6.153 Two water tanks, each with base area of $1 \mathrm{ft}^{2}$, are connected by a 0.5 -indiameter long-radius nozzle as in Fig. P6.153. If $\mathrm{h}=1 \mathrm{ft}$ as shown for $t=0$, estimate the time for $\mathrm{h}(\mathrm{t})$ to drop to 0.25 ft .

Solution: For water at $20^{\circ} \mathrm{C}$, take $\rho=$ 1.94 slug $/ \mathrm{ft}^{3}$ and $\mu=2.09 \mathrm{E}-5$ slug/ft•s. For a long-radius nozzle with $\beta \approx 0$, guess $\mathrm{Cd} \approx$ 0.98 and Kloss $\approx 0.9$ from Fig. 6.44. The elevation difference $h$ must balance the head losses in the nozzle and submerged exit:


Fig. P6. 153

$$
\begin{gathered}
\Delta \mathrm{z}=\sum \mathrm{h}_{\text {loss }}=\frac{\mathrm{V}_{\mathrm{t}}^{2}}{2 \mathrm{~g}} \sum \mathrm{~K}=\frac{\mathrm{V}_{\mathrm{t}}^{2}}{2(32.2)}\left(0.9_{\text {nozzle }}+1.0_{\text {exit }}\right)=\mathrm{h}, \quad \text { solve } \mathrm{V}_{\mathrm{t}}=5.82 \sqrt{\mathrm{~h}} \\
\text { hence } \mathrm{Q}=\mathrm{V}_{\mathrm{t}}\left(\frac{\pi}{4}\right)\left(\frac{1 / 2}{12}\right)^{2} \approx 0.00794 \sqrt{\mathrm{~h}}=-\frac{\mathbf{1}}{\mathbf{2}} \mathrm{A}_{\text {tank }} \frac{\mathrm{dh}}{\mathrm{dt}}=-0.5 \frac{\mathrm{dh}}{\mathrm{dt}}
\end{gathered}
$$

The boldface factor $\mathbf{1 / 2}$ accounts for the fact that, as the left tank falls by dh, the right tank rises by the same amount, hence $\mathrm{dh} / \mathrm{dt}$ changes twice as fast as for one tank alone. We can separate and integrate and find the time for $h$ to drop from 1 ft to 0.25 ft :

$$
\int_{0.25}^{1.0} \frac{\mathrm{dh}}{\sqrt{\mathrm{~h}}}=0.0159 \int_{0}^{\mathrm{t}_{\text {final }}} \mathrm{dt}, \quad \text { or: } \quad \mathrm{t}_{\text {final }}=\frac{2(\sqrt{1}-\sqrt{0.25})}{0.0159} \approx \mathbf{6 3} \mathbf{~ s} \quad A n s .
$$

6.154 Water at $20^{\circ} \mathrm{C}$ flows through the orifice in the figure, which is monitored by a mercury manometer. If $d=3 \mathrm{~cm}$, (a) what is $h$ when the flow is $20 \mathrm{~m}^{3} / \mathrm{h}$; and (b) what is $Q$ when $h=58 \mathrm{~cm}$ ?

Solution: (a) Evaluate $\mathrm{V}=\mathrm{Q} / \mathrm{A}=2.83 \mathrm{~m} / \mathrm{s}$ and $\operatorname{ReD}=\rho \mathrm{VD} / \mu=141,000, \beta=0.6$, thus $\mathrm{Cd} \approx 0.613$.


Fig. P6. 154

$$
Q=\frac{20}{3600}=C_{d} \frac{\pi}{4} d^{2} \sqrt{\frac{2 \Delta p}{\rho\left(1-\beta^{4}\right)}}=(0.613) \frac{\pi}{4}(0.03)^{2} \sqrt{\frac{2(13550-998)(9.81) h}{998\left(1-0.6^{4}\right)}}
$$

where we have introduced the manometer formula $\Delta \mathrm{p}=\left(\rho_{\text {mercury }}-\rho_{\text {water }}\right) \mathrm{gh}$.

$$
\text { Solve for: } \boldsymbol{h} \approx 0.58 \mathrm{~m}=\mathbf{5 8} \mathbf{~ c m} \quad \text { Ans. (a) }
$$

Solve this problem when $h=58 \mathrm{~cm}$ is known and $Q$ is the unknown. Well, we can see that the numbers are the same as part (a), and the solution is

$$
\text { Solve for: } \boldsymbol{Q} \approx 0.00556 \mathrm{~m}^{3} / \mathrm{s}=\mathbf{2 0} \mathbf{~ m}^{\mathbf{3}} / \mathbf{h} \quad \text { Ans. (b) }
$$

6.155 It is desired to meter a flow of $20^{\circ} \mathrm{C}$ gasoline in a 12 - cm -diameter pipe, using a modern venturi nozzle. In order for international standards to be valid (Fig. 6.40), what is the permissible range of (a) flow rates, (b) nozzle diameters, and (c) pressure drops? (d) For the highest pressure-drop condition, would compressibility be a problem?

Solution: For gasoline at $20^{\circ} \mathrm{C}$, take $\rho=680 \mathrm{~kg} / \mathrm{m}^{3}$ and $\mu=2.92 \mathrm{E}-4 \mathrm{~kg} / \mathrm{m} \cdot \mathrm{s}$. Examine the possible range of Reynolds number and beta ratio:

$$
\begin{aligned}
& 1.5 \mathrm{E} 5<\operatorname{Re}_{\mathrm{D}}=\frac{4 \rho \mathrm{Q}}{\pi \mu \mathrm{D}}=\frac{4(680) \mathrm{Q}}{\pi(2.92 \mathrm{E}-4)(0.12)}<2.0 \mathrm{E} 5, \\
& \text { or } \quad \mathbf{0 . 0 0 6 1}<\mathbf{Q}<\mathbf{0 . 0 0 8 1} \frac{\mathbf{m}^{3}}{\mathbf{s}} \text { Ans. (a) } \\
& 0.316<\beta=\mathrm{d} / \mathrm{D}<0.775, \quad \text { or: } \mathbf{3 . 8}<\mathbf{d}<\mathbf{9 . 3} \mathbf{~ c m} \quad \text { Ans. (b) }
\end{aligned}
$$

For estimating pressure drop, first compute $\mathrm{Cd}(\beta)$ from Eq. (6.116): $0.924<\mathrm{Cd}<0.985$ :

$$
\mathrm{Q}=\mathrm{C}_{\mathrm{d}} \frac{\pi}{4}(0.12 \beta)^{2} \sqrt{\frac{2 \Delta \mathrm{p}}{680\left(1-\beta^{4}\right)}}, \quad \text { or: } \quad \Delta \mathrm{p}=2.66 \mathrm{E} 6\left(1-\beta^{4}\right)\left[\frac{\mathrm{Q}}{\mathrm{C}_{\mathrm{d}} \beta^{2}}\right]^{2}
$$

put in large Q , small $\beta$, etc. to obtain the range $\mathbf{2 0 0}<\Delta \mathrm{p}<\mathbf{1 8 0 0 0} \mathbf{P a}$ Ans. (c)
6.156 Ethanol at $20^{\circ} \mathrm{C}$ flows down through a modern venturi nozzle as in Fig. P6.156. If the mercury manometer reading is 4 in, as shown, estimate the flow rate, in gal/min.

Solution: For ethanol at $20^{\circ} \mathrm{C}$, take $\rho=$ 1.53 slug $/ \mathrm{ft}^{3}$ and $\mu=2.51 \mathrm{E}-5$ slug/fts. Given $\beta=0.5$, the discharge coefficient is

$$
\mathrm{Cd}=0.9858-0.196(0.5)^{4.5} \approx 0.9771
$$



Fig. P6.156

