## ENSC 283 Week \# 3, Tutorial \# 2- Pressure Distribution in a Fluid

Problem 1: A manometer connects an oil pipeline and a water pipeline as shown in the figure. Determine the difference in pressure between the two pipelines using the readings on the manometer. Use $S G_{o i l}=0.86$ and $S G_{H g}=13.6$.


## Solution

Step 1: Write out what you are required to solve for (this is so you don't forget to answer everything the question is asking for)

Find:

- Pressure difference between water and oil pipelines


## Step 2: Prepare a data table

| Data | Value | Unit |
| :---: | :---: | :---: |
| $S G_{\text {oil }}$ | 0.86 | - |
| $S G_{H g}$ | 13.6 | - |

Also, required heights for computations are shown in the figure.

Step 3: State your assumptions (you may have to add to your list of assumptions as you proceed in the problem)

Assumptions:

1) The specific weight of water is assumed to be $9800 \mathrm{~N} / \mathrm{m}^{3}$.

## Step 4: Calculations

The points of interest have been positioned on the manometer in the figure. The pressure at point 2 is equal to the pressure at point 3 :

$$
\begin{gather*}
p_{2}=p_{3}  \tag{Eq1}\\
p_{\text {water }}+\gamma_{\text {water }} \times 0.04(\mathrm{~m})=p_{4}+\gamma_{H g} \times 0.08(\mathrm{~m}) \tag{Eq2}
\end{gather*}
$$

Note that the heights must be in meters. The pressure at point 4 is essentially the same as that at point 5 , since the specific weight of air is negligible compared with that of the oil. So,

$$
\begin{equation*}
p_{4}=p_{5}=p_{o i l}-\gamma_{o i l} \times 0.06(m) \tag{Eq3}
\end{equation*}
$$

Finally,

$$
\begin{align*}
p_{\text {water }}-p_{\text {oil }} & =-\gamma_{\text {water }} \times 0.04(\mathrm{~m})+\gamma_{H g} \times 0.08-\gamma_{\text {oil }} \times 0.06(\mathrm{~m})  \tag{Eq4}\\
& =-9800\left(\frac{N}{m^{3}}\right) \times 0.04(\mathrm{~m}) \\
+ & (13.6 \times 9800)\left(\frac{N}{m^{3}}\right) 0.08(\mathrm{~m}) \\
& -(0.86 \times 9800)\left(\frac{N}{m^{3}}\right) 0.06(\mathrm{~m})=9764.72 \mathrm{~Pa}\left(\mathrm{~N} / \mathrm{m}^{2}\right)
\end{align*}
$$

Note that $\gamma_{o i l}=S G_{o i l} \cdot \gamma_{\text {water }}$ and $\gamma_{H g}=S G_{H g} \cdot \gamma_{\text {water }}$.

Problem 2: The 6-ft-diameter drainage conduit, shown in the following figure is half full of water at rest. Determine the magnitude and the line of action of the resultant force that the water exerts on a $1-\mathrm{ft}$ length of the curved section BC of the curved section BC of the conduit wall.


We first isolate a volume of fluid bounded by the curved section BC, the horizontal surface AB, and the vertical surface AC, as shown in Fig. (b). The volume has length of 1 ft . The forces acting on the volume are horizontal force, $\mathrm{F}_{1}$, which acts on the vertical surface AC , The weight W , of fluid content within the volume, and horizontal and vertical components of force of conduit wall on the fluid, $\mathrm{F}_{\mathrm{H}}$ and $\mathrm{F}_{\mathrm{V}}$, respectively.
The magnitude of $F_{1}$ is found from the following equation (the pressure at the centroid of the vertical surface AC times its area):

$$
\mathrm{F}_{1}=\gamma \mathrm{h}_{\mathrm{c}} \mathrm{~A}=\left(62.4 \mathrm{lb} / f t^{3}\right)\left(\frac{3}{2} f t\right)\left(3 f t^{2}\right)=281 \mathrm{lb}
$$

and this force acts on the center of pressure of the vertical plate AC which is located $1 f t$ above C as shown. The fluid weight $W$, is

$$
\mathrm{W}=\gamma \mathrm{V}=\left(62.4 \mathrm{lb} / \mathrm{ft}^{3}\right)\left(9 \pi / 4 f t^{2}\right)(1 f t)=441 \mathrm{lb}
$$

and acts through center of gravity of the mass of fluid (located at $4 \mathrm{R} / 3 \pi$ from center of the pipe), that is 1.27 ft to the right of AC as shown. Therefore to satisfy equilibrium:

$$
\mathrm{F}_{\mathrm{H}}=\mathrm{F}_{1}=281 \mathrm{lb} \quad \& \quad \mathrm{~F}_{\mathrm{V}}=\mathrm{W}=441 \mathrm{lb}
$$

And magnitude of resultant force is:

$$
\begin{gathered}
F_{R}=\sqrt{\left(F_{H}^{2}\right)+\left(F_{V}^{2}\right)} \\
=\sqrt{\left(281 l b^{2}\right)+\left(441 l b^{2}\right)}=523 l b
\end{gathered}
$$

It is obvious that $\mathrm{F}_{\mathrm{R}}$ passes trough A (the center of the pipe), since at each point on the curved surface of the conduit, the elemental force due to the pressure is normal to the surface. Using the triangle shown in Fig. (c), one can calculate the angle of $\mathrm{F}_{\mathrm{R}}$ as follows:

$$
\theta=\tan ^{-1} \frac{F_{H}}{F_{V}}=\tan ^{-1} \frac{281 \mathrm{lb}}{441 \mathrm{lb}}=32.5 \text { degree }
$$

