Solution: Let the CV cut through the flanges and surround the pipe bend. The mass flow rate is \((230 \text{ N/s})/(9.81 \text{ m/s}^2) = 23.45 \text{ kg/s}\). The volume flow rate is \(Q = 230/9790 = 0.0235 \text{ m}^3/\text{s}\). Then the pipe inlet and exit velocities are the same magnitude:

\[
V_1 = V_2 = V = \frac{Q}{A} = \frac{0.0235 \text{ m}^3/\text{s}}{(\pi/4)(0.05 \text{ m})^2} \approx 12.0 \frac{\text{m}}{\text{s}}
\]

Subtract \(p_a\) everywhere, so only \(p_1\) and \(p_2\) are non-zero. The horizontal force balance is:

\[
\sum F_x = F_{x,\text{flange}} + (p_1 - p_a)A_1 + (p_2 - p_a)A_2 = m_2u_2 - m_1u_1
\]

\[
= F_{x,\text{fl}} + \left(\frac{64000}{4}(0.05)^2 + \frac{33000}{4}(0.05)^2\right) = (23.45)(-12.0 - 12.0 \text{ m/s})
\]

or: \(F_{x,\text{flange}} = -126 - 65 - 561 \approx -750 \text{ N} \quad \text{Ans.}\)

The total \(x\)-directed force on the flanges acts to the left. The vertical force balance is

\[
\sum F_y = F_{y,\text{flange}} = W_{\text{pipe}} + W_{\text{fluid}} = 0 + (9790)(0.05)^2(0.75) \approx 14 \text{ N} \quad \text{Ans.}
\]

Clearly the fluid weight is pretty small. The largest force is due to the \(180^\circ\) turn.

3.44 Consider uniform flow past a cylinder with a V-shaped wake, as shown. Pressures at (1) and (2) are equal. Let \(b\) be the width into the paper. Find a formula for the force \(F\) on the cylinder due to the flow. Also compute \(C_D = F/(\rho U^2 L b)\).

**Solution:** The proper CV is the entrance (1) and exit (2) plus streamlines above and below which hit the top and bottom of the wake, as shown. Then steady-flow continuity yields,

\[
0 = \int_2^\infty \rho u \, dA - \int_1^0 \rho u \, dA = 2 \int_0^L \rho \frac{U}{2} \left(1 + \frac{y}{L}\right) b \, dy - 2 \rho U b H,
\]
where $2H$ is the inlet height. Solve for $H = 3L/4$.

Now the linear momentum relation is used. Note that the drag force $F$ is to the right (force of the fluid on the body) thus the force $F$ of the body on fluid is to the left. We obtain,

$$
\sum F_x = 0 = \int u \rho u \, dA - \int u \rho u \, dA = 2 \int_0^L \left( \frac{1}{2} \left( 1 + \frac{y}{L} \right) \rho \frac{U}{2} \left( 1 + \frac{y}{L} \right) \right) b \, dy - 2H \rho U^2 b = -F_{\text{drag}}
$$

Use $H = \frac{3L}{4}$, then $F_{\text{drag}} = \frac{3}{2} \rho U^2 L b - \frac{7}{6} \rho U^2 L b \approx \frac{1}{3} \rho U^2 L b \quad \text{Ans.}$

The dimensionless force, or drag coefficient $F/(\rho U^2 L b)$, equals $C_D = 1/3$. \textit{Ans.}