Solution: (a) The total volume of oil in the vat is \((3 \text{ m})(7 \text{ m})(4 \text{ m} + 2 \text{ m})/2 = 63 \text{ m}^3\). Therefore the weight of oil in the vat is

\[ W = \gamma_{\text{oil}} (\text{Vol}) = (0.85)(9790 \text{ N/m}^3)(63 \text{ m}^3) = 524,000 \text{ N} \quad \text{Ans. (a)} \]

(b) The force on the horizontal bottom surface of the vat is

\[ F_{\text{bottom}} = \gamma_{\text{oil}} h_{\text{CG}} A_{\text{bottom}} = (0.85)(9790)(3 \text{ m})(2 \text{ m})(7 \text{ m}) = 350,000 \text{ N} \quad \text{Ans. (b)} \]

Note that \(F\) is less than the total weight of oil—the student might explain why they differ?

(c) I found in my statics book that the centroid of this trapezoid is 1.33 m below the surface, or 1.67 m above the bottom, as shown. Therefore the side-panel force is

\[ F_{\text{side}} = \gamma_{\text{oil}} h_{\text{CG}} A_{\text{side}} = (0.85)(9790)(1.33 \text{ m})(9 \text{ m}^2) = 100,000 \text{ N} \quad \text{Ans. (c)} \]

These are large forces. Big vats have to be strong!

\[ \text{2.51} \quad \text{Gate AB in Fig. P2.51 is 1.2 m long and 0.8 m into the paper. Neglecting atmospheric-pressure effects, compute the force } F \text{ on the gate and its center of pressure position } X. \]

Solution: The centroidal depth of the gate is

\[ h_{\text{CG}} = 4.0 + (1.0 + 0.6)\sin 40^\circ = 5.028 \text{ m}, \]

hence \( F_{\text{AB}} = \gamma_{\text{oil}} h_{\text{CG}} A_{\text{gate}} = (0.82 \times 9790)(5.028)(1.2 \times 0.8) = 38750 \text{ N} \quad \text{Ans.} \)

The line of action of \(F\) is slightly below the centroid by the amount

\[ y_{\text{CP}} = -\frac{I_{xx}\sin \theta}{h_{\text{CG}} A} = -\frac{(1/12)(0.8)(1.2)^3\sin 40^\circ}{(5.028)(1.2 \times 0.8)} = -0.0153 \text{ m} \]

Thus the position of the center of pressure is at \(X = 0.6 + 0.0153 \approx 0.615 \text{ m} \quad \text{Ans.} \)