A shower head emits a cylindrical jet of clean 20°C water into air. The pressure inside the jet is approximately 200 Pa greater than the air pressure. Estimate the jet diameter, in mm.

Solution: From Table A.5 the surface tension of water at 20°C is 0.0728 N/m. For a liquid cylinder, the internal excess pressure from Eq. (1.31) is $\Delta p = Y/R$. Thus, for our data,

$$\Delta p = Y/R = 200 \text{ N/m}^2 = (0.0728 \text{ N/m})/R,$$

$$\text{solve } R = 0.000364 \text{ m}, \quad \boldsymbol{D = 0.00073 \text{ m}} \quad \textit{Ans.}$$

1.65 The system in Fig. P1.65 is used to estimate the pressure p1 in the tank by measuring the 15-cm height of liquid in the 1-mm-diameter tube. The fluid is at 60°C. Calculate the true fluid height in the tube and the percent error due to capillarity if the fluid is (a) water; and (b) mercury.

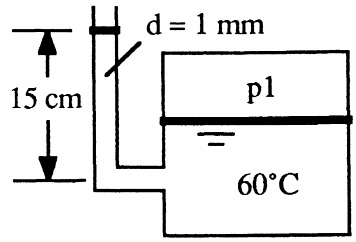


Fig. P1.65

Solution: This is a somewhat more realistic variation of Ex. 1.9. Use values from that example for contact angle θ:
(a) Water at 60°C: $\gamma \approx 9640\ \text{N/m}^3$, $\theta \approx 0°$:

$$h = \frac{4Y\cos\theta}{\gamma D} = \frac{4(0.0662\ \text{N/m})\cos(0°)}{(9640\ \text{N/m}^3)(0.001\ \text{m})} = 0.0275\ \text{m},$$

or: $\Delta h_{true} = 15.0 - 2.75\ \text{cm} \approx$ **12.25 cm (+22% error)** *Ans.* (a)

(b) Mercury at 60°C: $\gamma \approx 132200\ \text{N/m}^3$, $\theta \approx 130°$:

$$h = \frac{4Y\cos\theta}{\gamma D} = \frac{4(0.47\ \text{N/m})\cos 130°}{(132200\ \text{N/m}^3)(0.001\ \text{m})} = -0.0091\ \text{m},$$

or: $\Delta h_{true} = 15.0 + 0.91 \approx$ **15.91 cm (−6% error)** *Ans.* (b)

1.66 A thin wire ring, 3 cm in diameter, is lifted from a water surface at 20°C. What is the lift force required? Is this a good method? Suggest a ring material.

Solution: In the literature this ring-pull device is called a DuNouy Tensiometer. The forces are very small and may be measured by a calibrated soft-spring balance. Platinum-iridium is recommended for the ring, being noncorrosive and highly wetting to most liquids. There are two surfaces, inside and outside the ring, so the total force measured is

$$F = 2(Y\pi D) = 2Y\pi D$$

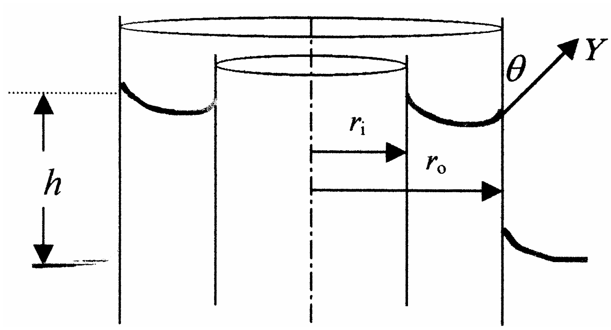
This is crude—commercial devices recommend multiplying this relation by a correction factor $f = O(1)$ which accounts for wire diameter and the distorted surface shape.

For the given data, $Y \approx 0.0728$ N/m (20°C water/air) and the estimated pull force is

$$F = 2\pi(0.0728\ \text{N/m})(0.03\ \text{m}) \approx \mathbf{0.0137\ N} \quad \textit{Ans.}$$

For further details, see, e.g., F. Daniels et al., *Experimental Physical Chemistry*, 7th ed., McGraw-Hill Book Co., New York, 1970.

1.67 A vertical concentric annulus, with outer radius r_o and inner radius r_i, is lowered into fluid of surface tension Y and contact angle $\theta < 90°$. Derive an expression for the capillary rise h in the annular gap, if the gap is very narrow.



Solution: For the figure above, the force balance on the annular fluid is

$$Y\cos\theta(2\pi r_o + 2\pi r_i) = \rho g \pi\left(r_o^2 - r_i^2\right)h$$

Cancel where possible and the result is

$$\boldsymbol{h = 2Y\cos\theta/\{\rho g(r_o - r_i)\}} \quad \textit{Ans.}$$

1.68* Analyze the shape $\eta(x)$ of the water-air interface near a wall, as shown. Assume small slope, $R^{-1} \approx d^2\eta/dx^2$. The pressure difference across the interface is $\Delta p \approx \rho g\eta$, with a contact angle θ at $x = 0$ and a horizontal surface at $x = \infty$. Find an expression for the maximum height h.

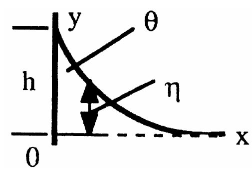


Fig. P1.68

Solution: This is a two-dimensional surface-tension problem, with single curvature. The surface tension rise is balanced by the weight of the film. Therefore the differential equation is

$$\Delta p = \rho g\eta = \frac{Y}{R} \approx Y\frac{d^2\eta}{dx^2} \quad \left(\frac{d\eta}{dx} \ll 1\right)$$

This is a second-order differential equation with the well-known solution,

$$\eta = C_1\exp[Kx] + C_2\exp[-Kx], \quad K = \sqrt{(\rho g/Y)}$$

To keep η from going infinite as $x = \infty$, it must be that $\mathbf{C_1 = 0}$. The constant C_2 is found from the maximum height at the wall:

$$\eta|_{x=0} = h = C_2\exp(0), \quad \text{hence } C_2 = h$$

Meanwhile, the contact angle shown above must be such that,

$$\frac{d\eta}{dx}\bigg|_{x=0} = -\cot(\theta) = -hK, \quad \text{thus } h = \frac{\cot\theta}{K}$$

The complete (small-slope) solution to this problem is:

$$\boldsymbol{\eta = h\exp[-(\rho g/Y)^{1/2}x]}, \quad \text{where } h = (Y/\rho g)^{1/2}\cot\theta \quad \textit{Ans.}$$

The formula clearly satisfies the requirement that $\eta = 0$ if $x = \infty$. It requires "small slope" and therefore the contact angle should be in the range $70° < \theta < 110°$.

1.69 A solid cylindrical needle of diameter d, length L, and density ρ_n may "float" on a liquid surface. Neglect buoyancy and assume a contact angle of 0°. Calculate the maximum diameter needle able to float on the surface.

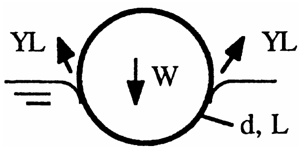


Fig. P1.69

Solution: The needle "dents" the surface downward and the surface tension forces are upward, as shown. If these tensions are nearly vertical, a vertical force balance gives:

$$\sum F_z = 0 = 2YL - \rho g\frac{\pi}{4}d^2L, \quad \text{or:} \quad \mathbf{d_{max}} \approx \sqrt{\frac{\mathbf{8Y}}{\boldsymbol{\pi\rho g}}} \quad \textit{Ans.}\text{ (a)}$$

(b) Calculate d_{max} for a steel needle (SG ≈ 7.84) in water at 20°C. The formula becomes:

$$d_{max} = \sqrt{\frac{8Y}{\pi\rho g}} = \sqrt{\frac{8(0.073\text{ N/m})}{\pi(7.84 \times 998\text{ kg/m}^3)(9.81\text{ m/s}^2)}} \approx 0.00156\text{ m} \approx \mathbf{1.6\ mm} \quad \textit{Ans.}\text{ (b)}$$

1.70 Derive an expression for the capillary-height change h, as shown, for a fluid of surface tension Y and contact angle θ between two parallel plates W apart. Evaluate h for water at 20°C if W = 0.5 mm.

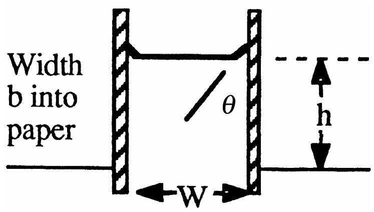


Fig. P1.70

Solution: With *b* the width of the plates into the paper, the capillary forces on each wall together balance the weight of water held above the reservoir free surface:

$$\rho gWhb = 2(Yb\cos\theta), \quad \text{or:} \quad h \approx \frac{\mathbf{2Y\cos\theta}}{\boldsymbol{\rho}\mathbf{gW}} \quad \textit{Ans.}$$

For water at 20°C, Y ≈ 0.0728 N/m, $\rho g \approx 9790\ N/m^3$, and θ ≈ 0°. Thus, for W = 0.5 mm,

$$h = \frac{2(0.0728\ N/m)\cos 0°}{(9790\ N/m^3)(0.0005\ m)} \approx 0.030\ m \approx \mathbf{30\ mm} \quad \textit{Ans.}$$

1.71* A soap bubble of diameter D_1 coalesces with another bubble of diameter D_2 to form a single bubble D_3 with the same amount of air. For an isothermal process, express D_3 as a function of D_1, D_2, p_{atm}, and surface tension Y.

Solution: The masses remain the same for an isothermal process of an ideal gas:

$$m_1 + m_2 = \rho_1\upsilon_1 + \rho_2\upsilon_2 = m_3 = \rho_3\upsilon_3,$$

$$\text{or:} \quad \left(\frac{p_a + 4Y/r_1}{RT}\right)\left(\frac{\pi}{6}D_1^3\right) + \left(\frac{p_a + 4Y/r_2}{RT}\right)\left(\frac{\pi}{6}D_2^3\right) = \left(\frac{p_a + 4Y/r_3}{RT}\right)\left(\frac{\pi}{6}D_3^3\right)$$

The temperature cancels out, and we may clean up and rearrange as follows:

$$\mathbf{p_a D_3^3 + 8YD_3^2 = \left(p_a D_2^3 + 8YD_2^2\right) + \left(p_a D_1^3 + 8YD_1^2\right)} \quad \textit{Ans.}$$

This is a cubic polynomial with a known right hand side, to be solved for D_3.

1.72 Early mountaineers boiled water to estimate their altitude. If they reach the top and find that water boils at 84°C, approximately how high is the mountain?

Solution: From Table A-5 at 84°C, vapor pressure $p_v \approx 55.4$ kPa. We may use this value to interpolate in the standard altitude, Table A-6, to estimate

$$z \approx \mathbf{4800\ m} \quad \textit{Ans.}$$

1.73 A small submersible moves at velocity V in 20°C water at 2-m depth, where ambient pressure is 131 kPa. Its critical cavitation number is Ca ≈ 0.25. At what velocity will cavitation bubbles form? Will the body cavitate if V = 30 m/s and the water is cold (5°C)?

Solution: From Table A-5 at 20°C read p_v = 2.337 kPa. By definition,

$$Ca_{crit} = 0.25 = \frac{2(p_a - p_v)}{\rho V^2} = \frac{2(131000 - 2337)}{(998\ kg/m^3)V^2}, \quad \text{solve } V_{crit} \approx \mathbf{32.1\ m/s} \quad \textit{Ans.}\ (a)$$

If we decrease water temperature to 5°C, the vapor pressure reduces to 863 Pa, and the density changes slightly, to 1000 kg/m^3. For this condition, if V = 30 m/s, we compute:

$$\mathrm{Ca} = \frac{2(131000 - 863)}{(1000)(30)^2} \approx 0.289$$

This is *greater* than 0.25, therefore the body **will not cavitate for these conditions**. *Ans*. (b)

1.74 Oil, with a vapor pressure of 20 kPa, is delivered through a pipeline by equally-spaced pumps, each of which increases the oil pressure by 1.3 MPa. Friction losses in the pipe are 150 Pa per meter of pipe. What is the maximum possible pump spacing to avoid cavitation of the oil?

Solution: The absolute maximum length L occurs when the pump inlet pressure is slightly greater than 20 kPa. The pump increases this by 1.3 MPa and friction drops the pressure over a distance L until it again reaches 20 kPa. In other words, quite simply,

$$1.3\ \text{MPa} = 1{,}300{,}000\ \text{Pa} = (150\ \text{Pa/m})\text{L}, \quad \text{or} \quad \text{L}_{\max} \approx \mathbf{8660\ m} \quad \textit{Ans.}$$

It makes more sense to have the pump inlet at 1 atm, not 20 kPa, dropping L to about 8 km.

1.75 An airplane flies at 555 mi/h. At what altitude in the standard atmosphere will the airplane's Mach number be exactly 0.8?

Solution: First convert V = 555 mi/h x 0.44704 = 248.1 m/s. Then the speed of sound is

$$a = \frac{V}{\mathrm{Ma}} = \frac{248.1\, m/s}{0.8} = 310\, m/s$$

Reading in Table A.6, we estimate the altitude to be approximately **7500 m**. *Ans*.

1.76 Estimate the speed of sound of steam at 200°C and 400 kPa, (a) by an ideal-gas approximation (Table A.4); and (b) using EES (or the Steam Tables) and making small isentropic changes in pressure and density and approximating Eq. (1.38).
Solution: (a) For steam, $k \approx 1.33$ and R = 461 m^2/s^2·K. The ideal gas formula predicts:

$$a \approx \sqrt{(\text{kRT})} = \sqrt{\{1.33(461\ \text{m}^2/\text{s}^2 \cdot \text{K})(200 + 273\ \text{K})\}} \approx \mathbf{539\ m/s} \quad \textit{Ans.}\ \text{(a)}$$

(b) We use the formula $a = \sqrt{(\partial p/\partial \rho)_s} \approx \sqrt{\{\Delta p|_s/\Delta \rho|_s\}}$ for small isentropic changes in p and ρ. From EES, at 200°C and 400 kPa, the entropy is s = 1.872 kJ/kg·K. Raise and lower the pressure 1 kPa at the same entropy. At p = 401 kPa, ρ = 1.87565 kg/m^3. At p = 399 kPa, ρ = 1.86849 kg/m^3. Thus $\Delta\rho$ = 0.00716 kg/m^3, and the formula for sound speed predicts:

$$a \approx \sqrt{\{\Delta p|_s/\Delta \rho|_s\}} = \sqrt{\{(2000\ \text{N/m}^2)/(0.00358\ \text{kg/m}^3)\}} = \mathbf{529\ m/s} \quad \textit{Ans.}\,\text{(b)}$$

Again, as in Prob. 1.34, the ideal gas approximation is within 2% of a Steam-Table solution.

1.77 The density of gasoline varies with pressure approximately as follows:

p, atm:	1	500	1000	1500
ρ, lbm/ft^3:	42.45	44.85	46.60	47.98

Estimate (a) its speed of sound, and (b) its bulk modulus at 1 atm.

Solution: For a crude estimate, we could just take differences of the first two points:

$$a \approx \sqrt{(\Delta p/\Delta \rho)} \approx \sqrt{\left\{\frac{(500-1)(2116)\ \text{lbf/ft}^2}{(44.85-42.45)/32.2\ \text{slug/ft}^3}\right\}} \approx 3760\ \frac{\text{ft}}{\text{s}} \approx \mathbf{1150}\ \frac{\mathbf{m}}{\mathbf{s}} \quad \textit{Ans.}\ \text{(a)}$$

$$B \approx \rho a^2 = [42.45/32.2\ \text{slug/ft}^3](3760\ \text{ft/s})^2 \approx 1.87\text{E}7\ \frac{\text{lbf}}{\text{ft}^2} \approx \mathbf{895\ MPa} \quad \textit{Ans.}\ \text{(b)}$$

For more accuracy, we could fit the data to the nonlinear equation of state for liquids, Eq. (1.22). The best-fit result for gasoline (data above) is $n \approx 8.0$ and $B \approx 900$.

Equation (1.22) is too simplified to show temperature or entropy effects, so we assume that it approximates "isentropic" conditions and thus differentiate:

$$\frac{p}{p_a} \approx (B+1)(\rho/\rho_a)^n - B, \quad \text{or:} \quad a^2 = \frac{dp}{d\rho} \approx \frac{n(B+1)p_a}{\rho_a}(\rho/\rho_a)^{n-1}$$

$$\text{or,} \quad \text{at 1 atm,} \quad a_{\text{liquid}} \approx \sqrt{n(B+1)p_a/\rho_a}$$

The bulk modulus of gasoline is thus approximately:

$$\text{“B”} = \rho \frac{dp}{d\rho}\Big|_{1\ \text{atm}} = n(B+1)p_a = (8.0)(901)(101350\ \text{Pa}) \approx \mathbf{731\ MPa} \quad \textit{Ans.}\ \text{(b)}$$

And the speed of sound in gasoline is approximately,

$$a_{1\ atm} = [(8.0)(901)(101350\ \text{Pa})/(680\ \text{kg/m}^3)]^{1/2} \approx \mathbf{1040\ \frac{m}{s}} \quad \textit{Ans. (a)}$$

1.78 Sir Isaac Newton measured sound speed by timing the difference between seeing a cannon's puff of smoke and hearing its boom. If the cannon is on a mountain 5.2 miles away, estimate the air temperature in °C if the time difference is (a) 24.2 s; (b) 25.1 s.

Solution: Cannon booms are finite (shock) waves and travel slightly faster than sound waves, but what the heck, assume it's close enough to sound speed:

$$\text{(a)}\quad a \approx \frac{\Delta x}{\Delta t} = \frac{5.2(5280)(0.3048)}{24.2} = 345.8\ \frac{m}{s} = \sqrt{1.4(287)T}, \quad T \approx 298\ K \approx \mathbf{25°C} \quad \textit{Ans. (a)}$$

$$\text{(b)}\quad a \approx \frac{\Delta x}{\Delta t} = \frac{5.2(5280)(0.3048)}{25.1} = 333.4\ \frac{m}{s} = \sqrt{1.4(287)T}, \quad T \approx 277\ K \approx \mathbf{4°C} \quad \textit{Ans. (b)}$$

1.79 Even a tiny amount of dissolved gas can drastically change the speed of sound of a gas-liquid mixture. By estimating the pressure-volume change of the mixture, Olson [40] gives the following approximate formula:

$$a_{mixture} \approx \sqrt{\frac{p_g K_l}{[x\rho_g + (1-x)\rho_l][xK_l + (1-x)p_g]}}$$

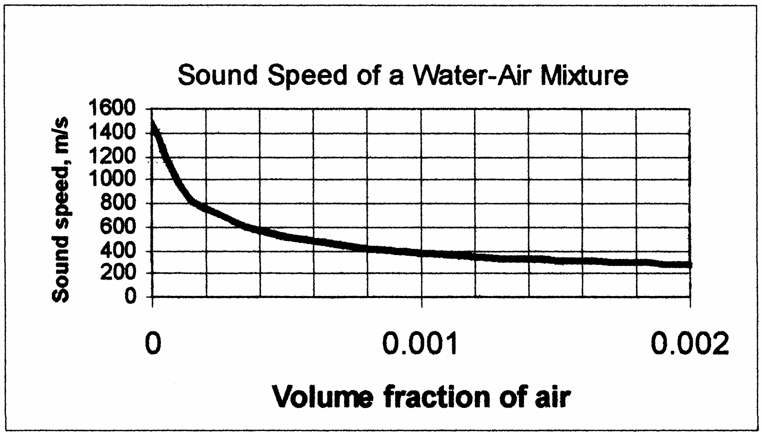
where x is the volume fraction of gas, K is the bulk modulus, and subscripts ℓ and g denote the liquid and gas, respectively. (a) Show that the formula is dimensionally homogeneous. (b) For the special case of air bubbles (density 1.7 kg/m^3 and pressure 150 kPa) in water (density 998 kg/m^3 and bulk modulus 2.2 GPa), plot the mixture speed of sound in the range $0 \le x \le 0.002$ and discuss.

Solution: (a) Since x is dimensionless and K dimensions cancel between the numerator and denominator, the remaining dimensions are pressure divided by density:

$$\{a_{\text{mixture}}\} = [\{p\}/\{\rho\}]^{1/2} = [(M/LT^2)/(M/L^3)]^{1/2} = [L^2/T^2]^{1/2}$$

$$= \mathbf{L/T} \quad \textit{Yes, homogeneous} \quad \textit{Ans. (a)}$$

(b) For the given data, a plot of sound speed versus gas volume fraction is as follows:



The difference in air and water compressibility is so great that the speed drop-off is quite sharp.

1.80* A two-dimensional steady velocity field is given by $u = x^2 - y^2$, $v = -2xy$. Find the streamline pattern and sketch a few lines. [*Hint*: The differential equation is exact.]

Solution: Equation (1.44) leads to the differential equation:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dx}{x^2 - y^2} = \frac{dy}{-2xy}, \quad \text{or:} \quad (2xy)dx + (x^2 - y^2)dy = 0$$

As hinted, this equation is *exact*, that is, it has the form $dF = (\partial F/\partial x)dx + (\partial F/\partial y)dy = 0$. We may check this readily by noting that $\partial/\partial y(2xy) = \partial/\partial x(x^2 - y^2) = 2x = \partial^2 F/\partial x\partial y$. Thus we may integrate to give the formula for streamlines:

$$\mathbf{F = x^2y - y^3/3 + constant} \quad \textit{Ans.}$$

This represents (inviscid) flow in a series of 60° corners, as shown in Fig. E4.7a of the text. [This flow is also discussed at length in Section 4.7.]

1.81 Repeat Ex. 1.13 by letting the velocity components increase linearly with time:

$$\mathbf{V} = Kxt\mathbf{i} - Kyt\mathbf{j} + 0\mathbf{k}$$

Solution: The flow is unsteady and two-dimensional, and Eq. (1.44) still holds:

$$\text{Streamline:} \quad \frac{dx}{u} = \frac{dy}{v}, \quad \text{or:} \quad \frac{dx}{Kxt} = \frac{dy}{-Kyt}$$

The terms K and t both vanish and leave us with the same result as in Ex. 1.13, that is,

$$\int dx/x = -\int dy/y, \quad \text{or:} \quad \mathbf{xy = C} \quad \textit{Ans.}$$

The streamlines have exactly the same "stagnation flow" shape as in Fig. 1.13. However, the flow *is* accelerating, and the mass flow between streamlines is constantly increasing.

1.82 A velocity field is given by u = V cosθ, v = V sinθ, and w = 0, where V and θ are constants. Find an expression for the streamlines of this flow.

Solution: Equation (1.44) may be used to find the streamlines:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dx}{V\cos\theta} = \frac{dy}{V\sin\theta}, \quad \text{or:} \quad \frac{dy}{dx} = \tan\theta$$

$$\text{Solution:} \quad \mathbf{y = (\tan\theta)x + constant} \quad \textit{Ans.}$$

The streamlines are straight parallel lines which make an angle θ with the *x* axis. In other words, this velocity field represents a *uniform stream V moving upward at angle θ.*

1.83* A two-dimensional *unsteady* velocity field is given by u = x(1 + 2t), v = y. Find the time-varying streamlines which pass through some reference point (x_o,y_o). Sketch some.

Solution: Equation (1.44) applies with time as a parameter:

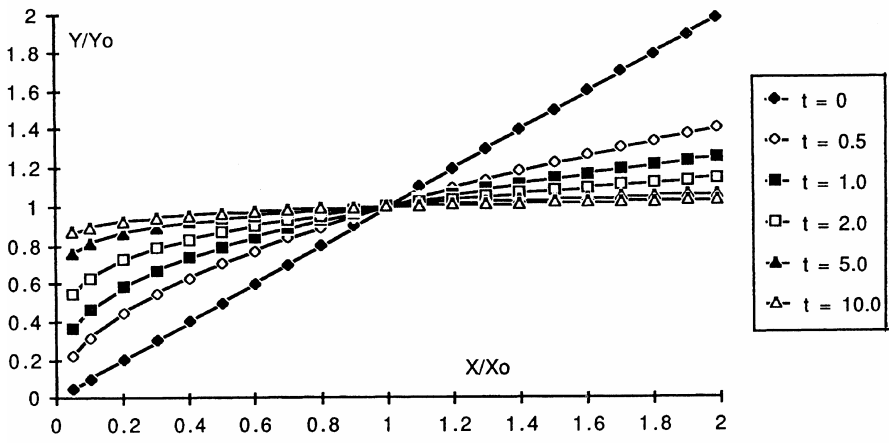
$$\frac{dx}{u} = \frac{dx}{x(1+2t)} = \frac{dy}{v} = \frac{dy}{y}, \quad \text{or:} \quad \ln(y) = \frac{1}{1+2t}\ln(x) + \text{constant}$$

$$\text{or:} \quad y = Cx^{1/(1+2t)}, \quad \text{where C is a constant}$$

In order for all streamlines to pass through y = y_o at x = x_o, the constant must be such that:

$$\boldsymbol{y = y_o(x/x_o)^{1/(1+2t)}} \quad \textit{Ans.}$$

Some streamlines are plotted on the next page and are seen to be strongly time-varying.



1.84* Modify Prob. 1.83 to find the equation of the <u>*pathline*</u> which passes through the point (xo, yo) at t = 0. Sketch this pathline.

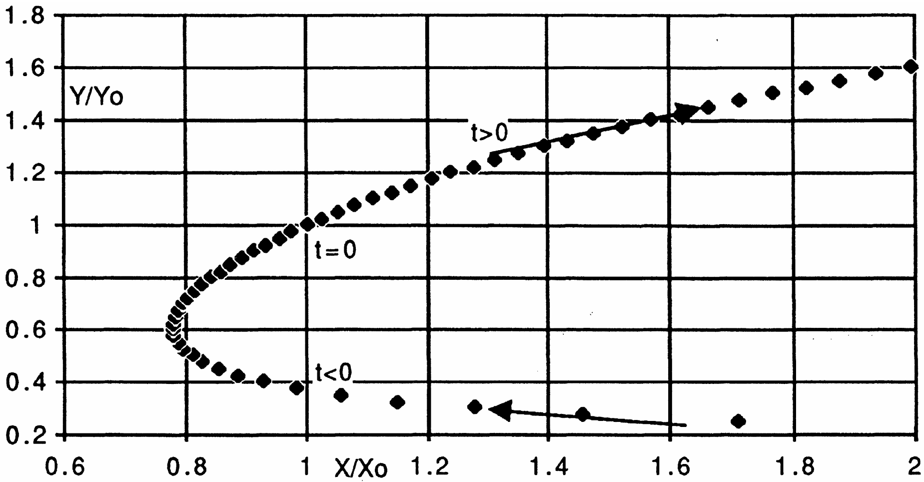
Solution: The pathline is computed by integration, over time, of the velocities:

$$\frac{dx}{dt} = u = x(1+2t), \quad \text{or:} \quad \int \frac{dx}{x} = \int (1+2t)\,dt, \quad \text{or:} \quad x = x_o e^{t+t^2}$$

$$\frac{dy}{dt} = v = y, \quad \text{or:} \quad \int \frac{dy}{y} = \int dt, \quad \text{or:} \quad y = y_o e^t$$

We have implemented the initial conditions (x, y) = (xo, yo) at t = 0. [We were very lucky, as *planned* for this problem, that u did not depend upon y and v did not depend upon x.] Now eliminate t between these two to get a geometric expression for this particular pathline:

$\mathbf{x = x_o \exp\{\ln(y/y_o) + \ln^2(y/y_o)\}}$ This pathline is shown in the sketch below.



1.85-a Report to the class on the achievements of *Evangelista Torricelli*.

Solution: Torricelli's biography is taken from a goldmine of information which I did not put in the references, preferring to let the students find it themselves: C. C. Gillespie (ed.), *Dictionary of Scientific Biography*, 15 vols., Charles Scribner's Sons, New York, 1976.

Torricelli (1608–1647) was born in Faenza, Italy, to poor parents who recognized his genius and arranged through Jesuit priests to have him study mathematics, philosophy, and (later) hydraulic engineering under Benedetto Castelli. His work on dynamics of projectiles attracted the attention of Galileo himself, who took on Torricelli as an assistant in 1641. Galileo died one year later, and Torricelli was appointed in his place as "mathematician and philosopher" by Duke Ferdinando II of Tuscany. He then took up residence in Florence, where he spent his five happiest years, until his death in 1647. In 1644 he published his only known printed work, *Opera Geometrica*, which made him famous as a mathematician and geometer.

In addition to many contributions to geometry and calculus, Torricelli was the first to show that a zero-drag projectile formed a *parabolic* trajectory. His tables of trajectories for various angles and initial velocities were used by Italian artillerymen. He was an excellent machinist and constructed—and sold—the very finest telescope lenses in Italy.

Torricelli's hydraulic studies were brief but stunning, leading Ernst Mach to proclaim him the 'founder of hydrodynamics.' He deduced his theorem that the velocity of efflux from a hole in a tank was equal to $\sqrt{(2gh)}$, where h is the height of the free surface above the hole. He also showed that the efflux jet was parabolic and even commented on water-droplet breakup and the effect of air resistance. By experimenting with various liquids in closed tubes—including mercury (from mines in Tuscany)—he thereby invented the *barometer*. From barometric pressure (about 30 feet of water) he was able to explain why siphons did not work if the elevation change was too large. He also was the first to explain that winds were produced by temperature and *density differences* in the atmosphere and not by "evaporation."

1.85-b Report to the class on the achievements of *Henri de Pitot*.

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a).

Pitot (1695–1771) was born in Aramon, France, to patrician parents. He hated to study and entered the military instead, but only for a short time. Chance reading of a textbook obtained in Grenoble led him back to academic studies of mathematics, astronomy, and engineering. In 1723 he became assistant to Réamur at the French Academy of Sciences and in 1740 became a civil engineer upon his appointment as a director of public works in Languedoc Province. He retired in 1756 and returned to Aramon until his death in 1771.

Pitot's research was apparently mediocre, described as "competent solutions to minor problems without lasting significance"—not a good recommendation for tenure nowadays! His *lasting* contribution was the invention, in 1735, of the instrument which bears his name: a glass tube bent at right angles and inserted into a moving stream with the opening facing upstream. The water level in the tube rises a distance *h* above the surface, and Pitot correctly deduced that the stream velocity ≈ √(2gh). This is still a basic instrument in fluid mechanics.

1.85-c Report to the class on the achievements of *Antoine Chézy.*

Solution: The following notes are from Rouse and Ince [Ref. 23].

Chézy (1718–1798) was born in Châlons-sur-Marne, France, studied engineering at the Ecole des Ponts et Chaussées and then spent his entire career working for this school, finally being appointed Director one year before his death. His chief contribution was to study the flow in open channels and rivers, resulting in a famous formula, used even today, for the average velocity:

$$V \approx \text{const}\sqrt{AS/P}$$

where A is the cross-section area, S the bottom slope, and P the wetted perimeter, i.e., the length of the bottom and sides of the cross-section. The "constant" depends primarily on the roughness of the channel bottom and sides. [See Chap. 10 for further details.]

1.85-d Report to the class on the achievements of *Gotthilf Heinrich Ludwig Hagen*.

Solution: The following notes are from Rouse and Ince [Ref. 23].

Hagen (1884) was born in Königsberg, East Prussia, and studied there, having among his teachers the famous mathematician Bessel. He became an engineer, teacher, and writer and published a handbook on hydraulic engineering in 1841. He is best known for his study in 1839 of pipe-flow resistance, for water flow at heads of 0.7 to 40 cm, diameters of 2.5 to 6 mm, and lengths of 47 to 110 cm. The measurements indicated that the pressure drop was proportional to Q at low heads and proportional (approximately) to Q^2 at higher heads, where "strong movements" occurred—turbulence. He also showed that Δp was approximately proportional to D^{-4}.

Later, in an 1854 paper, Hagen noted that the difference between laminar and turbulent flow was clearly visible in the efflux jet, which was either "smooth or fluctuating," and in glass tubes, where sawdust particles either "moved axially" or, at higher Q, "came into whirling motion." Thus Hagen was a true pioneer in fluid mechanics experimentation. Unfortunately, his achievements were somewhat overshadowed by the more widely publicized 1840 tube-flow studies of J. L. M. Poiseuille, the French physician.

1.85-e Report to the class on the achievements of *Julius Weisbach*.

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a) and also from Rouse and Ince [Ref. 23].

Weisbach (1806–1871) was born near Annaberg, Germany, the 8th of nine children of working-class parents. He studied mathematics, physics, and mechanics at Göttingen and Vienna and in 1931 became instructor of mathematics at Freiberg Gymnasium. In 1835 he was promoted to full professor at the Bergakademie in Freiberg. He published 15 books and 59 papers, primarily on hydraulics. He was a skilled laboratory worker and summarized his results in *Experimental-Hydraulik* (Freiberg, 1855) and in the *Lehrbuch der Ingenieur- und Maschinen-Mechanik* (Brunswick, 1845), which was still in print 60 years later. There were 13 chapters on hydraulics in this latter treatise. Weisbach modernized the subject of fluid mechanics, and his discussions and drawings of flow patterns would be welcome in any 20th century textbook—see Rouse and Ince [23] for examples.

Weisbach was the first to write the pipe-resistance head-loss formula in modern form: $h_{f(pipe)} = f(L/D)(V^2/2g)$, where f was the dimensionless 'friction factor,' which Weisbach noted was not a constant but related to the pipe flow parameters [see Sect. 6.4]. He was also the first to derive the "weir equation" for volume flow rate Q over a dam of crest length L:

$$Q \approx \frac{2}{3}C_w(2g)^{1/2}\left[\left(H+\frac{V^2}{2g}\right)^{3/2}-\left(\frac{V^2}{2g}\right)^{3/2}\right] \approx \frac{2}{3}C_w(2g)^{1/2}H^{3/2}$$

where H is the upstream water head level above the dam crest and C_W is a dimensionless weir coefficient ≈ O(unity). [see Sect. 10.7] In 1860 Weisbach received the first Honorary Membership awarded by the German engineering society, the *Verein Deutscher Ingenieure*.

1.85-f Report to the class on the achievements of *George Gabriel Stokes*.

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a).

Stokes (1819–1903) was born in Skreen, County Sligo, Ireland, to a clergical family associated for generations with the Church of Ireland. He attended Bristol College and Cambridge University and, upon graduation in 1841, was elected Fellow of Pembroke College, Cambridge. In 1849, he became Lucasian Professor at Cambridge, a post once held by Isaac Newton. His 60-year career was spent primarily at Cambridge and resulted in many honors: President of the Cambridge Philosophical Society (1859), secretary (1854) and president (1885) of the Royal Society of London, member of Parliament (1887–1891), knighthood (1889), the Copley Medal (1893), and Master of Pembroke College (1902). A true 'natural philosopher,' Stokes systematically explored hydrodynamics, elasticity, wave mechanics, diffraction, gravity, acoustics, heat, meteorology, and chemistry. His primary research output was from 1840–1860, for he later became tied down with administrative duties.

In hydrodynamics, Stokes has several formulas and fields named after him:

(1) The equations of motion of a linear viscous fluid: the *Navier-Stokes equations*.
(2) The motion of nonlinear deep-water surface waves: *Stokes waves*.
(3) The drag on a sphere at low Reynolds number: *Stokes' formula,* $F = 3\pi\mu VD$.
(4) Flow over immersed bodies for $Re \ll 1$: *Stokes flow*.
(5) A metric (CGS) unit of kinematic viscosity, ν: $1\ cm^2/s = 1$ *stoke.*
(6) A relation between the 1st and 2nd coefficients of viscosity: *Stokes' hypothesis.*
(7) A stream function for axisymmetric flow: *Stokes' stream function* [see Chap. 8].

Although Navier, Poisson, and Saint-Venant had made derivations of the equations of motion of a viscous fluid in the 1820's and 1830's, Stokes was quite unfamiliar with the French literature. He published a completely independent derivation in 1845 of the *Navier-Stokes equations* [see Sect. 4.3], using a 'continuum-calculus' rather than a 'molecular' viewpoint, and showed that these equations were directly analogous to the motion of elastic solids. Although not really new, Stokes' equations were notable for being the first to replace the mysterious French 'molecular coefficient' ε by the coefficient of absolute viscosity, μ.

1.85-g Report to the class on the achievements of *Moritz Weber*.

Solution: The following notes are from Rouse and Ince [Ref. 23].

Weber (1871–1951) was professor of naval mechanics at the Polytechnic Institute of Berlin. He clarified the principles of similitude (dimensional analysis) in the form used today. It was he who named the Froude number and the Reynolds number in honor of those workers. In a 1919 paper, he developed a dimensionless surface-tension (capillarity) parameter [see Sect. 5.4] which was later named the *Weber number* in his honor.

1.85-h Report to the class on the achievements of *Theodor von Kármán.*

Solution: The following notes are abstracted from the *Dictionary of Scientific Biography* (see Prob. 1.85-a). Another good reference is his ghost-written (by Lee Edson) autobiography, *The Wind and Beyond*, Little-Brown, Boston, 1967.

Kármán (1881–1963) was born in Budapest, Hungary, to distinguished and well-educated parents. He attended the Technical University of Budapest and in 1906 received a fellowship to Göttingen, where he worked for six years with Ludwig Prandtl, who had just developed boundary layer theory. He received a doctorate in 1912 from Göttingen and was then appointed director of aeronautics at the Polytechnic Institute of Aachen. He remained at Aachen until 1929, when he was named director of the newly formed Guggenheim Aeronautical Laboratory at the California Institute of Technology. Kármán developed CalTech into a premier research center for aeronautics. His leadership spurred the growth of the aerospace industry in southern California. He helped found the Jet Propulsion Laboratory and the Aerojet General Corporation. After World War II, Kármán founded a research arm for NATO, the Advisory Group for Aeronautical Research and Development, whose renowned educational institute in Brussels is now called the Von Kármán Center.

Kármán was uniquely skilled in integrating physics, mathematics, and fluid mechanics into a variety of phenomena. His most famous paper was written in 1912 to explain the puzzling alternating vortices shed behind cylinders in a steady-flow experiment conducted by K. Hiemenz, one of Kármán's students—these are now called *Kármán vortex streets* [see Fig. 5.2a]. Shed vortices are thought to have caused the destruction by winds of the Tacoma Narrows Bridge in 1940 in Washington State.

Kármán wrote 171 articles and 5 books and his methods had a profound influence on fluid mechanics education in the 20th century.

1.85-i Report to the class on the achievements of *Paul Richard Heinrich Blasius.*

Solution: The following notes are from Rouse and Ince [Ref. 23].

Blasius (1883–1970) was Ludwig Prandtl's first graduate student at Göttingen. His 1908 dissertation gave the analytic solution for the laminar boundary layer on a flat plate [see Sect. 7.4]. Then, in two papers in 1911 and 1913, he gave the first demonstration that pipe-flow resistance could be nondimensionalized as a plot of friction factor versus Reynolds number—the first "Moody-type" chart. His correlation, $f \approx 0.316\ Re_d^{-1/4}$, is still is use today. He later worked on analytical solutions of boundary layers with variable pressure gradients.

1.85-j Report to the class on the achievements of *Ludwig Prandtl*.

Solution: The following notes are from Rouse and Ince [Ref. 23].

Ludwig Prandtl (1875–1953) is described by Rouse and Ince [23] as the father of modern fluid mechanics. Born in Munich, the son of a professor, Prandtl studied engineering and received a doctorate in elasticity. But his first job as an engineer made him aware of the lack of correlation between theory and experiment in fluid mechanics. He conducted research from 1901–1904 at the Polytechnic Institute of Hanover and presented a seminal paper in 1904, outlining the new concept of "boundary layer theory." He was promptly hired as professor and director of applied mechanics at the University of Gottingen, where he remained throughout his career. He, and his dozens of famous students, started a new "engineering science" of fluid mechanics, emphasizing (1) mathematical analysis based upon by physical reasoning; (2) new experimental techniques; and (3) new and inspired flow-visualization schemes which greatly increased our understanding of flow phenomena.

In addition to boundary-layer theory, Prandtl made important contributions to (1) wing theory; (2) turbulence modeling; (3) supersonic flow; (4) dimensional analysis; and (5) instability and transition of laminar flow. He was a legendary engineering professor.

1.85-k Report to the class on the achievements of *Osborne Reynolds*.

Solution: The following notes are from Rouse and Ince [Ref. 23].

Osborne Reynolds (1842–1912) was born in Belfast, Ireland, to a clerical family and studied mathematics at Cambridge University. In 1868 he was appointed chair of engineering at a college which is now known as the University of Manchester Institute of Science and Technology (UMIST). He wrote on wide-ranging topics—mechanics, electricity, navigation—and developed a new hydraulics laboratory at UMIST. He was the first person to demonstrate cavitation, that is, formation of vapor bubbles due to high velocity and low pressure. His most famous experiment, still performed in the undergraduate laboratory at UMIST (see Fig. 6.5 in the text) demonstrated transition of laminar pipe flow into turbulence. He also showed in this experiment that the viscosity was very important and led him to the dimensionless stability parameter $\rho VD/\mu$ now called the *Reynolds number* in his honor. Perhaps his most important paper, in 1894, extended the Navier-Stokes equations (see Eqs. 4.38 of the text) to time-averaged randomly fluctuating turbulent flow, with a result now called the *Reynolds equations* of turbulence. Reynolds also contributed to the concept of the *control volume* which forms the basis of integral analysis of flow (Chap. 3).

1.85-l Report to the class on the achievements of *John William Strutt, Lord Rayleigh.*

Solution: The following notes are from Rouse and Ince [Ref. 23].

John William Strutt (1842–1919) was born in Essex, England, and inherited the title Lord Rayleigh. He studied at Cambridge University and was a traditional hydrodynamicist in the spirit of Euler and Stokes. He taught at Cambridge most of his life and also served as president of the Royal Society. He is most famous for his work (and his textbook) on the theory of sound. In 1904 he won the Nobel Prize for the discovery of argon gas. He made at least five important contributions to hydrodynamics: (1) the equations of bubble dynamics in liquids, now known as *Rayleigh-Plesset theory*; (2) the theory of nonlinear surface waves; (3) the capillary (surface tension) instability of jets; (4) the "heat-transfer analogy" to laminar flow; and (5) dimensional similarity, especially related to viscosity data for argon gas and later generalized into group theory which previewed Buckingham's Pi Theorem. He ended his career as president, in 1909, of the first British committee on aeronautics.

1.85-m Report to the class on the achievements of *Daniel Bernoulli.*

Solution: The following notes are from Rouse and Ince [Ref. 23].

Daniel Bernoulli (1700–1782) was born in Groningen, Holland, his father, Johann, being a Dutch professor. He studied at the University of Basel, Switzerland, and taught mathematics for a few years at St. Petersburg, Russia. There he wrote, and published in 1738, his famous treatise *Hydrodynamica*, for which he is best known. This text contained numerous ingenious drawings illustrating various flow phenomena. Bernoulli used energy concepts to establish proportional relations between kinetic and potential energy, with pressure work added only in the abstract. Thus he never actually derived the famous equation now bearing his name (Eq. 3.77 of the text), later derived in 1755 by his friend Leonhard Euler. Daniel Bernoulli never married and thus never contributed additional members to his famous family of mathematicians.

1.85-n Report to the class on the achievements of *Leonhard Euler.*

Solution: The following notes are from Rouse and Ince [Ref. 23].

Leonhard Euler (1707–1783) was born in Basel, Switzerland, and studied mathematics under Johann Bernoulli, Daniel's father. He succeeded Daniel Bernoulli as professor of mathematics at the St. Petersburg Academy, leaving there in 1741 to join the faculty of Berlin University. He lost his sight in 1766 but continued to work, aided by a prodigious memory, and produced a vast output of scientific papers, dealing with mathematics, optics, mechanics, hydrodynamics, and celestial mechanics (for which he is most famous today). His famous paper of 1755 on fluid flow derived the full inviscid equations of fluid motion (Eqs. 4.36 of the text) now called *Euler's equations*. He used a fixed coordinate system, now called the *Eulerian frame of reference*. The paper also presented, for the first time, the correct form of Bernoulli's equation (Eq. 3.77 of the text). Separately, in 1754 he produced a seminal paper on the theory of reaction turbines, leading to *Euler's turbine equation* (Eq. 11.11 of the text).

P1.86 A right circular cylinder volume υ is to be calculated from the measured base radius R and height H. If the uncertainty in R is 2% and the uncertainty in H is 3%, estimate the overall uncertainty in the calculated volume.

Solution: The formula for volume is, of course, $\upsilon = \pi R^2 H$. There are two terms to be calculated on the right-hand side of Eq. (1.43):

$$\frac{\partial \upsilon}{\partial R}\delta R = 2\pi RH\,\delta R \quad ; \quad \frac{\partial \upsilon}{\partial H}\delta H = \pi R^2\,\delta H$$

Expressed as a ratio to the volume, these become

$$\frac{\delta \upsilon}{\upsilon}\Big|_{due\,to\,R} = \frac{2\pi RH\,\delta R}{\pi R^2 H} = 2\frac{\delta R}{R} \quad ; \quad \frac{\delta \upsilon}{\upsilon}\Big|_{due\,to\,H} = \frac{\pi R^2\,\delta H}{\pi R^2 H} = \frac{\delta H}{H}$$

Note $\delta R/R = 2\%$ and $\delta H/H = 3\%$. The overall uncertainty in the volume is a root-mean-square:

$$\frac{\delta \upsilon}{\upsilon} = [(2\frac{\delta R}{R})^2 + (\frac{\delta H}{H})^2]^{1/2} = [\{2(2\%)\}^2 + (3\%)^2]^{1/2} = (16+9)^{1/2} = (25)^{1/2} = \mathbf{5\%} \quad \textit{Ans.}$$

Because it is doubled, the error in radius contributes more to the overall uncertainty.

P1.87 Use the theory of Prob. 1.49 for a shaft 8 cm long, rotating at 1200 r/min, with $r_i = 2.00$ cm and $r_o = 2.05$ cm. (*a*) If the measured torque is 0.293 N·m, what is the fluid viscosity? (*b*) Suppose that the uncertainties in the experiment are as follows: L (±0.5 mm), M (±0.003 N-m), Ω (±1 percent), r_i and r_o (±0.02 mm). Estimate the overall uncertainty of the measured viscosity.

Solution: First convert 1200 r/min to (1200)(2π/60) = 125.7 rad/s. The analytical solution for viscosity, from Prob. 1.49, is to be evaluated for the given data:

$$\mu = \frac{M\,(r_o - r_i)}{2\pi\,\Omega\,r_i^3\,L} = \frac{(0.293\,N-m)(0.0205-0.0200\,m)}{2\pi(125.7\,rad/s)(0.02m)^3(0.08m)} = \mathbf{0.29\frac{kg}{m-s}} \quad \textit{Ans.(a)}$$

This could be SAE 30W oil! (*b*) Now express the uncertainties as fractions. The only tricky thing is the *clearance* ($r_o - r_i$): It is highly unlike that the variations in r will exactly oppose each other. There we adopt a root-mean-square approach for the clearance:

$$\frac{\delta r_o}{(r_o - r_i)} = \frac{\delta r_i}{(r_o - r_i)} = \frac{0.02mm}{0.5\,mm} = 0.04 \;;\; \text{Then } \frac{\delta(r_o - r_i)}{(r_o - r_i)} = [(0.04)^2 + (0.04)^2]^{1/2} = 0.0566$$

$$\frac{\delta L}{L} = \frac{0.05\,mm}{80\,mm} = 0.000625 \;;\; \frac{\delta M}{M} = \frac{0.003}{0.293} = 0.0102 \;;\; \frac{\delta\Omega}{\Omega} = 0.01 \;;\; \frac{\delta r_i^3}{r_i^3} = \frac{3\delta r_i}{r_i} = 3(\frac{0.02mm}{20\,mm}) = 0.003$$

$$\text{Thus } \frac{\delta\mu}{\mu} = [(0.0566)^2 + (0.000625)^2 + (0.0102)^2 + (0.01)^2 + (0.003)^2]^{1/2} = 0.059 \text{ or } \mathbf{5.9\%} \quad \textit{Ans.(b)}$$

By far the largest contribution to the uncertainty comes from the error in clearance. The writer is aware that others may have a different approach and result for this problem.

P1.88 The device in Fig. P1.54 is called a *rotating disk viscometer* [28]. Suppose that $R = 5$ cm and $h = 1$ mm. (*a*) If the torque required to rotate the disk at 900 r/min is 0.537 N·m, what is the viscosity of the fluid? (*b*) If the uncertainty in each parameter (M, R, h, Ω) is ±1%, what is the overall uncertainty in the viscosity?

Solution: From Prob. 1.54, the analytical result for the required torque M is

$$M = \frac{\pi \mu \Omega R^4}{h}, \quad \text{rewrite this as} \quad \mu = \frac{M h}{\pi \Omega R^4}$$

(a) First convert Ω to radians: Ω = 900 r/min x 2π/60 = 94.2 rad/s. Then apply the formula:

$$\mu = \frac{M h}{\pi \Omega R^4} = \frac{(0.537\,N-m)(0.001m)}{\pi(94.2rad/s)(0.05m)^4} = 0.29\frac{N-s}{m^2} = \mathbf{0.29\frac{kg}{m-s}} \quad Ans.(a)$$

This could be SAE 30W oil!

(*b*) in calculating uncertainty, the only complication is the term R^4, whose uncertainty, from Eq. (1.44), is 4 times the uncertainty in R:

$$\frac{\delta R^4}{R^4} \approx 4\frac{\delta R}{R}$$

That said, the overall uncertainty in viscosity is calculated as

$$\frac{\delta\mu}{\mu} = [(\frac{\delta M}{M})^2 + (\frac{\delta h}{h})^2 + (\frac{\delta\Omega}{\Omega})^2 + (\frac{\delta R^4}{R^4})^2]^{1/2}$$

$$= [(1\%)^2 + (1\%)^2 + (1\%)^2 + \{4(1\%)\}^2]^{1/2} = \sqrt{19} = \mathbf{4.4\%} \quad Ans.(b)$$

Clearly the error in R is responsible for almost all of the uncertainty in the viscosity.

P1.89 For the cone-plate viscometer of Fig. P1.56, suppose $R = 6$ cm and $\theta = 3°$. (*a*) If the torque M required to rotate the cone at 600 r/min is 0.157 N·m, what is the viscosity of the fluid? (*b*) If the uncertainty in each parameter (M, R, θ, Ω) is ±2%, what is the overall uncertainty in the viscosity?

Solution: First convert 600 r/min to (600)(2π/60) = 62.8 rad/s. (a) From Prob. 1.5, the analytical result for the measured viscosity μ is calculated as

$$\mu = \frac{3M\sin\theta}{2\pi\Omega R^3} = \frac{3(0.157N-m)\sin(3^\circ)}{2\pi(62.8\,rad/s)(0.06m)^3} = 0.29\frac{N-s}{m^2} = \mathbf{0.29\frac{kg}{m-s}} \quad Ans.(a)$$

Once again, this could be SAE 30W oil! (*b*) The only complication in the uncertainty calculation is that the error in R^3 is three times the error in *R*:

$$\frac{\delta R^3}{R^3} \approx \frac{(\frac{\partial R^3}{\partial R})\delta R}{R^3} = \frac{3R^2\delta R}{R^3} = 3\frac{\delta R}{R}$$

Then the overall uncertainty is computed as

$$\frac{\delta\mu}{\mu} = [(\frac{\delta M}{M})^2 + (\frac{\delta\theta}{\theta})^2 + (\frac{\delta\Omega}{\Omega})^2 + (\frac{\delta R^3}{R^3})^2]^{1/2}$$
$$= [(2\%)^2 + (2\%)^2 + (2\%)^2 + \{3(2\%)\}^2]^{1/2} = \sqrt{48} = \mathbf{6.9\%} \quad Ans.(b)$$

The radius R, whose error is tripled, dominates the uncertainty.

P1.90 The dimensionless *drag coefficient* C_D of a sphere, to be studied in Chaps. 5 and 7, is

$$C_D = \frac{F}{(1/2)\rho V^2(\pi/4)D^2}$$

where *F* is the drag force, ρ the fluid density, *V* the fluid velocity, and *D* the sphere diameter. If the uncertainties of these variables are *F* (±3%), ρ (±1.5%), *V* (±2%), and *D* (±1%), what is the overall uncertainty in the measured drag coefficient?

Solution: Since *F* and ρ occur alone, i.e. to the 1st power, their uncertainties are as stated. However, both *V* and *D* are squared, so the relevant uncertainties are doubled:

$$\frac{\delta V^2}{V^2} = 2\frac{\delta V}{V} \quad ; \quad \frac{\delta D^2}{D^2} = 2\frac{\delta D}{D}$$

Putting together the four different uncertainties in the definition of C_D, we obtain

$$\frac{\delta C_D}{C_D} = [(\frac{\delta F}{F})^2 + (\frac{\delta\rho}{\rho})^2 + (\frac{\delta V^2}{V^2})^2 + (\frac{\delta D^2}{D^2})^2]^{1/2}$$
$$= [(3\%)^2 + (1.5\%)^2 + \{2(2\%)\}^2 + \{2(1\%)\}^2] = \sqrt{31.25} = \mathbf{5.6\%} \quad Ans.$$

The largest contribution comes from the uncertainty in velocity.

FUNDAMENTALS OF ENGINEERING EXAM PROBLEMS: <u>Answers</u>

FE-1.1 The absolute viscosity μ of a fluid is primarily a function of
(a) density **(b) <u>temperature</u>** (c) pressure (d) velocity (e) surface tension

FE-1.2 If a uniform solid body weighs 50 N in air and 30 N in water, its specific gravity is
(a) 1.5 (b) 1.67 **(c) <u>2.5</u>** (d) 3.0 (e) 5.0

FE-1.3 Helium has a molecular weight of 4.003. What is the weight of 2 cubic meters of helium at 1 atmosphere and 20°C?
(a) <u>3.3 N</u> (b) 6.5 N (c) 11.8 N (d) 23.5 N (e) 94.2 N

FE-1.4 An oil has a kinematic viscosity of 1.25E–4 m^2/s and a specific gravity of 0.80. What is its dynamic (absolute) viscosity in kg/(m·s)?
(a) 0.08 **(b) <u>0.10</u>** (c) 0.125 (d) 1.0 (e) 1.25

FE-1.5 Consider a soap bubble of diameter 3 mm. If the surface tension coefficient is 0.072 N/m and external pressure is 0 Pa gage, what is the bubble's internal gage pressure?
(a) −24 Pa (b) +48 Pa (c) +96 Pa **(d) <u>+192 Pa</u>** (e) −192 Pa

FE-1.6 The only possible dimensionless group which combines velocity V, body size L, fluid density ρ, and surface tension coefficient σ is:
(a) $L\rho\sigma/V$ (b) $\rho VL^2/\sigma$ (c) $\rho\sigma V^2/L$ (d) $\sigma LV^2/\rho$ **(e) <u>$\rho LV^2/\sigma$</u>**

FE-1.7 Two parallel plates, one moving at 4 m/s and the other fixed, are separated by a 5-mm-thick layer of oil of specific gravity 0.80 and kinematic viscosity 1.25E–4 m^2/s. What is the average shear stress in the oil?
(a) <u>80 Pa</u> (b) 100 Pa (c) 125 Pa (d) 160 Pa (e) 200 Pa

FE-1.8 Carbon dioxide has a specific heat ratio of 1.30 and a gas constant of 189 J/(kg·°C). If its temperature rises from 20°C to 45°C, what is its internal energy rise?
(a) 12.6 kJ/kg **(b) <u>15.8 kJ/kg</u>** (c) 17.6 kJ/kg (d) 20.5 kJ/kg (e) 25.1 kJ/kg

(b)

FE-1.9 A certain water flow at 20°C has a critical cavitation number, where bubbles form, Ca ≈ 0.25, where $Ca = 2(p_a - p_{vap})/(\rho V^2)$. If p_a = 1 atm and the vapor pressure is 0.34 psia, for what water velocity will bubbles form?
(a) 12 mi/hr (b) 28 mi/hr (c) 36 mi/hr (d) 55 mi/hr **(e) <u>63 mi/hr</u>**

FE-1.10 Example 1.10 gave an analysis which predicted that the viscous moment on a rotating disk was $M = \pi\mu\Omega R^4/(2h)$. If the uncertainty of each of the four variables (μ, Ω, R, h) is 1.0 %, what is the estimated overall uncertainty of the moment M?
(*a*) 4.0 % , (*b*) **<u>4.4 %</u>** , (*c*) 5.0 % , (*d*) 6.0 % , (*e*) 7.0 %

COMPREHENSIVE PROBLEMS

C1.1 Sometimes equations can be developed and practical problems solved by knowing nothing more than the dimensions of the key parameters. For example, consider the heat loss through a window in a building. Window efficiency is rated in terms of "R value," which has units of $ft^2·hr·°F/Btu$. A certain manufacturer offers a double-pane window with R = 2.5 and also a triple-pane window with R = 3.4. Both windows are 3 ft by 5 ft. On a given winter day, the temperature difference between inside and outside is 45°F. (a) Develop and equation for window heat loss Q, in time period Δt, as a function of window area A, R value, and temperature difference ΔT. How much heat is lost through the above (a) double-pane window, or (b) triple-pane window, in 24 hours? (c) Suppose the building is heated with propane gas, at $3.25 per gallon, burning at 80% efficiency. Propane has 90,000 Btu of available energy per gallon. In a 24-hour period, how much money would a homeowner save, per window, by installing a triple-pane rather than a double-pane window? (d) Finally, suppose the homeowner buys 20 such triple-pane windows for the house. A typical winter equals about 120 heating days at ΔT = 45°F. Each triple-pane window costs $85 more than the double-pane window. Ignoring interest and inflation, how many years will it take the homeowner to make up the additional cost of the triple-pane windows from heating bill savings?

Solution: (a) The function Q = fcn(Δt, R, A, ΔT) must have units of Btu. The only combination of units which accomplishes this is:

$$Q = \frac{\Delta \mathbf{t} \Delta T A}{R} \quad Ans. \qquad Thus\ Q_{lost} = \frac{(24\ hr)(45°F)(3\ ft \cdot 5\ ft)}{2.5\ ft^2 \cdot hr \cdot °F/Btu} = \mathbf{6480\ Btu} \quad Ans.\,(a)$$

(b) Triple-pane window: use R = 3.4 instead of 2.5 to obtain $Q_{3\text{-pane}}$ = **4760 Btu** *Ans*. (b)

(c) The savings, using propane, for one triple-pane window for one 24-hour period is:

$$\Delta Cost = \frac{\$3.25/gal}{90000\,Btu/gal}(6480 - 4760\,Btu)\frac{1}{0.80_{efficiency}} = \$0.078 = \mathbf{7.8\ cents} \quad Ans.(c)$$

(d) Extrapolate to 20 windows, 120 cold days per year, and $85 extra cost per window:

$$Pay-back\ time = \frac{\$85/window}{(0.078\,\$/window/day)(120\,days/year)} = \mathbf{9\ years} \qquad Ans.(d)$$

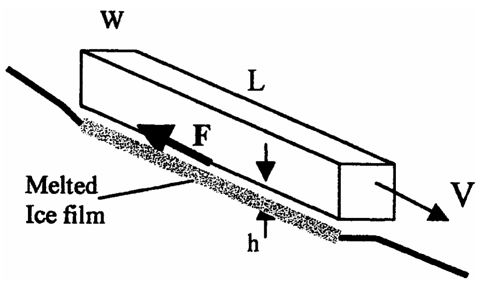
Not a very good investment. We are using '$' and 'windows' as "units" in our equations!

C1.2 When a person ice-skates, the ice surface actually melts beneath the blades, so that he or she skates on a thin film of water between the blade and the ice. (a) Find an expression for total friction force F on the bottom of the blade as a function of skater velocity V, blade length L, water film thickness h, water viscosity μ, and blade width W. (b) Suppose a skater of mass *m*, moving at constant speed V_o, suddenly stands stiffly with skates pointed directly forward and allows herself to coast to a stop. Neglecting air resistance, how far will she travel (on *two* blades) before she stops? Give the answer X as a function of (V_o, *m*, L, h, μ, W). (c) Compute X for the case V_o = 4 m/s, m = 100 kg, L = 30 cm, W = 5 mm, and h = 0.1 mm. Do you think our assumption of negligible air resistance was a good one?

Solution: (a) The skate bottom and the melted ice are like two parallel plates:

$$\tau = \mu\frac{V}{h},\quad F = \tau A = \frac{\mu VLW}{h}\quad \textit{Ans. (a)}$$

(b) Use **F** = m**a** to find the stopping distance:



$$\Sigma F_x = -F = -\frac{2\mu VLW}{h} = ma_x = m\frac{dV}{dt}$$

(*the* '2' *is for two blades*)

Separate and integrate once to find the velocity, once again to find the distance traveled:

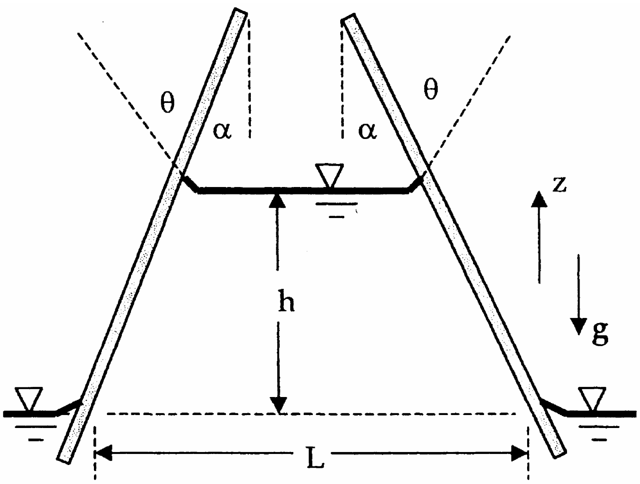
$$\int\frac{dV}{V} = -\int\frac{2\mu LW}{mh}dt,\quad or:\quad V = V_o e^{\frac{-2\mu LW}{mh}t},\quad X = \int_0^\infty V\,dt = \frac{V_o mh}{2\mu LW}\quad \textit{Ans. (b)}$$

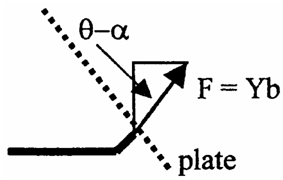
(c) Apply our specific numerical values to a 100-kg (!) person:

$$X = \frac{(4.0\ m/s)(100\ kg)(0.0001\ m)}{2(1.788E\text{–}3\ kg/m\cdot s)(0.3\ m)(0.005\ m)} = \mathbf{7460\ m\,(!)}\quad \textit{Ans. (c)}$$

We could coast to the next town on ice skates! It appears that our assumption of negligible air drag was grossly incorrect.

C1.3 Two thin flat plates are tilted at an angle α and placed in a tank of known surface tension Y and contact angle θ, as shown. At the free surface of the liquid in the tank, the two plates are a distance *L* apart, and of width *b* into the paper. (a) What is the total *z*-directed force, due to surface tension, acting on the liquid column between plates? (b) If the liquid density is ρ, find an expression for Y in terms of the other variables.





Solution: (a) Considering the right side of the liquid column, the surface tension acts tangent to the local surface, that is, along the dashed line at right. This force has magnitude F = Yb, as shown. Its vertical component is $F\cos(\theta-\alpha)$, as shown. There are two plates. Therefore, the total *z*-directed force on the liquid column is

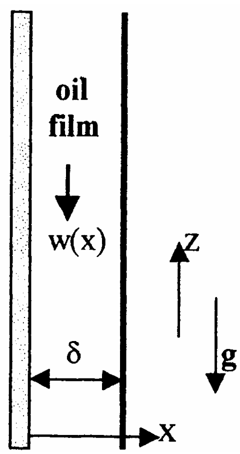
$$\mathbf{F_{vertical} = 2Yb\cos(\theta - \alpha)} \quad \textit{Ans.}\ \text{(a)}$$

(b) The vertical force in (a) above holds up the entire weight of the liquid column between plates, which is $W = \rho g\{bh(L - h\tan\alpha)\}$. Set W equal to F and solve for

$$\mathbf{U = [\rho gbh(L - h\tan\alpha)]/[2\cos(\theta - \alpha)]} \quad \textit{Ans.}\ \text{(b)}$$

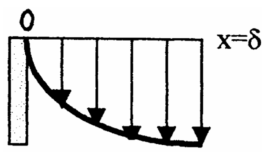
C1.4 Oil of viscosity μ and density ρ drains steadily down the side of a tall, wide vertical plate, as shown. The film is fully developed, that is, its thickness δ and velocity profile w(x) are independent of distance *z* down the plate. Assume that the atmosphere offers no shear resistance to the film surface.

(a) Sketch the approximate shape of the velocity profile w(x), keeping in mind the boundary conditions.

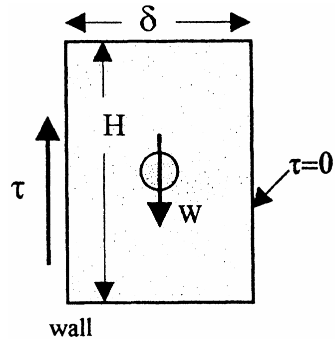


(b) Suppose film thickness d is measured, along with the slope of the velocity profile at the wall, $(dw/dx)_{wall}$, with a laser-Doppler anemometer (Chap. 6). Find an expression for μ as a function of ρ, δ, $(dw/dx)_{wall}$, and g. Note that both w and $(dw/dx)_{wall}$ will be negative as shown.

Solution: (a) The velocity profile must be such that there is no slip (w = 0) at the wall and no shear (dw/dx = 0) at the film surface. This is shown at right. *Ans.* (a)



(b) Consider a freebody of any vertical length H of film, as at right. Since there is no acceleration (fully developed film), the weight of the film must exactly balance the shear force on the wall:



$$W = \rho g(H\delta b) = \tau_{wall}(Hb), \quad \tau_{wall} = -\mu \frac{dw}{dx}\Big|_{wall}$$

Solve this equality for the fluid viscosity:

$$\boldsymbol{\mu = \frac{-\rho g \delta}{(dw/dx)_{wall}}} \quad \textit{Ans.}\text{ (b)}$$

C1.5 Viscosity can be measured by flow through a thin-bore or *capillary* tube if the flow rate is low. For length L, (small) diameter $D \ll L$, pressure drop Δp, and (low) volume flow rate Q, the formula for viscosity is $\mu = D^4\Delta p/(CLQ)$, where C is a constant. (a) Verify that C is dimensionless. The following data are for water flowing through a 2-mm-diameter tube which is 1 meter long. The pressure drop is held constant at Δp = 5 kPa.

T, °C:	10.0	40.0	70.0
Q, L/min:	0.091	0.179	0.292

(b) Using proper SI units, determine an average value of C by accounting for the variation with temperature of the viscosity of water.

Solution: (a) Check the dimensions of the formula and solve for $\{C\}$:

$$\{\mu\} = \left\{\frac{M}{LT}\right\} = \left\{\frac{D^4\Delta p}{CLQ}\right\} = \left\{\frac{L^4(ML^{-1}T^{-2})}{\{C\}(L)(L^3/T)}\right\} = \left\{\frac{M}{LT\{C\}}\right\},$$

therefore **{C} = {1}** ***Dimensionless*** *Ans.* (a)

(b) Use the given data, with values of μ_{water} from Table A.1, to evaluate C, with L = 1 m, D = 0.002 m, and Δp = 5000 Pa. Convert the flow rate from L/min to m^3/s.

T, °C:	10.0	40.0	70.0
Q, m^3/s:	1.52E–6	2.98E–6	4.87E–6
μ_{water}, kg/m-s:	1.307E–3	0.657E–3	0.405E–3
$\boldsymbol{C = D^4\Delta p/(\mu LQ)}$:	40.3	40.9	40.6

The estimated value of $\boldsymbol{C = 40.6 \pm 0.3}$. The theoretical value (Chap. 4) is C = 128/π = 40.74.

C1.6 The *rotating-cylinder viscometer* in Fig. C1.6 shears the fluid in a narrow clearance, Δr, as shown. Assume a linear velocity distribution in the gaps. If the driving torque M is measured, find an expression for μ by (a) neglecting, and (b) including the bottom friction.

Solution: (a) The fluid in the annular region has the same shear stress analysis as Prob. 1.49:

$$M = \int R\,dF = \int (R)(\tau)\,dA \int_0^{2\pi} R\left(\mu\frac{\Omega R}{\Delta R}\right) RL\,d\theta = 2\pi\mu\frac{\Omega R^3 L}{\Delta R},$$

$$or: \quad \boldsymbol{\mu = \frac{M\Delta R}{2\pi\Omega R^3 L}} \quad \textit{Ans.}\text{ (a)}$$

(b) Now add in the moment of the (variable) shear stresses on the bottom of the cylinder:

$$M_{bottom} = \int r\tau\,dA = \int_0^R r\left(\mu\frac{\Omega r}{\Delta R}\right) 2\pi r\,dr$$

$$= \frac{2\pi\Omega\mu}{\Delta R}\int_0^R r^3\,dr = \frac{2\pi\Omega\mu R^4}{4\Delta R}$$

$$Thus \quad M_{total} = \frac{2\pi\Omega\mu R^3 L}{\Delta R} + \frac{2\pi\Omega\mu R^4}{4\Delta R}$$

$$Solve\ for \quad \mu = \boldsymbol{\frac{M\Delta R}{2\pi\Omega R^3(L + R/4)}} \quad \textit{Ans.}\text{ (b)}$$

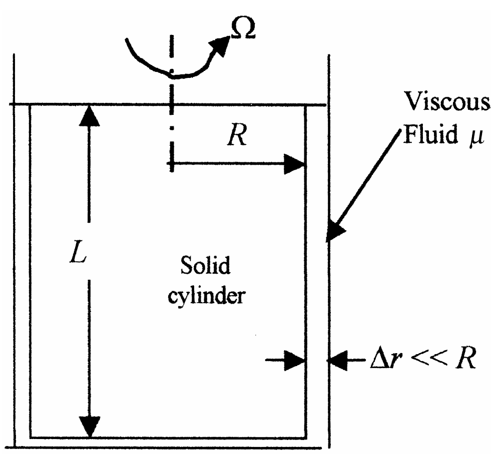


Fig. C1.6

C1.7 SAE 10W oil at 20°C flows past a flat surface, as in Fig. 1.4(b). The velocity profile $u(y)$ is measured, with the following results:

y, m:	0.0	0.003	0.006	0.009	0.012	0.015
u, m/s:	0.0	1.99	3.94	5.75	7.29	8.46

Using your best interpolating skills, estimate the shear stress in the oil (a) at the wall (y = 0); and (b) at y = 15 mm.

Solution: For SAE10W oil, from Table A.3, read μ = 0.104 kg/m·s. We need to estimate the derivative (du/dy) at the two values of y, then compute $\tau = \mu(du/dy)$.

Method 1: Use a Newton-Raphson three-point derivative estimate.
At three equally-spaced points, $du/dy|_{y_o} \approx (-3u_o + 4u_1 - u_2)/(2\Delta y)$. Thus

(a) at y = 0: $du/dy|_{y=0} \approx [-3(0.00) + 4(1.99) - (3.94)]/(2\{0.003\}) = 670\ s^{-1}$

$$\text{Then}\quad \tau = \mu(du/dy) = (670\ s^{-1})(0.104\ kg/m\cdot s) \approx \mathbf{70\ Pa} \quad \textit{Ans.}\ (a)$$

(b) at y = 0.015 m: $du/dy|_{y=0} \approx [3(8.46) - 4(7.29) + (5.75)]/(2\{0.003\}) = 328\ s^{-1}$

$$\text{Then}\quad \tau = \mu(du/dy) = (328\ s^{-1})(0.104\ kg/m\cdot s) \approx \mathbf{34\ Pa} \quad \textit{Ans.}\ (b)$$

Method 2: Type the six data points into Excel and run a cubic "trendline" fit. The result is

$$u \approx 656.2y + 4339.8y^2 - 699163y^3$$

Differentiating this polynomial at $y = 0$ gives $du/dy \approx 656.2\ s^{-1}$, $\tau \approx$ **68 Pa** *Ans*. (a)

Differentiating this polynomial at $y = 0.015$ gives $du/dy \approx 314\ s^{-1}$, $\tau \approx$ **33 Pa** *Ans*. (b)

C1.8 A mechanical device, which uses the rotating cylinder of Fig. C1.6, is the *Stormer viscometer* [Ref. 29 of Chap. 1]. Instead of being driven at constant Ω, a cord is wrapped around the shaft and attached to a falling weight W. The time t to turn the shaft a given number of revolutions (usually 5) is measured and correlated with viscosity. The Stormer formula is

$$t = A\mu/(W - B)$$

where A and B are constants which are determined by calibrating the device with a known fluid. Here are calibration data for a Stormer viscometer tested in glycerol, using a weight of 50 N:

μ, kg/m·s:	0.23	0.34	0.57	0.84	1.15
t, sec:	15	23	38	56	77

(a) Find reasonable values of A and B to fit this calibration data. [*Hint*: The data are not very sensitive to the value of B.] (b) A more viscous fluid is tested with a 100-N weight and the measured time is 44 s. Estimate the viscosity of this fluid.

Solution: (a) The data fit well, with a standard deviation of about 0.17 s in the value of t, to the values

$$A \approx \mathbf{3000} \quad \text{and} \quad B \approx \mathbf{3.5} \quad \textit{Ans.}\ \text{(a)}$$

(b) With a new fluid and a new weight, the values of A and B should nevertheless be the same:

$$t = 44\ s \approx \frac{A\mu}{W-B} = \frac{3000\mu}{100\ N - 3.5}, \quad \textit{solve for}\ \mu_{new\ fluid} \approx \mathbf{1.42}\ \frac{kg}{m \cdot s} \quad \textit{Ans.}\ \text{(b)}$$

C1.9 The lever in Fig. P1.101 has a weight W at one end and is tied to a cylinder at the other end. The cylinder has negligible weight and buoyancy and slides upward through a film of heavy oil of viscosity μ. (*a*) If there is no acceleration (uniform lever rotation) derive a formula for the rate of fall V_2 of the weight. Neglect the lever weight. Assume a linear velocity profile in the oil film.

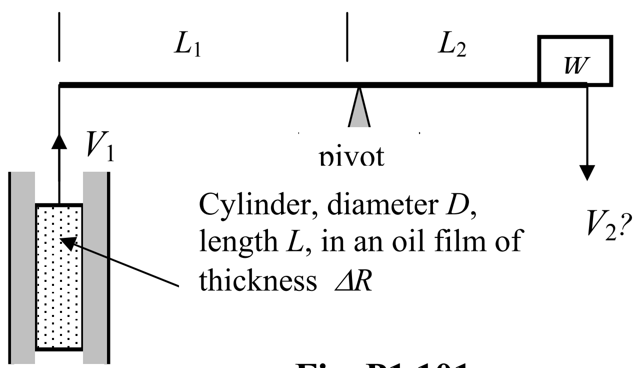


Fig. P1.101

(*b*) Estimate the fall velocity of the weight if W = 20 N, L_1 = 75 cm, L_2 = 50 cm, D = 100 cm, L = 22 cm, ΔR = 1 mm, and the oil is glycerin at 20°C.

Solution: (a) If the motion is uniform, no acceleration, then the moments balance about the pivot:

$$\sum M_{pivot} = 0 = WL_2 - F_1 L_1, \quad \text{where} \quad F_1 = \tau_w A_w = (\mu \frac{V_1}{\Delta R})(\pi DL)$$

Since the lever is rigid, the endpoint velocities vary according to their lengths from the pivot:

$$V_1 = \Omega L_1 \ ; \ V_2 = \Omega L_2 \qquad \therefore \quad V_1 / L_1 = V_2 / L_2$$

Combine these two relations to obtain the desired solution for a highly viscous fluid:

$$V_2 = V_1(L_2/L_1) = \frac{W\,\Delta R}{\mu\pi DL}(L_2/L_1)^2 \qquad Ans.(a)$$

(*b*) For glycerin at 20°C, from Table A.3, $\mu = 1.49$ kg/m-s. The formula above yields

$$V_2 = \frac{W\,\Delta R}{\mu\pi DL}\left(\frac{L_2}{L_1}\right)^2 = \frac{(20\,N)(0.001\,m)}{(1.49N-s/m^2)\pi(0.1\,m)(0.22\,m)}\left(\frac{75\,cm}{50\,cm}\right)^2 \approx \mathbf{0.44}\ \frac{\mathbf{m}}{\mathbf{s}} \quad Ans.(b)$$

C1.10 A popular gravity-driven instrument is the Cannon-Ubbelohde viscometer, shown in Fig. C1.10. The test liquid is drawn up above the bulb on the right side and allowed to drain by gravity through the capillary tube below the bulb. The time *t* for the meniscus to pass from upper to lower timing marks is recorded. The kinematic viscosity is computed by the simple formula $\nu = Ct$, where C is a calibration constant. For ν in the range of 100-500 mm^2/s, the recommended constant is C = 0.50 mm^2/s^2, with an accuracy less than 0.5%.

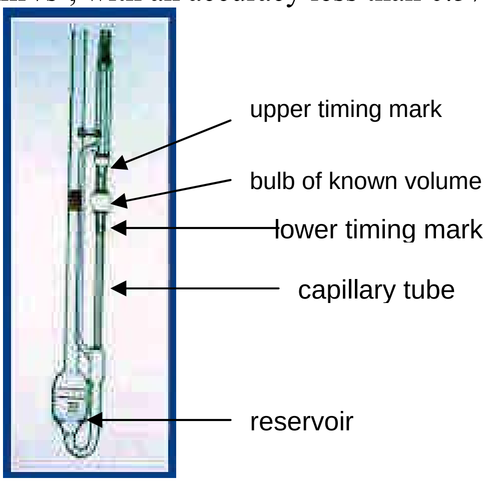


Fig. C1.10

The Cannon-Ubbelohde viscometer.

(*a*) What liquids from Table A.3 are in this viscosity range? (*b*) Is the calibration formula dimensionally consistent? (*c*) What system properties might the constant C depend upon? (*d*) What problem in this chapter hints at a formula for estimating the viscosity?

Solution: (*a*) Very hard to tell, because values of ν are not listed – sorry, I'll add these values if I remember. It turns out that only *three* of these 17 liquids are in the 100-500 mm^2/s range: SAE 10W, 10W30, and 30W oils, at 120, 194, and 326 mm^2/s respectively. (*b*) No, the formula is dimensionally inconsistent because C has units; thus C = 0.5 is appropriate *only* for a mm^2/s result. (*c*) Hidden in C are the length and diameter of the capillary tube and the acceleration of gravity - see Eq. (6.12) later. (*d*) Problem P1.58 gives a formula for flow through a capillary tube. If the flow is vertical and gravity driven, $\Delta p = \rho g L$ and $Q = (\pi/4)d^2V$, leaving a new formula as follows: $\nu \approx gd^2/(32V)$.

C1.11 Mott [Ref. 50, p. 38] discusses a simple falling-ball viscometer, which we can analyze later in Chapter 7. A small ball of diameter D and density ρ_b falls though a tube of test liquid. The fall velocity V is calculated by the time to fall a measured distance. The formula for calculating the viscosity of the fluid is
This result is limited by the requirement that the Reynolds number ($\rho VD/\mu$) be less than 1.0. Suppose a steel ball (SG = 7.87) of diameter 2.2 mm falls in SAE 25W oil (SG = 0.88) at 20°C.

$$\mu = \frac{(\rho_b - \rho)\, g\, D^2}{18\, V}$$

The measured fall velocity is 8.4 cm/s. (*a*) What is the viscosity of the oil, in kg/m-s? (*b*) Is the Reynolds number small enough for a valid estimate?

Solution: Relating SG to water, Eq. (1.7), the steel density is 7.87(1000) = 7870 kg/m^3 and the oil density is 0.88(1000) = 880 kg/m^3. Using SI units, the formula predicts

$$\mu_{oil} = \frac{(7870 - 880\, kg/m^3)(9.81\, m/s^2)(0.0022\, m)^2}{18(0.084\, m/s)} \approx \mathbf{0.22\ \frac{kg}{m-s}} \quad \textit{Ans.}$$

$$\text{Check} \quad \text{Re} = \frac{\rho VD}{\mu} = \frac{(880)(0.084)(0.0022)}{0.22} = 0.74 < 1.0 \quad \textit{OK}$$

As mentioned, we shall analyze this falling sphere problem in Chapter 7.
