symmetrically along the barrier. Calculate the horizontal force $F$ needed, per unit thickness into the paper, to hold the barrier stationary.

Solution: For water take $\rho=998 \mathrm{~kg} / \mathrm{m}$. The control volume (see figure) cuts through all four jets, which are numbered. The velocity of all jets follows from the weight flow at (1):

$$
\begin{aligned}
& V_{1,2,3,4}=V_{1}=\frac{\dot{w}_{1}}{\rho g A_{1}}=\frac{1960 \mathrm{~N} / \mathrm{s}}{\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)\left(998 \mathrm{~kg} / \mathrm{m}^{3}\right)(0.04 \mathrm{~m})(1 \mathrm{~m})}=5.0 \frac{\mathrm{~m}}{\mathrm{~s}} \\
& \dot{m}_{1}=\frac{\dot{w}_{1}}{g}=\frac{1960 \mathrm{~N} / \mathrm{s}-\mathrm{m}}{9.81 \mathrm{~N} / \mathrm{s}^{2}}=200 \frac{\mathrm{~kg}}{\mathrm{~s}-\mathrm{m}} ; \dot{m}_{2}=0.3 \dot{m}_{1}=60 \frac{\mathrm{~kg}}{\mathrm{~s}-\mathrm{m}} ; \dot{m}_{3}=\dot{m}_{4}=70 \frac{\mathrm{~kg}}{\mathrm{~s}-\mathrm{m}}
\end{aligned}
$$

Then the $x$-momentum relation for this control volume yields

$$
\begin{aligned}
& \Sigma F_{x}=-F=\dot{m}_{2} u_{2}+\dot{m}_{3} u_{3}+\dot{m}_{4} u_{4}-\dot{m}_{1} u_{1}= \\
& -F=(60)(5.0)+(70)\left(-5.0 \cos 55^{\circ}\right)+(70)\left(-5.0 \cos 55^{\circ}\right)-200(5.0), \text { or }: \\
& \quad F=1000+201+201-300 \approx \mathbf{1 1 0 0} \mathbf{N} \text { per meter of width Ans. }
\end{aligned}
$$

3.31 A bellows may be modeled as a deforming wedge-shaped volume as in Fig. P3.31. The check valve on the left (pleated) end is closed during the stroke. If $b$ is the bellows width into the paper, derive an expression for outlet mass flow $\dot{m}_{o}$ as a function of stroke $\theta(t)$.

Solution: For a control volume enclosing the bellows and the outlet flow, we obtain


Fig. P3.31

$$
\frac{\mathrm{d}}{\mathrm{dt}}(\rho v)+\dot{\mathrm{m}}_{\text {out }}=0, \quad \text { where } v=\mathrm{bhL}=\mathrm{bL}^{2} \tan \theta
$$

since L is constant, solve for $\dot{\mathrm{m}}_{\mathrm{o}}=-\frac{\mathrm{d}}{\mathrm{dt}}\left(\rho b \mathrm{~L}^{2} \tan \theta\right)=-\rho \mathbf{b} \mathrm{L}^{2} \sec ^{2} \theta \frac{\mathbf{d} \theta}{\mathbf{d t}}$ Ans.

