Problem 1

A completely reversible heat pump produces heat at a rate of 100 kW to warm a house maintained at 21 °C. The exterior air, which is at 10°C, serves as the source. Calculate the rate of entropy change of the two reservoirs and determine if this heat pump satisfies the second law according to the increase of entropy principle.
Problem 1:

Known:

*Hot and cold reservoirs temperature*

*Heat Produced by heat pump*

Find:

- The rate of entropy change of the two reservoirs.

Assumptions:

- The heat pump operates steadily.

Analysis:

Since the heat pump is completely reversible, the combination of the coefficient of performance expression, first Law, and thermodynamic temperature scale gives:

\[
COP_{HP,rev} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - 283[K]/294[K]}
\]

The power required to drive this heat pump, according to the coefficient of performance, is then:
\[
\dot{W}_{\text{net, in}} = \frac{\dot{Q}_H}{COP_{HP, rev}} = \frac{100 \,[kW]}{26.73} = 3.741 \,[kW]
\]

According to the first law, the rate at which heat is removed from the low-temperature energy reservoir is:

\[
Q_L = Q_H - W_{\text{net, in}} = 100 \,[kW] - 3.741[kW] = 96.26 \,[kW]
\]

The rate, at which the entropy of the high temperature reservoir changes, according to the definition of the entropy, is:

\[
\Delta \hat{S}_H = \frac{\dot{Q}_H}{T_H} = \frac{100 \,[kW]}{294 \,[K]} = 0.340 \,\left[\frac{kW}{K}\right]
\]

and that of the low-temperature reservoir is:

\[
\Delta \hat{S}_L = \frac{\dot{Q}_L}{T_L} = \frac{-96.26 \,[kW]}{283 \,[K]} = -0.340 \,\left[\frac{kW}{K}\right]
\]

The net rate of entropy change of everything in this system is

\[
\Delta \hat{S}_{\text{total}} = \Delta \hat{S}_H + \Delta \hat{S}_L = 0.340 - 0.340 = 0 \,\left[\frac{kW}{K}\right]
\]

as it must be since the heat pump is completely reversible.