Solutions Manual

for

Introduction to Thermodynamics and Heat Transfer

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2nd Edition, 2008

Chapter 11

TRANSIENT HEAT CONDUCTION

Lumped System Analysis

11-1C In heat transfer analysis, some bodies are observed to behave like a "lump" whose entire body temperature remains essentially uniform at all times during a heat transfer process. The temperature of such bodies can be taken to be a function of time only. Heat transfer analysis which utilizes this idealization is known as the lumped system analysis. It is applicable when the Biot number (the ratio of conduction resistance within the body to convection resistance at the surface of the body) is less than or equal to 0.1.

11-2C The lumped system analysis is more likely to be applicable for the body cooled naturally since the Biot number is proportional to the convection heat transfer coefficient, which is proportional to the air velocity. Therefore, the Biot number is more likely to be less than 0.1 for the case of natural convection.

11-3C The lumped system analysis is more likely to be applicable for the body allowed to cool in the air since the Biot number is proportional to the convection heat transfer coefficient, which is larger in water than it is in air because of the larger thermal conductivity of water. Therefore, the Biot number is more likely to be less than 0.1 for the case of the solid cooled in the air

11-4C The temperature drop of the potato during the second minute will be less than 4°C since the temperature of a body approaches the temperature of the surrounding medium asymptotically, and thus it changes rapidly at the beginning, but slowly later on.

11-5C The temperature rise of the potato during the second minute will be less than 5°C since the temperature of a body approaches the temperature of the surrounding medium asymptotically, and thus it changes rapidly at the beginning, but slowly later on.

11-6C Biot number represents the ratio of conduction resistance within the body to convection resistance at the surface of the body. The Biot number is more likely to be larger for poorly conducting solids since such bodies have larger resistances against heat conduction.

11-7C The heat transfer is proportional to the surface area. Two half pieces of the roast have a much larger surface area than the single piece and thus a higher rate of heat transfer. As a result, the two half pieces will cook much faster than the single large piece.

11-8C The cylinder will cool faster than the sphere since heat transfer rate is proportional to the surface area, and the sphere has the smallest area for a given volume.

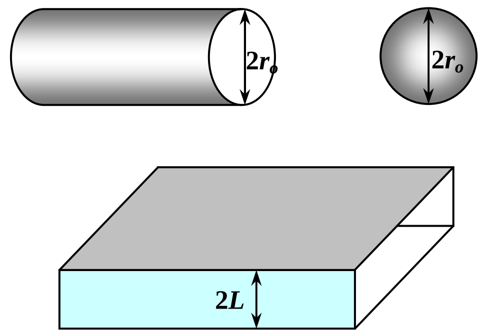
11-9C The lumped system analysis is more likely to be applicable in air than in water since the convection heat transfer coefficient and thus the Biot number is much smaller in air.

11-10C The lumped system analysis is more likely to be applicable for a golden apple than for an actual apple since the thermal conductivity is much larger and thus the Biot number is much smaller for gold.

11-11C The lumped system analysis is more likely to be applicable to slender bodies than the well-rounded bodies since the characteristic length (ratio of volume to surface area) and thus the Biot number is much smaller for slender bodies.

11-12 Relations are to be obtained for the characteristic lengths of a large plane wall of thickness $2L$, a very long cylinder of radius r_o and a sphere of radius r_o.

Analysis Relations for the characteristic lengths of a large plane wall of thickness $2L$, a very long cylinder of radius r_o and a sphere of radius r_o are



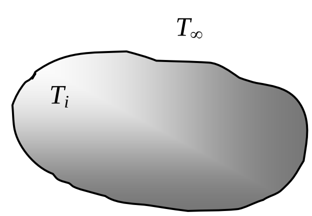
$$L_{c,wall} = \frac{V}{A_{\text{surface}}} = \frac{2LA}{2A} = L$$

$$L_{c,cylinder} = \frac{V}{A_{\text{surface}}} = \frac{\pi r_o^2 h}{2\pi r_o h} = \frac{r_o}{2}$$

$$L_{c,sphere} = \frac{V}{A_{\text{surface}}} = \frac{4\pi r_o^3/3}{4\pi r_o^2} = \frac{r_o}{3}$$

11-13 A relation for the time period for a lumped system to reach the average temperature $(T_i + T_\infty)/2$ is to be obtained.

Analysis The relation for time period for a lumped system to reach the average temperature $(T_i + T_\infty)/2$ can be determined as



$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{\frac{T_i + T_\infty}{2} - T_\infty}{T_i - T_\infty} = e^{-bt}$$

$$\frac{T_i - T_\infty}{2(T_i - T_\infty)} = e^{-bt} \longrightarrow \frac{1}{2} = e^{-bt}$$

$$-bt = -\ln 2 \longrightarrow t = \mathbf{\frac{\ln 2}{b} = \frac{0.693}{b}}$$

11-14 The temperature of a gas stream is to be measured by a thermocouple. The time it takes to register 99 percent of the initial ΔT is to be determined.

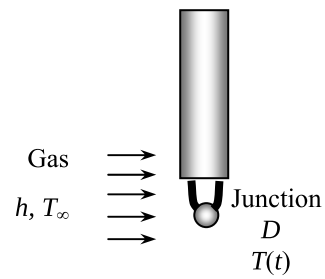
Assumptions **1** The junction is spherical in shape with a diameter of D = 0.0012 m. **2** The thermal properties of the junction are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** Radiation effects are negligible. **5** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of the junction are given to be $k = 35 \text{ W/m.°C}$, $\rho = 8500 \text{ kg/m}^3$, and $c_p = 320 \text{ J/kg.°C}$.

Analysis The characteristic length of the junction and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.0012 \text{ m}}{6} = 0.0002 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(90 \text{ W/m}^2.\text{°C})(0.0002 \text{ m})}{(35 \text{ W/m.°C})} = 0.00051 < 0.1$$



Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the thermocouple to read 99% of the initial temperature difference is determined from

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$$

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{90 \text{ W/m}^2.\text{°C}}{(8500 \text{ kg/m}^3)(320 \text{ J/kg.°C})(0.0002 \text{ m})} = 0.1654 \text{ s}^{-1}$$

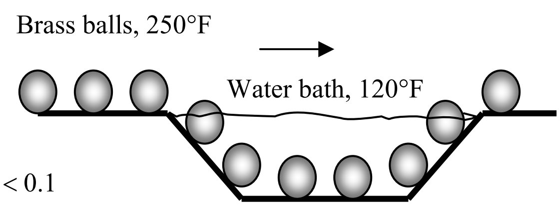
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow 0.01 = e^{-(0.1654 \text{ s}^{-1})t} \longrightarrow t = \mathbf{27.8 \text{ s}}$$

11-15E A number of brass balls are to be quenched in a water bath at a specified rate. The temperature of the balls after quenching and the rate at which heat needs to be removed from the water in order to keep its temperature constant are to be determined.

Assumptions **1** The balls are spherical in shape with a radius of r_o = 1 in. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the brass balls are given to be k = 64.1 Btu/h.ft.°F, ρ = 532 lbm/ft^3, and c_p = 0.092 Btu/lbm.°F.

Analysis (*a*) The characteristic length and the Biot number for the brass balls are



$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{2/12\text{ ft}}{6} = 0.02778\text{ ft}$$

$$Bi = \frac{hL_c}{k} = \frac{(42\text{ Btu/h.ft}^2\text{.°F})(0.02778\text{ ft})}{(64.1\text{ Btu/h.ft.°F})} = 0.01820 < 0.1$$

The lumped system analysis is applicable since Bi < 0.1. Then the temperature of the balls after quenching becomes

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{42\text{ Btu/h.ft}^2\text{.°F}}{(532\text{ lbm/ft}^3)(0.092\text{ Btu/lbm.°F})(0.02778\text{ ft})} = 30.9\text{ h}^{-1} = 0.00858\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t)-120}{250-120} = e^{-(0.00858\text{ s}^{-1})(120\text{ s})} \longrightarrow T(t) = \mathbf{166\ °F}$$

(*b*) The total amount of heat transfer from a ball during a 2-minute period is

$$m = \rho V = \rho\frac{\pi D^3}{6} = (532\text{ lbm/ft}^3)\frac{\pi(2/12\text{ ft})^3}{6} = 1.290\text{ lbm}$$

$$Q = mc_p[T_i - T(t)] = (1.29\text{ lbm})(0.092\text{ Btu/lbm.°F})(250-166)\text{°F} = 9.97\text{ Btu}$$

Then the rate of heat transfer from the balls to the water becomes

$$\dot{Q}_{total} = \dot{n}_{\text{ball}}Q_{\text{ball}} = (120\text{ balls/min})\times(9.97\text{ Btu}) = \mathbf{1196\ Btu/min}$$

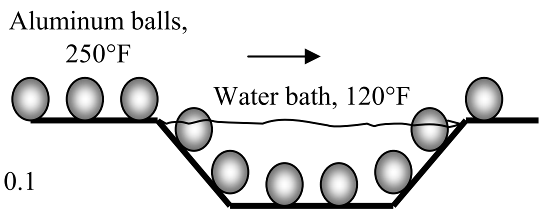
Therefore, heat must be removed from the water at a rate of 1196 Btu/min in order to keep its temperature constant at 120°F.

11-16E A number of aluminum balls are to be quenched in a water bath at a specified rate. The temperature of balls after quenching and the rate at which heat needs to be removed from the water in order to keep its temperature constant are to be determined.

Assumptions **1** The balls are spherical in shape with a radius of r_o = 1 in. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the aluminum balls are k = 137 Btu/h.ft.°F, ρ = 168 lbm/ft^3, and c_p = 0.216 Btu/lbm.°F (Table A-24E).

Analysis (*a*) The characteristic length and the Biot number for the aluminum balls are



$$L_c = \frac{V}{A} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{2/12 \text{ ft}}{6} = 0.02778 \text{ ft}$$

$$Bi = \frac{hL_c}{k} = \frac{(42 \text{ Btu/h.ft}^2.°\text{F})(0.02778 \text{ ft})}{(137 \text{ Btu/h.ft.°F})} = 0.00852 < 0.1$$

The lumped system analysis is applicable since Bi < 0.1. Then the temperature of the balls after quenching becomes

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{42 \text{ Btu/h.ft}^2.°\text{F}}{(168 \text{ lbm/ft}^3)(0.216 \text{ Btu/lbm.°F})(0.02778 \text{ ft})} = 41.66 \text{ h}^{-1} = 0.01157 \text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t)-120}{250-120} = e^{-(0.01157 \text{ s}^{-1})(120 \text{ s})} \longrightarrow T(t) = \mathbf{152°F}$$

(*b*) The total amount of heat transfer from a ball during a 2-minute period is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (168 \text{ lbm/ft}^3)\frac{\pi(2/12 \text{ ft})^3}{6} = 0.4072 \text{ lbm}$$

$$Q = mc_p[T_i - T(t)] = (0.4072 \text{ lbm})(0.216 \text{ Btu/lbm.°F})(250-152)°\text{F} = 8.62 \text{ Btu}$$

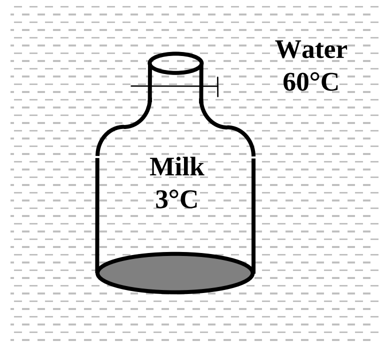
Then the rate of heat transfer from the balls to the water becomes

$$\dot{Q}_{total} = \dot{n}_{\text{ball}} Q_{\text{ball}} = (120 \text{ balls/min}) \times (8.62 \text{ Btu}) = \mathbf{1034 \text{ Btu/min}}$$

Therefore, heat must be removed from the water at a rate of 1034 Btu/min in order to keep its temperature constant at 120°F.

11-17 Milk in a thin-walled glass container is to be warmed up by placing it into a large pan filled with hot water. The warming time of the milk is to be determined.

Assumptions **1** The glass container is cylindrical in shape with a radius of r_0 = 3 cm. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.



Properties The thermal conductivity, density, and specific heat of the milk at 20°C are k = 0.598 W/m.°C, ρ = 998 kg/m^3, and c_p = 4.182 kJ/kg.°C (Table A-15).

Analysis The characteristic length and Biot number for the glass of milk are

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(0.03\text{ m})^2(0.07\text{ m})}{2\pi(0.03\text{ m})(0.07\text{ m}) + 2\pi(0.03\text{ m})^2} = 0.01050\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(120\text{ W/m}^2.°\text{C})(0.0105\text{ m})}{(0.598\text{ W/m}.°\text{C})} = 2.107 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the milk to warm up to 38°C:

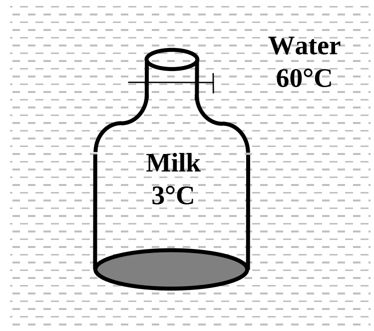
$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{120\text{ W/m}^2.°\text{C}}{(998\text{ kg/m}^3)(4182\text{ J/kg}.°\text{C})(0.0105\text{ m})} = 0.002738\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{38 - 60}{3 - 60} = e^{-(0.002738\text{ s}^{-1})t} \longrightarrow t = \mathbf{348\text{ s} = 5.8\text{ min}}$$

Therefore, it will take about 6 minutes to warm the milk from 3 to 38°C.

11-18 A thin-walled glass containing milk is placed into a large pan filled with hot water to warm up the milk. The warming time of the milk is to be determined.

Assumptions **1** The glass container is cylindrical in shape with a radius of r_0 = 3 cm. **2** The thermal properties of the milk are taken to be the same as those of water. **3** Thermal properties of the milk are constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the milk is stirred constantly, so that its temperature remains uniform at all times.



Properties The thermal conductivity, density, and specific heat of the milk at 20°C are k = 0.598 W/m.°C, ρ = 998 kg/m^3, and c_p = 4.182 kJ/kg.°C (Table A-15).

Analysis The characteristic length and Biot number for the glass of milk are

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi (0.03\text{ m})^2 (0.07\text{ m})}{2\pi (0.03\text{ m})(0.07\text{ m}) + 2\pi (0.03\text{ m})^2} = 0.01050\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(240\text{ W/m}^2.°\text{C})(0.0105\text{ m})}{(0.598\text{ W/m.°C})} = 4.21 > 0.1$$

For the reason explained above we can use the lumped system analysis to determine how long it will take for the milk to warm up to 38°C:

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{240\text{ W/m}^2.°\text{C}}{(998\text{ kg/m}^3)(4182\text{ J/kg.°C})(0.0105\text{ m})} = 0.005477\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{38 - 60}{3 - 60} = e^{-(0.005477\text{ s}^{-1})t} \longrightarrow t = \mathbf{174\text{ s} = 2.9\text{ min}}$$

Therefore, it will take about 3 minutes to warm the milk from 3 to 38°C.

11-19 A long copper rod is cooled to a specified temperature. The cooling time is to be determined.

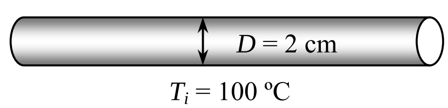
Assumptions **1** The thermal properties of the geometry are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of copper are k = 401 W/m·ºC, ρ = 8933 kg/m^3, and c_p = 0.385 kJ/kg·ºC (Table A-24).

Analysis For cylinder, the characteristic length and the Biot number are

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.02\text{ m}}{4} = 0.005\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(200\text{ W/m}^2.°\text{C})(0.005\text{ m})}{(401\text{ W/m.°C})} = 0.0025 < 0.1$$



Since $Bi < 0.1$, the lumped system analysis is applicable. Then the cooling time is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{200\text{ W/m}^2.°\text{C}}{(8933\text{ kg/m}^3)(385\text{ J/kg.°C})(0.005\text{ m})} = 0.01163\text{ s}^{-1}$$

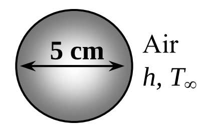
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25 - 20}{100 - 20} = e^{-(0.01163\text{ s}^{-1})t} \longrightarrow t = 238\text{ s} = \mathbf{4.0\text{ min}}$$

11-20 The heating times of a sphere, a cube, and a rectangular prism with similar dimensions are to be determined.

Assumptions **1** The thermal properties of the geometries are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of silver are given to be k = 429 W/m·ºC, ρ = 10,500 kg/m³, and c_p = 0.235 kJ/kg·ºC.

Analysis For sphere, the characteristic length and the Biot number are



$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.05\text{ m}}{6} = 0.008333\text{ m}$$

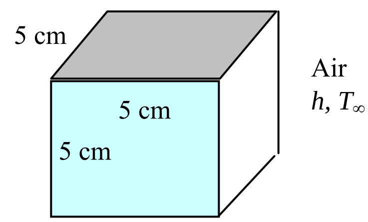
$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.008333\text{ m})}{(429\text{ W/m}.°\text{C})} = 0.00023 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the time period for the sphere temperature to reach to 25ºC is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{12\text{ W/m}^2.°\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg}.°\text{C})(0.008333\text{ m})} = 0.0005836\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005836\text{ s}^{-1})t} \longrightarrow t = 2428\text{ s} = \mathbf{40.5\text{ min}}$$

Cube:



$$L_c = \frac{V}{A_{\text{surface}}} = \frac{L^3}{6L^2} = \frac{L}{6} = \frac{0.05\text{ m}}{6} = 0.008333\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.008333\text{ m})}{(429\text{ W/m}.°\text{C})} = 0.00023 < 0.1$$

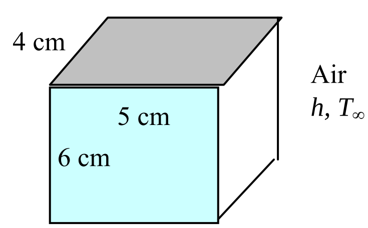
$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{12\text{ W/m}^2.°\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg}.°\text{C})(0.008333\text{ m})} = 0.0005836\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005836\text{ s}^{-1})t} \longrightarrow t = 2428\text{ s} = \mathbf{40.5\text{ min}}$$

Rectangular prism:

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{(0.04\text{ m})(0.05\text{ m})(0.06\text{ m})}{2(0.04\text{ m})(0.05\text{ m}) + 2(0.04\text{ m})(0.06\text{ m}) + 2(0.05\text{ m})(0.06\text{ m})} = 0.008108\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\text{ W/m}^2.°\text{C})(0.008108\text{ m})}{(429\text{ W/m}.°\text{C})} = 0.00023 < 0.1$$



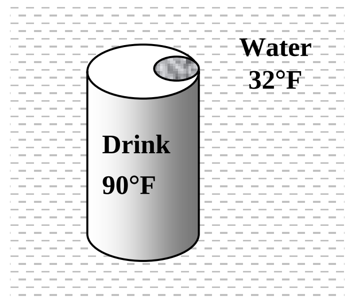
$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

$$= \frac{12\text{ W/m}^2.°\text{C}}{(10{,}500\text{ kg/m}^3)(235\text{ J/kg}.°\text{C})(0.008108\text{ m})} = 0.0005998\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{25-33}{0-33} = e^{-(0.0005998\text{ s}^{-1})t} \longrightarrow t = 2363\text{ s} = \mathbf{39.4\text{ min}}$$

The heating times are same for the sphere and cube while it is smaller in rectangular prism.

11-21E A person shakes a can of drink in a iced water to cool it. The cooling time of the drink is to be determined.



Assumptions **1** The can containing the drink is cylindrical in shape with a radius of r_o = 1.25 in. **2** The thermal properties of the drink are taken to be the same as those of water. **3** Thermal properties of the drinkare constant at room temperature. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Biot number in this case is large (much larger than 0.1). However, the lumped system analysis is still applicable since the drink is stirred constantly, so that its temperature remains uniform at all times.

Properties The density and specific heat of water at room temperature are ρ = 62.22 lbm/ft^3, and c_p = 0.999 Btu/lbm.°F (Table A-15E).

Analysis Application of lumped system analysis in this case gives

$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L + 2\pi r_o^2} = \frac{\pi(1.25/12\text{ ft})^2(5/12\text{ ft})}{2\pi(1.25/12\text{ ft})(5/12\text{ ft}) + 2\pi(1.25/12\text{ ft})^2} = 0.04167\text{ ft}$$

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{30\text{ Btu/h.ft}^2.\text{°F}}{(62.22\text{ lbm/ft}^3)(0.999\text{ Btu/lbm.°F})(0.04167\text{ ft})} = 11.583\text{ h}^{-1} = 0.00322\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{40-32}{90-32} = e^{-(0.00322\text{ s}^{-1})t} \longrightarrow t = \mathbf{615\text{ s}}$$

Therefore, it will take 10 minutes and 15 seconds to cool the canned drink to 40°F.

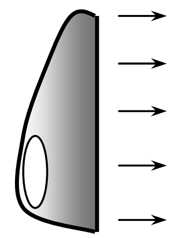
11-22 An iron whose base plate is made of an aluminum alloy is turned on. The time for the plate temperature to reach 140°C and whether it is realistic to assume the plate temperature to be uniform at all times are to be determined.

Assumptions **1** 85 percent of the heat generated in the resistance wires is transferred to the plate. **2** The thermal properties of the plate are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The density, specific heat, and thermal diffusivity of the aluminum alloy plate are given to be $\rho = 2770$ kg/m^3, $c_p = 875$ kJ/kg.°C, and $\alpha = 7.3\times10^{-5}$ m^2/s. The thermal conductivity of the plate can be determined from $k = \alpha\rho c_p = 177$ W/m.°C (or it can be read from Table A-24).

Analysis The mass of the iron's base plate is

$$m = \rho V = \rho LA = (2770\ \text{kg/m}^3)(0.005\ \text{m})(0.03\ \text{m}^2) = 0.4155\ \text{kg}$$



Noting that only 85 percent of the heat generated is transferred to the plate, the rate of heat transfer to the iron's base plate is

$$\dot{Q}_{\text{in}} = 0.85\times1000\ \text{W} = 850\ \text{W}$$

The temperature of the plate, and thus the rate of heat transfer from the plate, changes during the process. Using the average plate temperature, the average rate of heat loss from the plate is determined from

$$\dot{Q}_{\text{loss}} = hA(T_{\text{plate, ave}} - T_\infty) = (12\ \text{W/m}^2.°\text{C})(0.03\ \text{m}^2)\left(\frac{140+22}{2} - 22\right)°\text{C} = 21.2\ \text{W}$$

Energy balance on the plate can be expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{plate}} \rightarrow \dot{Q}_{\text{in}}\Delta t - \dot{Q}_{\text{out}}\Delta t = \Delta E_{\text{plate}} = mc_p\Delta T_{\text{plate}}$$

Solving for Δt and substituting,

$$\Delta t = \frac{mc_p\Delta T_{\text{plate}}}{\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}} = \frac{(0.4155\ \text{kg})(875\ \text{J/kg.°C})(140-22)°\text{C}}{(850-21.2)\ \text{J/s}} = \mathbf{51.8\ s}$$

which is the time required for the plate temperature to reach $140\,°\text{C}$. To determine whether it is realistic to assume the plate temperature to be uniform at all times, we need to calculate the Biot number,

$$L_c = \frac{V}{A_s} = \frac{LA}{A} = L = 0.005\ \text{m}$$

$$Bi = \frac{hL_c}{k} = \frac{(12\ \text{W/m}^2.°\text{C})(0.005\ \text{m})}{(177.0\ \text{W/m.°C})} = 0.00034 < 0.1$$

It is realistic to assume uniform temperature for the plate since Bi < 0.1.

Discussion This problem can also be solved by obtaining the differential equation from an energy balance on the plate for a differential time interval, and solving the differential equation. It gives

$$T(t) = T_\infty + \frac{\dot{Q}_{\text{in}}}{hA}\left(1 - \exp(-\frac{hA}{mc_p}t)\right)$$

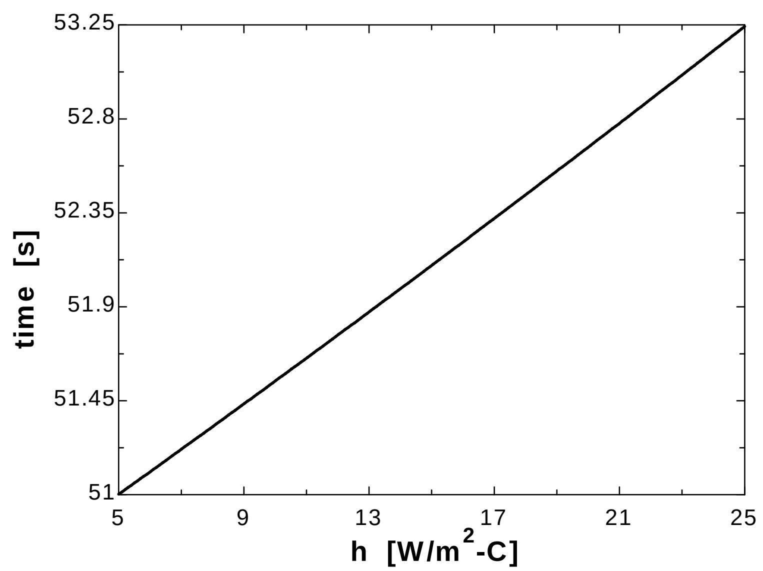
Substituting the known quantities and solving for t again gives 51.8 s.

11-23 EES Prob. 11-22 is reconsidered. The effects of the heat transfer coefficient and the final plate temperature on the time it will take for the plate to reach this temperature are to be investigated.

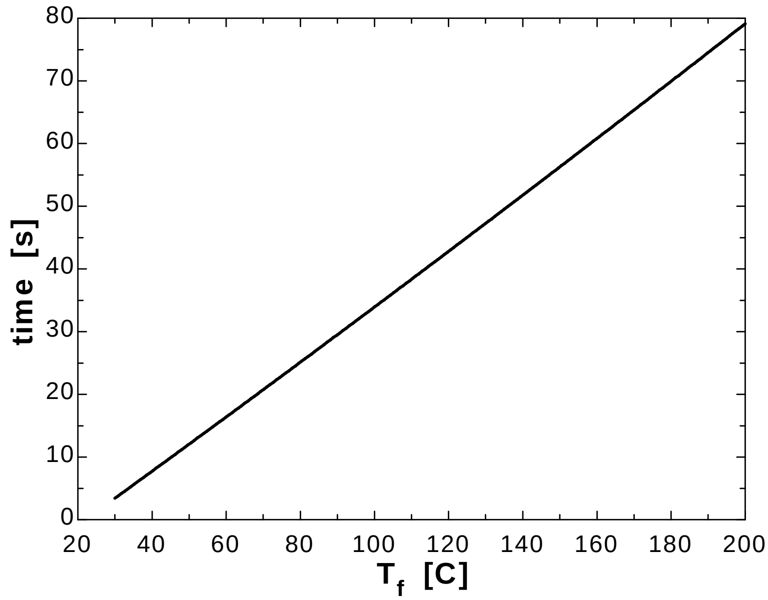
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
E_dot=1000 [W]
L=0.005 [m]
A=0.03 [m^2]
T_infinity=22 [C]
T_i=T_infinity
h=12 [W/m^2-C]
f_heat=0.85
T_f=140 [C]

"PROPERTIES"
rho=2770 [kg/m^3]
c_p=875 [J/kg-C]
alpha=7.3E-5 [m^2/s]

"ANALYSIS"
V=L*A
m=rho*V
Q_dot_in=f_heat*E_dot
Q_dot_out=h*A*(T_ave-T_infinity)
T_ave=1/2*(T_i+T_f)
(Q_dot_in-Q_dot_out)*time=m*c_p*(T_f-T_i) "energy balance on the plate"
```

h [W/m^2.C]	time [s]
5	51
7	51.22
9	51.43
11	51.65
13	51.88
15	52.1
17	52.32
19	52.55
21	52.78
23	53.01
25	53.24



T_f [C]	time [s]
30	3.428
40	7.728
50	12.05
60	16.39
70	20.74
80	25.12
90	29.51
100	33.92
110	38.35
120	42.8
130	47.28
140	51.76
150	56.27
160	60.8
170	65.35
180	69.92
190	74.51
200	79.12



11-24 Ball bearings leaving the oven at a uniform temperature of 900°C are exposed to air for a while before they are dropped into the water for quenching. The time they can stand in the air before their temperature falls below 850°C is to be determined.

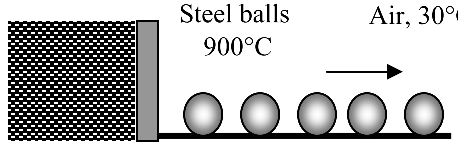
Assumptions **1** The bearings are spherical in shape with a radius of r_o = 0.6 cm. **2** The thermal properties of the bearings are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the bearings are given to be k = 15.1 W/m.°C, ρ = 8085 kg/m^3, and c_p = 0.480 kJ/kg.°F.

Analysis The characteristic length of the steel ball bearings and Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.012\text{ m}}{6} = 0.002\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(125\text{ W/m}^2.°\text{C})(0.002\text{ m})}{(15.1\text{ W/m.°C})} = 0.0166 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the allowable time is determined to be

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{125\text{ W/m}^2.°\text{C}}{(8085\text{ kg/m}^3)(480\text{ J/kg.°C})(0.002\text{ m})} = 0.01610\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{850-30}{900-30} = e^{-(0.0161\text{ s}^{-1})t} \longrightarrow t = \mathbf{3.68\text{ s}}$$

The result indicates that the ball bearing can stay in the air about 4 s before being dropped into the water.

11-25 A number of carbon steel balls are to be annealed by heating them first and then allowing them to cool slowly in ambient air at a specified rate. The time of annealing and the total rate of heat transfer from the balls to the ambient air are to be determined.

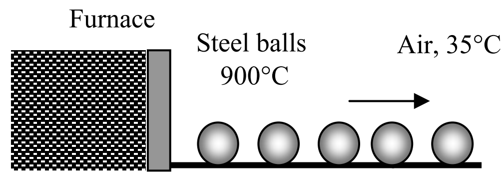
Assumptions **1** The balls are spherical in shape with a radius of r_o = 4 mm. **2** The thermal properties of the balls are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the balls are given to be k = 54 W/m.°C, ρ = 7833 kg/m^3, and c_p = 0.465 kJ/kg.°C.

Analysis The characteristic length of the balls and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.008\text{ m}}{6} = 0.0013\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(75\text{ W/m}^2.°\text{C})(0.0013\text{ m})}{(54\text{ W/m.°C})} = 0.0018 < 0.1$$



Therefore, the lumped system analysis is applicable. Then the time for the annealing process is determined to be

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{75\text{ W/m}^2.°\text{C}}{(7833\text{ kg/m}^3)(465\text{ J/kg.°C})(0.0013\text{ m})} = 0.01584\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{100-35}{900-35} = e^{-(0.01584\text{ s}^{-1})t} \longrightarrow t = \mathbf{163\text{ s}} = \mathbf{2.7\text{ min}}$$

The amount of heat transfer from a single ball is

$$m = \rho V = \rho \frac{\pi D^3}{6} = (7833\text{ kg/m}^3)\frac{\pi(0.008\text{ m})^3}{6} = 0.0021\text{ kg}$$

$$Q = mc_p[T_f - T_i] = (0.0021\text{ kg})(465\text{ J/kg.°C})(900-100)°\text{C} = 781\text{ J} = 0.781\text{ kJ (per ball)}$$

Then the total rate of heat transfer from the balls to the ambient air becomes

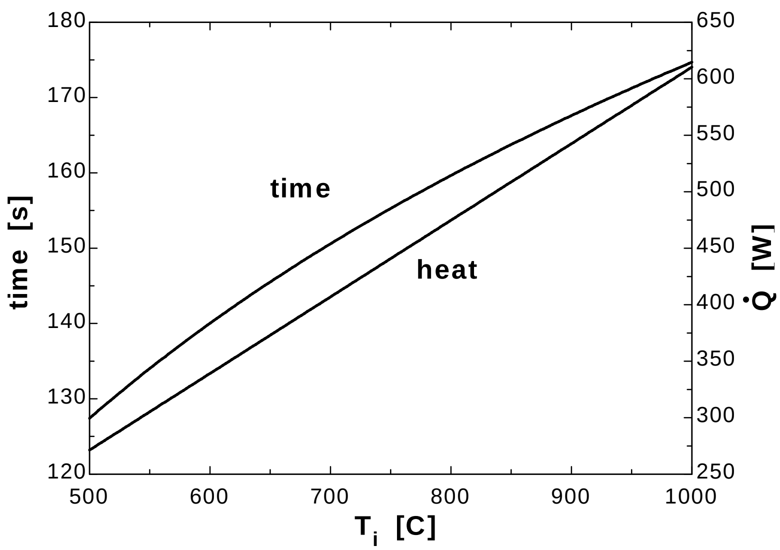
$$\dot{Q} = \dot{n}_{\text{ball}}Q = (2500\text{ balls/h})\times(0.781\text{ kJ/ball}) = 1{,}953\text{ kJ/h} = \mathbf{543\text{ W}}$$

11-26 EES Prob. 11-25 is reconsidered. The effect of the initial temperature of the balls on the annealing time and the total rate of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.008 [m];   T_i=900 [C]
T_f=100 [C];   T_infinity=35 [C]
h=75 [W/m^2-C];   n_dot_ball=2500 [1/h]

"PROPERTIES"
rho=7833 [kg/m^3];   k=54 [W/m-C]
c_p=465 [J/kg-C];   alpha=1.474E-6 [m^2/s]

"ANALYSIS"
A=pi*D^2
V=pi*D^3/6
L_c=V/A
Bi=(h*L_c)/k "if Bi < 0.1, the lumped sytem analysis is applicable"
b=(h*A)/(rho*c_p*V)
(T_f-T_infinity)/(T_i-T_infinity)=exp(-b*time)
m=rho*V
Q=m*c_p*(T_i-T_f)
Q_dot=n_dot_ball*Q*Convert(J/h, W)
```

T_i [C]	time [s]	Q [W]
500	127.4	271.2
550	134	305.1
600	140	339
650	145.5	372.9
700	150.6	406.9
750	155.3	440.8
800	159.6	474.7
850	163.7	508.6
900	167.6	542.5
950	171.2	576.4
1000	174.7	610.3



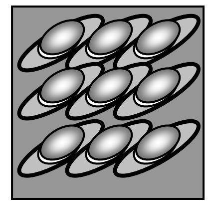
11-27 An electronic device is on for 5 minutes, and off for several hours. The temperature of the device at the end of the 5-min operating period is to be determined for the cases of operation with and without a heat sink.

Assumptions **1** The device and the heat sink are isothermal. **2** The thermal properties of the device and of the sink are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The specific heat of the device is given to be c_p = 850 J/kg.°C. The specific heat of the aluminum sink is 903 J/kg.°C (Table A-24), but can be taken to be 850 J/kg.°C for simplicity in analysis.

Analysis **(*a*) Approximate solution**

This problem can be solved approximately by using an average temperature for the device when evaluating the heat loss. An energy balance on the device can be expressed as



$$E_{\text{in}} - E_{\text{out}} + E_{\text{generation}} = \Delta E_{\text{device}} \longrightarrow -\dot{Q}_{\text{out}}\Delta t + \dot{E}_{\text{generation}}\Delta t = mc_p \Delta T_{\text{device}}$$

or, $$\dot{E}_{\text{generation}}\Delta t - hA_s\left(\frac{T+T_\infty}{2} - T_\infty\right)\Delta t = mc_p(T - T_\infty)$$

Substituting the given values,

$$(20\text{ J/s})(5\times 60\text{ s}) - (12\text{ W/m}^2.°\text{C})(0.0004\text{ m}^2)\left(\frac{T-25}{2}\right)^{\circ}\text{C}(5\times 60\text{ s}) = (0.02\text{ kg})(850\text{ J/kg.°C})(T-25)°\text{C}$$

which gives T = **363.6°C**

If the device were attached to an aluminum heat sink, the temperature of the device would be

$$(20\text{ J/s})(5\times 60\text{ s}) - (12\text{ W/m}^2.°\text{C})(0.0084\text{ m}^2)\left(\frac{T-25}{2}\right)°\text{C}(5\times 60\text{ s})$$
$$= (0.20+0.02)\text{kg}\times(850\text{ J/kg.°C})(T-25)°\text{C}$$

which gives T = **54.7°C**

Note that the temperature of the electronic device drops considerably as a result of attaching it to a heat sink.

(*b*) Exact solution

This problem can be solved exactly by obtaining the differential equation from an energy balance on the device for a differential time interval *dt.* We will get

$$\frac{d(T-T_\infty)}{dt} + \frac{hA_s}{mc_p}(T-T_\infty) = \frac{\dot{E}_{\text{generation}}}{mc_p}$$

It can be solved to give

$$T(t) = T_\infty + \frac{\dot{E}_{\text{generation}}}{hA_s}\left(1 - \exp(-\frac{hA_s}{mc_p}t)\right)$$

Substituting the known quantities and solving for t gives **363.4°C** for the first case and **54.6°C** for the second case, which are practically identical to the results obtained from the approximate analysis.

Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres with Spatial Effects

11-28C A cylinder whose diameter is small relative to its length can be treated as an infinitely long cylinder. When the diameter and length of the cylinder are comparable, it is not proper to treat the cylinder as being infinitely long. It is also not proper to use this model when finding the temperatures near the bottom or top surfaces of a cylinder since heat transfer at those locations can be two-dimensional.

11-29C Yes. A plane wall whose one side is insulated is equivalent to a plane wall that is twice as thick and is exposed to convection from both sides. The midplane in the latter case will behave like an insulated surface because of thermal symmetry.

11-30C The solution for determination of the one-dimensional transient temperature distribution involves many variables that make the graphical representation of the results impractical. In order to reduce the number of parameters, some variables are grouped into dimensionless quantities.

11-31C The Fourier number is a measure of heat conducted through a body relative to the heat stored. Thus a large value of Fourier number indicates faster propagation of heat through body. Since Fourier number is proportional to time, doubling the time will also double the Fourier number.

11-32C This case can be handled by setting the heat transfer coefficient h to infinity ∞ since the temperature of the surrounding medium in this case becomes equivalent to the surface temperature.

11-33C The maximum possible amount of heat transfer will occur when the temperature of the body reaches the temperature of the medium, and can be determined from $Q_{\max} = mc_p(T_\infty - T_i)$.

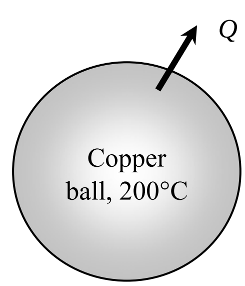
11-34C When the Biot number is less than 0.1, the temperature of the sphere will be nearly uniform at all times. Therefore, it is more convenient to use the lumped system analysis in this case.

11-35 A student calculates the total heat transfer from a spherical copper ball. It is to be determined whether his/her result is reasonable.

Assumptions The thermal properties of the copper ball are constant at room temperature.

Properties The density and specific heat of the copper ball are ρ = 8933 kg/m^3, and c_p = 0.385 kJ/kg.°C (Table A-24).

Analysis The mass of the copper ball and the maximum amount of heat transfer from the copper ball are



$$m = \rho V = \rho\left(\frac{\pi D^3}{6}\right) = (8933\ \text{kg/m}^3)\left[\frac{\pi(0.18\ \text{m})^3}{6}\right] = 27.28\ \text{kg}$$

$$Q_{\max} = mc_p[T_i - T_\infty] = (27.28\ \text{kg})(0.385\ \text{kJ/kg.°C})(200-25)\text{°C} = 1838\ \text{kJ}$$

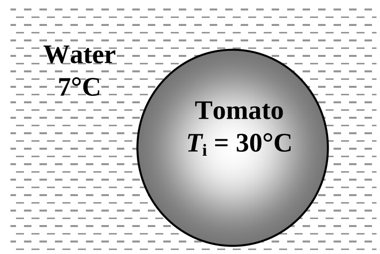
Discussion The student's result of 3150 kJ is **not reasonable** since it is greater than the maximum possible amount of heat transfer.

11-36 Tomatoes are placed into cold water to cool them. The heat transfer coefficient and the amount of heat transfer are to be determined.

Assumptions **1** The tomatoes are spherical in shape. **2** Heat conduction in the tomatoes is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the tomatoes are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the tomatoes are given to be $k = 0.59$ W/m.°C, $\alpha = 0.141\times10^{-6}$ m^2/s, $\rho = 999$ kg/m^3 and $c_p = 3.99$ kJ/kg.°C.

Analysis The Fourier number is



$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.141\times10^{-6}\text{ m}^2/\text{s})(2\times3600\text{ s})}{(0.04\text{ m})^2} = 0.635$$

which is greater than 0.2. Therefore one-term solution is applicable. The ratio of the dimensionless temperatures at the surface and center of the tomatoes are

$$\frac{\theta_{s,sph}}{\theta_{0,sph}} = \frac{\dfrac{T_s - T_\infty}{T_i - T_\infty}}{\dfrac{T_0 - T_\infty}{T_i - T_\infty}} = \frac{T_s - T_\infty}{T_0 - T_\infty} = \frac{A_1 e^{-\lambda_1^2\tau}\dfrac{\sin(\lambda_1)}{\lambda_1}}{A_1 e^{-\lambda_1^2\tau}} = \frac{\sin(\lambda_1)}{\lambda_1}$$

Substituting,

$$\frac{7.1-7}{10-7} = \frac{\sin(\lambda_1)}{\lambda_1} \longrightarrow \lambda_1 = 3.0401$$

From Table 11-2, the corresponding Biot number and the heat transfer coefficient are

$$\text{Bi} = 31.1$$

$$\text{Bi} = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.59\text{ W/m.°C})(31.1)}{(0.04\text{ m})} = \mathbf{459\ W/m^2.°C}$$

The maximum amount of heat transfer is

$$m = 8\rho V = 8\rho\pi D^3/6 = 8(999\text{ kg/m}^3)[\pi(0.08\text{ m})^3/6] = 2.143\text{ kg}$$

$$Q_{max} = mc_p[T_i - T_\infty] = (2.143\text{ kg})(3.99\text{ kJ/kg.°C})(30-7)\text{°C} = 196.6\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\left(\frac{Q}{Q_{max}}\right)_{cyl} = 1 - 3\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3} = 1 - 3\left(\frac{10-7}{30-7}\right)\frac{\sin(3.0401) - (3.0401)\cos(3.0401)}{(3.0401)^3} = 0.9565$$

$$Q = 0.9565 Q_{max}$$

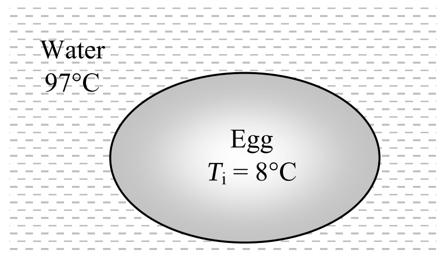
$$Q = 0.9565(196.6\text{ kJ}) = \mathbf{188\ kJ}$$

11-37 An egg is dropped into boiling water. The cooking time of the egg is to be determined.

Assumptions **1** The egg is spherical in shape with a radius of r_o = 2.75 cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is τ > 0.2 so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs are given to be k = 0.6 W/m.°C and $\alpha = 0.14\times10^{-6}$ m²/s.

Analysis The Biot number for this process is



$$Bi = \frac{hr_o}{k} = \frac{(1400\ \text{W/m}^2.°\text{C})(0.0275\ \text{m})}{(0.6\ \text{W/m.°C})} = 64.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 3.0877 \quad \text{and} \quad A_1 = 1.9969$$

Then the Fourier number becomes

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70-97}{8-97} = (1.9969)e^{-(3.0877)^2\tau} \longrightarrow \tau = 0.198 \approx 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the time required for the temperature of the center of the egg to reach 70°C is determined to be

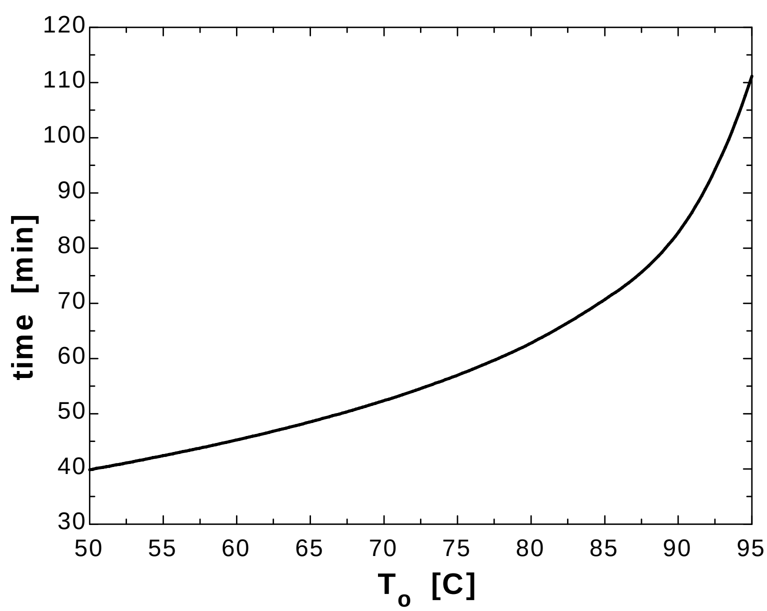
$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.198)(0.0275\ \text{m})^2}{(0.14\times10^{-6}\ \text{m}^2/\text{s})} = 1070\ \text{s} = \mathbf{17.8\ min}$$

11-38 EES Prob. 11-37 is reconsidered. The effect of the final center temperature of the egg on the time it will take for the center to reach this temperature is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.055 [m]
T_i=8 [C]
T_o=70 [C]
T_infinity=97 [C]
h=1400 [W/m^2-C]

"PROPERTIES"
k=0.6 [W/m-C]
alpha=0.14E-6 [m^2/s]

"ANALYSIS"
Bi=(h*r_o)/k
r_o=D/2
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1=1.9969
A_1=3.0863
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)
time=(tau*r_o^2)/alpha*Convert(s, min)
```

T_o [C]	time [min]
50	39.86
55	42.4
60	45.26
65	48.54
70	52.38
75	57
80	62.82
85	70.68
90	82.85
95	111.1



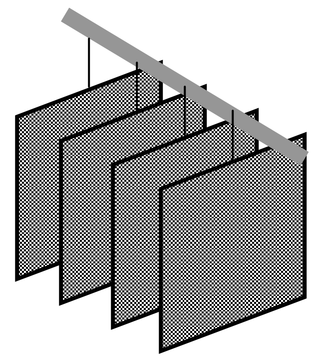
11-39 Large brass plates are heated in an oven. The surface temperature of the plates leaving the oven is to be determined.

Assumptions **1** Heat conduction in the plate is one-dimensional since the plate is large relative to its thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the plate are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is τ > 0.2 so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of brass at room temperature are given to be k = 110 W/m.°C, $\alpha = 33.9\times10^{-6}$ m^2/s

Analysis The Biot number for this process is

$$Bi = \frac{hL}{k} = \frac{(80\ \text{W/m}^2.°\text{C})(0.015\ \text{m})}{(110\ \text{W/m.°C})} = 0.0109$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 0.1035 \quad \text{and} \quad A_1 = 1.0018$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(33.9\times10^{-6}\ \text{m}^2/\text{s})(10\ \text{min}\times 60\ \text{s/min})}{(0.015\ \text{m})^2} = 90.4 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the temperature at the surface of the plates becomes

$$\theta(L,t)_{wall} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) = (1.0018)e^{-(0.1035)^2(90.4)}\cos(0.1035) = 0.378$$

$$\frac{T(L,t)-700}{25-700} = 0.378 \longrightarrow T(L,t) = \mathbf{445\ °C}$$

Discussion This problem can be solved easily using the lumped system analysis since Bi < 0.1, and thus the lumped system analysis is applicable. It gives

$$\alpha = \frac{k}{\rho c_p} \rightarrow \rho c_p = \frac{k}{\alpha} = \frac{110\ \text{W/m}\cdot°\text{C}}{33.9\times10^{-6}\ \text{m}^2/s} = 3.245\times10^6\ \text{W}\cdot\text{s/m}^3\cdot°\text{C}$$

$$b = \frac{hA}{\rho V c_p} = \frac{hA}{\rho(LA)c_p} = \frac{h}{\rho L c_p} = \frac{h}{L(k/\alpha)} = \frac{80\ \text{W/m}^2\cdot°\text{C}}{(0.015\ \text{m})(3.245\times10^6\ \text{W}\cdot\text{s/m}^3\cdot°\text{C})} = 0.001644\ \text{s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \quad\rightarrow\quad T(t) = T_\infty + (T_i - T_\infty)e^{-bt} = 700°\text{C} + (25 - 700°\text{C})e^{-(0.001644\ \text{s}^{-1})(600\ \text{s})} = \mathbf{448\ °C}$$

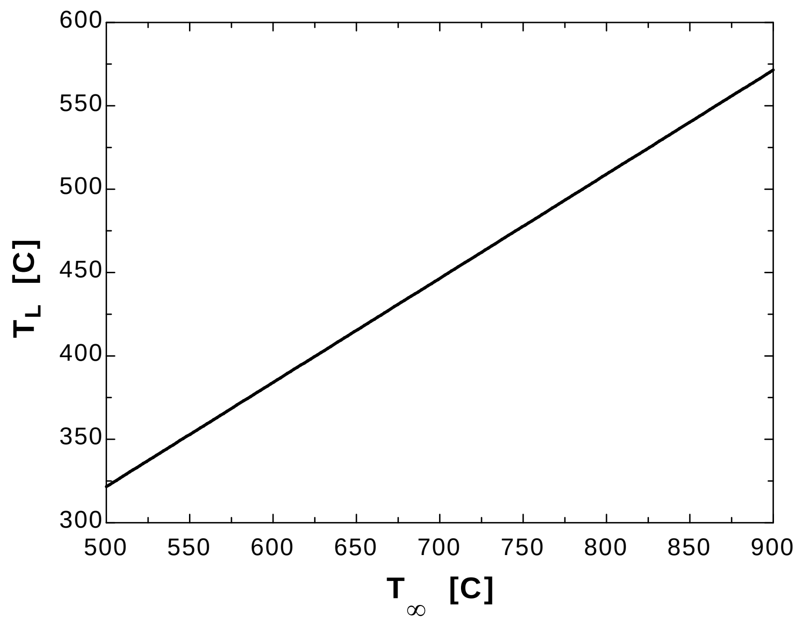
which is almost identical to the result obtained above.

11-40 EES Prob. 11-39 is reconsidered. The effects of the temperature of the oven and the heating time on the final surface temperature of the plates are to be investigated.

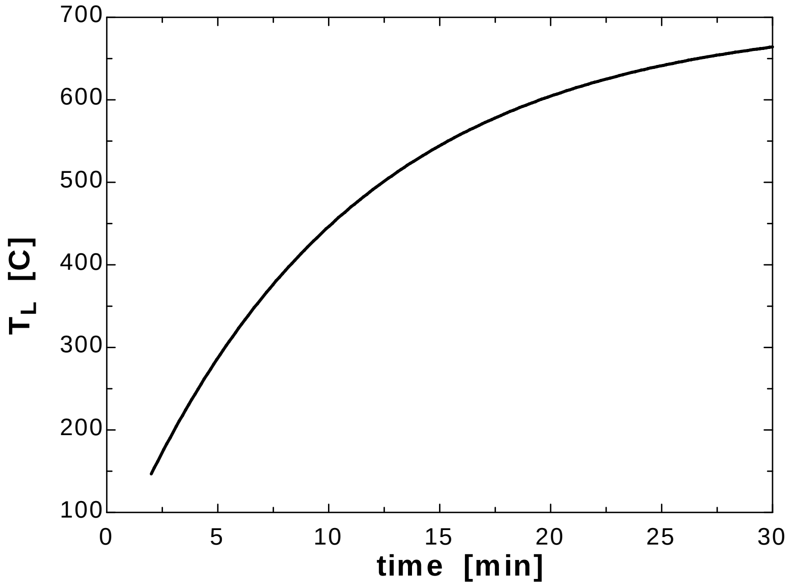
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=(0.03/2) [m]
T_i=25 [C]
T_infinity=700 [C]
time=10 [min]
h=80 [W/m^2-C]

"PROPERTIES"
k=110 [W/m-C]
alpha=33.9E-6 [m^2/s]

"ANALYSIS"
Bi=(h*L)/k
"From Table 11-2, corresponding to this Bi number, we read"
lambda_1=0.1039
A_1=1.0018
tau=(alpha*time*Convert(min, s))/L^2
(T_L-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)*Cos(lambda_1*L/L)
```

T_∞ [C]	T_L [C]
500	321.6
525	337.2
550	352.9
575	368.5
600	384.1
625	399.7
650	415.3
675	430.9
700	446.5
725	462.1
750	477.8
775	493.4
800	509
825	524.6
850	540.2
875	555.8
900	571.4



time [min]	T_L [C]
2	146.7
4	244.8
6	325.5
8	391.9
10	446.5
12	491.5
14	528.5
16	558.9
18	583.9
20	604.5
22	621.4
24	635.4
26	646.8
28	656.2
30	664



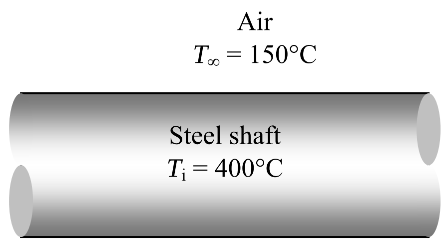
11-41 A long cylindrical shaft at 400°C is allowed to cool slowly. The center temperature and the heat transfer per unit length of the cylinder are to be determined.

Assumptions **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the shaft are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of stainless steel 304 at room temperature are given to be k = 14.9 W/m.°C, ρ = 7900 kg/m^3, c_p = 477 J/kg.°C, $\alpha = 3.95\times10^{-6}$ m^2/s

Analysis First the Biot number is calculated to be

$$Bi = \frac{hr_o}{k} = \frac{(60\ \text{W/m}^2.°\text{C})(0.175\ \text{m})}{(14.9\ \text{W/m}.°\text{C})} = 0.705$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.0904 \quad \text{and} \quad A_1 = 1.1548$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.95\times10^{-6}\ \text{m}^2/\text{s})(20\times60\ \text{s})}{(0.175\ \text{m})^2} = 0.1548$$

which is very close to the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the temperature at the center of the shaft becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.1548)e^{-(1.0904)^2(0.1548)} = 0.9607$$

$$\frac{T_0 - 150}{400 - 150} = 0.9607 \longrightarrow T_0 = \mathbf{390\ °C}$$

The maximum heat can be transferred from the cylinder per meter of its length is

$$m = \rho V = \rho\pi r_o^2 L = (7900\ \text{kg/m}^3)[\pi(0.175\ \text{m})^2(1\ \text{m})] = 760.1\ \text{kg}$$

$$Q_{\max} = mc_p[T_\infty - T_i] = (760.1\ \text{kg})(0.477\ \text{kJ/kg.°C})(400-150)°\text{C} = 90{,}640\ \text{kJ}$$

Once the constant J_1 = 0.4679 is determined from Table 11-3 corresponding to the constant λ_1 =1.0904, the actual heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\left(\frac{T_o - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\left(\frac{390-150}{400-150}\right)\frac{0.4679}{1.0904} = 0.1761$$

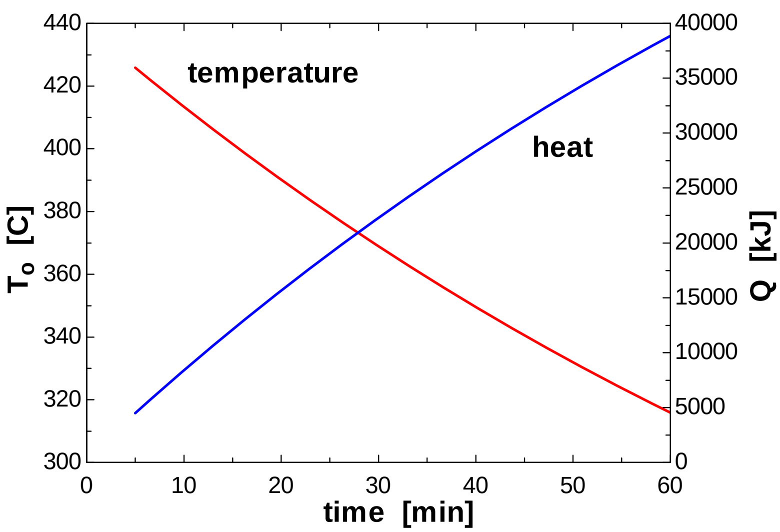
$$Q = 0.1761(90{,}640\ \text{kJ}) = \mathbf{15{,}960\ kJ}$$

11-42 EES Prob. 11-41 is reconsidered. The effect of the cooling time on the final center temperature of the shaft and the amount of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
r_o=(0.35/2) [m]
T_i=400 [C]
T_infinity=150 [C]
h=60 [W/m^2-C]
time=20 [min]

 "PROPERTIES"
k=14.9 [W/m-C]
rho=7900 [kg/m^3]
c_p=477 [J/kg-C]
alpha=3.95E-6 [m^2/s]

"ANALYSIS"
Bi=(h*r_o)/k
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1=1.0935
A_1=1.1558
J_1=0.4709 "From Table 11-3, corresponding to lambda_1"
tau=(alpha*time*Convert(min, s))/r_o^2
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)
L=1 "[m], 1 m length of the cylinder is considered"
V=pi*r_o^2*L
m=rho*V
Q_max=m*c_p*(T_i-T_infinity)*Convert(J, kJ)
Q/Q_max=1-2*(T_o-T_infinity)/(T_i-T_infinity)*J_1/lambda_1
```

time [min]	T_o [C]	Q [kJ]
5	425.9	4491
10	413.4	8386
15	401.5	12105
20	390.1	15656
25	379.3	19046
30	368.9	22283
35	359	25374
40	349.6	28325
45	340.5	31142
50	331.9	33832
55	323.7	36401
60	315.8	38853



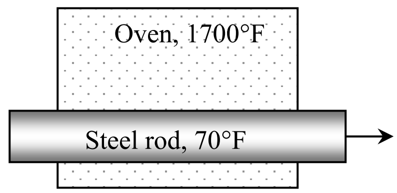
11-43E Long cylindrical steel rods are heat-treated in an oven. Their centerline temperature when they leave the oven is to be determined.

Assumptions **1** Heat conduction in the rods is one-dimensional since the rods are long and they have thermal symmetry about the center line. **2** The thermal properties of the rod are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of AISI stainless steel rods are given to be $k = 7.74$ Btu/h.ft.°F, $\alpha = 0.135$ ft^2/h.

Analysis The time the steel rods stays in the oven can be determined from

$$t = \frac{\text{length}}{\text{velocity}} = \frac{21\text{ ft}}{7\text{ ft/min}} = 3\text{ min} = 180\text{ s}$$



The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(20\text{ Btu/h.ft}^2.°\text{F})(2/12\text{ ft})}{(7.74\text{ Btu/h.ft.°F})} = 0.4307$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 0.8790 \quad \text{and} \quad A_1 = 1.0996$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.135\text{ ft}^2/\text{h})(3/60\text{ h})}{(2/12\text{ ft})^2} = 0.243$$

Then the temperature at the center of the rods becomes

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0996)e^{-(0.8790)^2(0.243)} = 0.911$$

$$\frac{T_0 - 1700}{70 - 1700} = 0.911 \longrightarrow T_o = \mathbf{215°F}$$

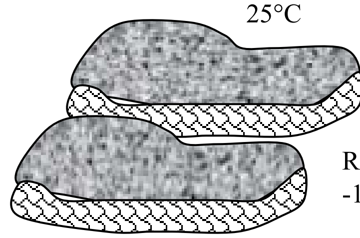
11-44 Steaks are cooled by passing them through a refrigeration room. The time of cooling is to be determined.

Assumptions **1** Heat conduction in the steaks is one-dimensional since the steaks are large relative to their thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the steaks are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of steaks are given to be $k = 0.45$ W/m.°C and $\alpha = 0.91\times10^{-7}$ m^2/s

Analysis The Biot number is

$$Bi = \frac{hL}{k} = \frac{(9 \text{ W/m}^2.°\text{C})(0.01 \text{ m})}{(0.45 \text{ W/m.°C})} = 0.200$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 0.4328 \quad \text{and} \quad A_1 = 1.0311$$

The Fourier number is

$$\frac{T(L,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 L / L)$$

$$\frac{2-(-11)}{25-(-11)} = (1.0311)e^{-(0.4328)^2 \tau} \cos(0.4328) \longrightarrow \tau = 5.085 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the length of time for the steaks to be kept in the refrigerator is determined to be

$$t = \frac{\tau L^2}{\alpha} = \frac{(5.085)(0.01 \text{ m})^2}{0.91\times10^{-7} \text{ m}^2/\text{s}} = 5590 \text{ s} = \mathbf{93.1\,min}$$

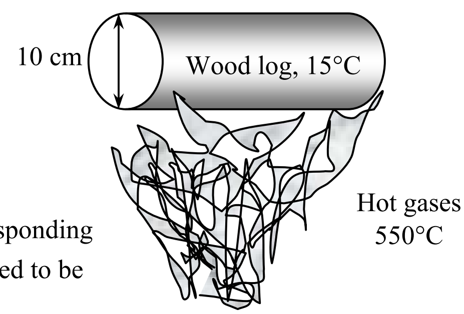
11-45 A long cylindrical wood log is exposed to hot gases in a fireplace. The time for the ignition of the wood is to be determined.

Assumptions **1** Heat conduction in the wood is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the wood are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is τ > 0.2 so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of wood are given to be k = 0.17 W/m.°C, $\alpha = 1.28\times10^{-7}$ m²/s

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(13.6\ \text{W/m}^2.°\text{C})(0.05\ \text{m})}{(0.17\ \text{W/m}.°\text{C})} = 4.00$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.9081 \quad \text{and} \quad A_1 = 1.4698$$

Once the constant J_0 is determined from Table 11-3 corresponding to the constant λ_1=1.9081, the Fourier number is determined to be

$$\frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o / r_o)$$

$$\frac{420-550}{15-550} = (1.4698)e^{-(1.9081)^2\tau}(0.2771) \longrightarrow \tau = 0.142$$

which is not above the value of 0.2 but it is close. We use one-term approximate solution (or the transient temperature charts) knowing that the result may be somewhat in error. Then the length of time before the log ignites is

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.142)(0.05\ \text{m})^2}{(1.28\times10^{-7}\ \text{m}^2/\text{s})} = 2770\ \text{s} = \mathbf{46.2\ min}$$

11-46 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is rare done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

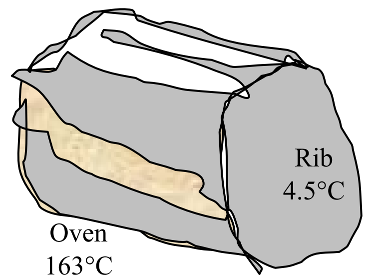
Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the rib are given to be $k = 0.45$ W/m.°C, $\rho = 1200$ kg/m^3, $c_p = 4.1$ kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s.

Analysis (*a*) The radius of the roast is determined to be

$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.002667\text{ m}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.002667\text{ m}^3)}{4\pi}} = 0.08603\text{ m}$$



The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\text{ m}^2/\text{s})(2\times3600+45\times60)\text{s}}{(0.08603\text{ m})^2} = 0.1217$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{60-163}{4.5-163} = 0.65 = A_1 e^{-\lambda_1^2(0.1217)}$$

It is determined from Table 11-2 by trial and error that this equation is satisfied when $Bi = 30$, which corresponds to $\lambda_1 = 3.0372$ and $A_1 = 1.9898$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\text{ W/m.°C})(30)}{(0.08603\text{ m})} = \mathbf{156.9\ W/m^2.°C}$$

This value seems to be larger than expected for problems of this kind. This is probably due to the Fourier number being less than 0.2.

(*b*) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9898)e^{-(3.0372)^2(0.1217)}\frac{\sin(3.0372\text{ rad})}{3.0372}$$

$$\frac{T(r_o,t)-163}{4.5-163} = 0.0222 \longrightarrow T(r_o,t) = \mathbf{159.5\ °C}$$

(*c*) The maximum possible heat transfer is

$$Q_{\max} = mc_p(T_\infty - T_i) = (3.2\text{ kg})(4.1\text{ kJ/kg.°C})(163-4.5)\text{°C} = 2080\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1)-\lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1-3(0.65)\frac{\sin(3.0372)-(3.0372)\cos(3.0372)}{(3.0372)^3} = 0.783$$

$$Q = 0.783Q_{\max} = (0.783)(2080\text{ kJ}) = \mathbf{1629\ kJ}$$

(*d*) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{71-163}{4.5-163} = (1.9898)e^{-(3.0372)^2\tau} \longrightarrow \tau = 0.1336$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1336)(0.08603\text{ m})^2}{(0.91\times10^{-7}\text{ m}^2/\text{s})} = 10{,}866\text{ s} = 181\text{ min} \cong \mathbf{3\ hr}$$

This result is close to the listed value of 3 hours and 20 minutes. The difference between the two results is due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

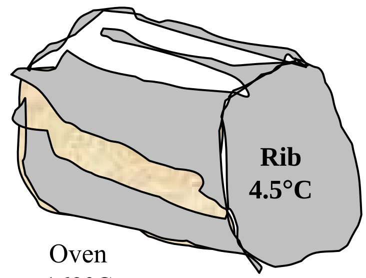
Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

11-47 A rib is roasted in an oven. The heat transfer coefficient at the surface of the rib, the temperature of the outer surface of the rib and the amount of heat transfer when it is well-done are to be determined. The time it will take to roast this rib to medium level is also to be determined.

Assumptions **1** The rib is a homogeneous spherical object. **2** Heat conduction in the rib is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the rib are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the rib are given to be $k = 0.45$ W/m.°C, $\rho = 1200$ kg/m^3, $c_p = 4.1$ kJ/kg.°C, and $\alpha = 0.91\times10^{-7}$ m^2/s

Analysis (*a*) The radius of the rib is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{3.2\text{ kg}}{1200\text{ kg/m}^3} = 0.00267\text{ m}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.00267\text{ m}^3)}{4\pi}} = 0.08603\text{ m}$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.91\times10^{-7}\text{ m}^2/\text{s})(4\times3600+15\times60)\text{s}}{(0.08603\text{ m})^2} = 0.1881$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the one-term solution formulation can be written in the form

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{77-163}{4.5-163} = 0.543 = A_1 e^{-\lambda_1^2 (0.1881)}$$

It is determined from Table 11-2 by trial and error that this equation is satisfied when $Bi = 4.3$, which corresponds to $\lambda_1 = 2.4900$ and $A_1 = 1.7402$. Then the heat transfer coefficient can be determined from.

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.45\text{ W/m.°C})(4.3)}{(0.08603\text{ m})} = \mathbf{22.5\text{ W/m}^2.°C}$$

(*b*) The temperature at the surface of the rib is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.7402)e^{-(2.49)^2(0.1881)} \frac{\sin(2.49)}{2.49}$$

$$\frac{T(r_o,t)-163}{4.5-163} = 0.132 \longrightarrow T(r_o,t) = \mathbf{142.1\text{ °C}}$$

(*c*) The maximum possible heat transfer is

$$Q_{\max} = mc_p(T_\infty - T_i) = (3.2\text{ kg})(4.1\text{ kJ/kg.°C})(163-4.5)\text{°C} = 2080\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.543)\frac{\sin(2.49) - (2.49)\cos(2.49)}{(2.49)^3} = 0.727$$

$$Q = 0.727 Q_{\max} = (0.727)(2080\text{ kJ}) = \mathbf{1512\text{ kJ}}$$

(*d*) The cooking time for medium-done rib is determined to be

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{71-163}{4.5-163} = (1.7402)e^{-(2.49)^2 \tau} \longrightarrow \tau = 0.177$$

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.177)(0.08603\text{ m})^2}{(0.91\times10^{-7}\text{ m}^2/\text{s})} = 14{,}400\text{ s} = 240\text{ min} = \mathbf{4\text{ hr}}$$

This result is close to the listed value of 4 hours and 15 minutes. The difference between the two results is probably due to the Fourier number being less than 0.2 and thus the error in the one-term approximation.

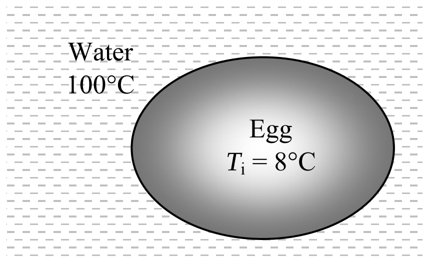
Discussion The temperature of the outer parts of the rib is greater than that of the inner parts of the rib after it is taken out of the oven. Therefore, there will be a heat transfer from outer parts of the rib to the inner parts as a result of this temperature difference. The recommendation is logical.

11-48 An egg is dropped into boiling water. The cooking time of the egg is to be determined.

Assumptions **1** The egg is spherical in shape with a radius of r_0 = 2.75 cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is τ > 0.2 so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs can be approximated by those of water at room temperature to be $k = 0.607$ W/m.°C, $\alpha = k/\rho c_p = 0.146\times10^{-6}$ m^2/s (Table A-15).

Analysis The Biot number is



$$Bi = \frac{hr_o}{k} = \frac{(800 \text{ W/m}^2.°\text{C})(0.0275 \text{ m})}{(0.607 \text{ W/m.°C})} = 36.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 3.0533 \quad \text{and} \quad A_1 = 1.9925$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{60-100}{8-100} = (1.9925)e^{-(3.0533)^2 \tau} \longrightarrow \tau = 0.1633$$

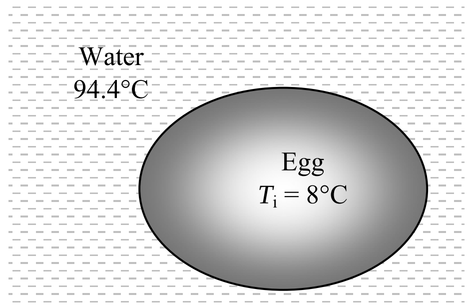
which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the length of time for the egg to be kept in boiling water is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1633)(0.0275 \text{ m})^2}{0.146\times10^{-6} \text{ m}^2\text{/s}} = 846 \text{ s} = \mathbf{14.1\,min}$$

11-49 An egg is cooked in boiling water. The cooking time of the egg is to be determined for a location at 1610-m elevation.

Assumptions **1** The egg is spherical in shape with a radius of r_o = 2.75 cm. **2** Heat conduction in the egg is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the egg and heat transfer coefficient are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and diffusivity of the eggs can be approximated by those of water at room temperature to be $k = 0.607$ W/m.°C, $\alpha = k/\rho c_p = 0.146\times10^{-6}$ m^2/s (Table A-15).



Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(800\ \text{W/m}^2.°\text{C})(0.0275\ \text{m})}{(0.607\ \text{W/m.°C})} = 36.2$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 3.0533 \quad \text{and} \quad A_1 = 1.9925$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{60 - 94.4}{8 - 94.4} = (1.9925)e^{-(3.0533)^2 \tau} \longrightarrow \tau = 0.1727$$

which is somewhat below the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can still be used, with the understanding that the error involved will be a little more than 2 percent. Then the length of time for the egg to be kept in boiling water is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.1727)(0.0275\ \text{m})^2}{(0.146\times10^{-6}\ \text{m}^2/\text{s})} = 895\ \text{s} = \mathbf{14.9\ min}$$

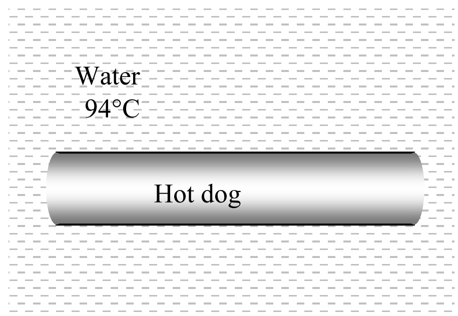
11-50 A hot dog is dropped into boiling water, and temperature measurements are taken at certain time intervals. The thermal diffusivity and thermal conductivity of the hot dog and the convection heat transfer coefficient are to be determined.

Assumptions **1** Heat conduction in the hot dog is one-dimensional since it is long and it has thermal symmetry about the centerline. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of hot dog available are given to be $\rho = 980$ kg/m^3 and $c_p = 3900$ J/kg.°C.

Analysis (*a*) From Fig. 11-16b we have

$$\left.\begin{array}{l} \dfrac{T-T_\infty}{T_0-T_\infty} = \dfrac{88-94}{59-94} = 0.17 \\ \dfrac{r}{r_o} = \dfrac{r_o}{r_o} = 1 \end{array}\right\} \dfrac{1}{Bi} = \dfrac{k}{hr_o} = 0.15$$



The Fourier number is determined from Fig. 11-16a to be

$$\left.\begin{array}{l} \dfrac{1}{Bi} = \dfrac{k}{hr_o} = 0.15 \\ \dfrac{T_0-T_\infty}{T_i-T_\infty} = \dfrac{59-94}{20-94} = 0.47 \end{array}\right\} \tau = \dfrac{\alpha t}{r_o^{\,2}} = 0.20$$

The thermal diffusivity of the hot dog is determined to be

$$\frac{\alpha t}{r_o^2} = 0.20 \longrightarrow \alpha = \frac{0.2 r_o^2}{t} = \frac{(0.2)(0.011\text{ m})^2}{120\text{ s}} = \mathbf{2.017 \times 10^{-7}\ m^2/s}$$

(*b*) The thermal conductivity of the hot dog is determined from

$$k = \alpha \rho c_p = (2.017\times10^{-7}\text{ m}^2\text{/s})(980\text{ kg/m}^3)(3900\text{ J/kg.°C}) = \mathbf{0.771\ W/m.°C}$$

(*c*) From part (*a*) we have $\dfrac{1}{Bi} = \dfrac{k}{hr_o} = 0.15$. Then,

$$\frac{k}{h} = 0.15 r_o = (0.15)(0.011\text{ m}) = 0.00165\text{ m}$$

Therefore, the heat transfer coefficient is

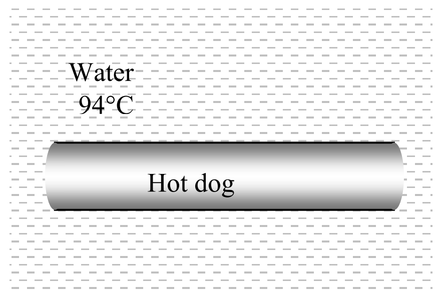
$$\frac{k}{h} = 0.00165 \longrightarrow h = \frac{0.771\text{ W/m.°C}}{0.00165\text{ m}} = \mathbf{467\ W/m^2.°C}$$

11-51 Using the data and the answers given in Prob. 11-50, the center and the surface temperatures of the hot dog 4 min after the start of the cooking and the amount of heat transferred to the hot dog are to be determined.

Assumptions **1** Heat conduction in the hot dog is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of hot dog and the convection heat transfer coefficient are given or obtained in P11-50 to be $k = 0.771$ W/m.°C, $\rho = 980$ kg/m^3, $c_p = 3900$ J/kg.°C, $\alpha = 2.017\times10^{-7}$ m^2/s, and $h = 467$ W/m^2.°C.

Analysis The Biot number is



$$Bi = \frac{hr_o}{k} = \frac{(467\text{ W/m}^2.°\text{C})(0.011\text{ m})}{(0.771\text{ W/m.°C})} = 6.66$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 2.0785 \quad \text{and} \quad A_1 = 1.5357$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(2.017\times10^{-7}\text{ m}^2\text{/s})(4\text{ min}\times 60\text{ s/min})}{(0.011\text{ m})^2} = 0.4001 > 0.2$$

Then the temperature at the center of the hot dog is determined to be

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5357)e^{-(2.0785)^2(0.4001)} = 0.2727$$

$$\frac{T_0 - 94}{20 - 94} = 0.2727 \longrightarrow T_o = \mathbf{73.8\ °C}$$

From Table 11-3 we read J_0 =0.1789 corresponding to the constant λ_1 =2.0785. Then the temperature at the surface of the hot dog becomes

$$\frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r_o / r_o) = (1.5357)e^{-(2.0785)^2(0.4001)}(0.1789) = 0.04878$$

$$\frac{T(r_o,t) - 94}{20 - 94} = 0.04878 \longrightarrow T(r_o,t) = \mathbf{90.4\ °C}$$

The maximum possible amount of heat transfer is

$$m = \rho V = \rho \pi r_o^2 L = (980\text{ kg/m}^3)\left[\pi(0.011\text{ m})^2 (0.125\text{ m})\right] = 0.04657\text{ kg}$$

$$Q_{\max} = mc_p(T_i - T_\infty) = (0.04657\text{ kg})(3900\text{ J/kg.°C})(94 - 20)°\text{C} = 13{,}440\text{ J}$$

From Table 11-3 we read J_1 = 0.5701 corresponding to the constant λ_1 =2.0785. Then the actual heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.2727)\frac{0.5701}{2.0785} = 0.8504$$

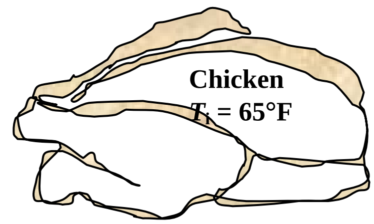
$$Q = 0.8504(13{,}440\text{ kJ}) = \mathbf{11{,}430\ kJ}$$

11-52E Whole chickens are to be cooled in the racks of a large refrigerator. Heat transfer coefficient that will enable to meet temperature constraints of the chickens while keeping the refrigeration time to a minimum is to be determined.

Assumptions **1** The chicken is a homogeneous spherical object. **2** Heat conduction in the chicken is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the chicken are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the chicken are given to be k = 0.26 Btu/h.ft.°F, ρ = 74.9 lbm/ft^3, c_p = 0.98 Btu/lbm.°F, and α = 0.0035 ft^2/h.

Analysis The radius of the chicken is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{5\text{ lbm}}{74.9\text{ lbm/ft}^3} = 0.06676\text{ ft}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.06676\text{ ft}^3)}{4\pi}} = 0.2517\text{ ft}$$

From Fig. 11-17b we have

$$\left.\begin{aligned} \frac{T - T_\infty}{T_0 - T_\infty} &= \frac{35-5}{45-5} = 0.75 \\ \frac{x}{r_o} &= \frac{r_o}{r_o} = 1 \end{aligned}\right\} \frac{1}{Bi} = \frac{k}{hr_o} = 2$$

Then the heat transfer coefficients becomes

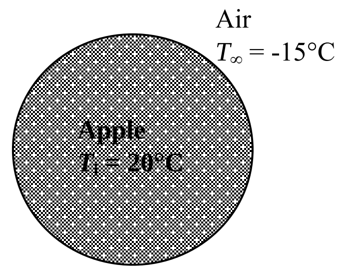
$$h = \frac{k}{2r_o} = \frac{0.26\text{ Btu/.ft.°F}}{2(0.2517\text{ ft})} = \mathbf{0.516\ Btu/h.ft^2.°F}$$

11-53 A person puts apples into the freezer to cool them quickly. The center and surface temperatures of the apples, and the amount of heat transfer from each apple in 1 h are to be determined.

Assumptions **1** The apples are spherical in shape with a diameter of 9 cm. **2** Heat conduction in the apples is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the apples are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the apples are given to be $k = 0.418$ W/m.°C, $\rho = 840$ kg/m^3, $c_p = 3.81$ kJ/kg.°C, and $\alpha = 1.3\times10^{-7}$ m^2/s.

Analysis The Biot number is



$$Bi = \frac{hr_o}{k} = \frac{(8\text{ W/m}^2.°\text{C})(0.045\text{ m})}{(0.418\text{ W/m.°C})} = 0.861$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.476 \quad \text{and} \quad A_1 = 1.2390$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.3\times10^{-7}\text{ m}^2\text{/s})(1\text{ h}\times3600\text{ s/h})}{(0.045\text{ m})^2} = 0.231 > 0.2$$

Then the temperature at the center of the apples becomes

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{T_0 - (-15)}{20-(-15)} = (1.239)e^{-(1.476)^2(0.231)} = 0.749 \longrightarrow T_0 = \mathbf{11.2°C}$$

The temperature at the surface of the apples is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.239)e^{-(1.476)^2(0.231)}\frac{\sin(1.476\text{ rad})}{1.476} = 0.505$$

$$\frac{T(r_o,t) - (-15)}{20-(-15)} = 0.505 \longrightarrow T(r_o,t) = \mathbf{2.7°C}$$

The maximum possible heat transfer is

$$m = \rho \boldsymbol{V} = \rho\frac{4}{3}\pi r_o^3 = (840\text{ kg/m}^3)\left[\frac{4}{3}\pi(0.045\text{ m})^3\right] = 0.3206\text{ kg}$$

$$Q_{\max} = mc_p(T_i - T_\infty) = (0.3206\text{ kg})(3.81\text{ kJ/kg.°C})[20-(-15)]°\text{C} = 42.75\text{ kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.749)\frac{\sin(1.476\text{ rad}) - (1.476)\cos(1.476\text{ rad})}{(1.476)^3} = 0.402$$

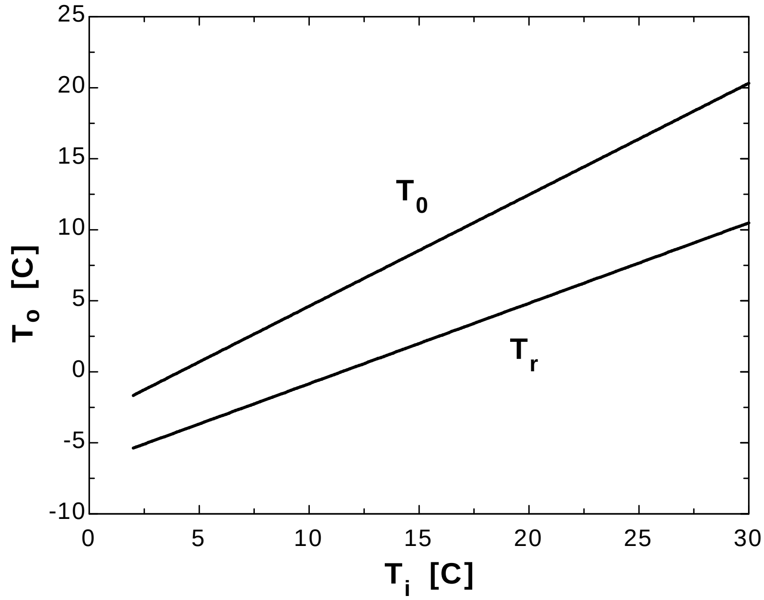
$$Q = 0.402Q_{\max} = (0.402)(42.75\text{ kJ}) = \mathbf{17.2\text{ kJ}}$$

11-54 EES Prob. 11-53 is reconsidered. The effect of the initial temperature of the apples on the final center and surface temperatures and the amount of heat transfer is to be investigated.

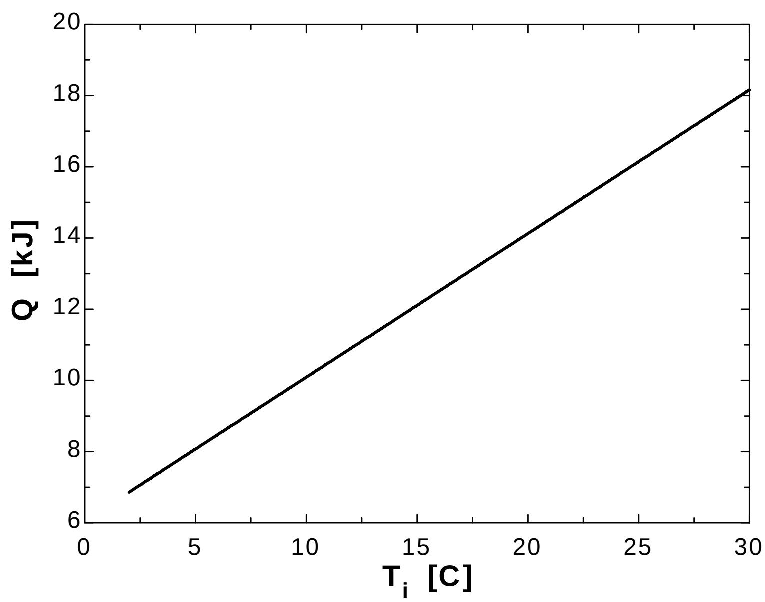
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_infinity=-15 [C]
T_i=20 [C]
h=8 [W/m^2-C]
r_o=0.09/2 [m]
time=1*3600 [s]

"PROPERTIES"
k=0.513 [W/m-C]
rho=840 [kg/m^3]
c_p=3.6 [kJ/kg-C]
alpha=1.3E-7 [m^2/s]

"ANALYSIS"
Bi=(h*r_o)/k
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1=1.3525
A_1=1.1978
tau=(alpha*time)/r_o^2
(T_o-T_infinity)/(T_i-T_infinity)=A_1*exp(-lambda_1^2*tau)

(T_r-T_infinity)/(T_i-T_infinity)=A_1*exp(-
lambda_1^2*tau)*Sin(lambda_1*r_o/r_o)/(lambda_1*r_o/r_o)

V=4/3*pi*r_o^3
m=rho*V
Q_max=m*c_p*(T_i-T_infinity)
Q/Q_max=1-3*(T_o-T_infinity)/(T_i-T_infinity)*(Sin(lambda_1)-
lambda_1*Cos(lambda_1))/lambda_1^3
```

T_i [C]	T_o [C]	T_r [C]	Q [kJ]
2	-1.658	-5.369	6.861
4	-0.08803	-4.236	7.668
6	1.482	-3.103	8.476
8	3.051	-1.97	9.283
10	4.621	-0.8371	10.09
12	6.191	0.296	10.9
14	7.76	1.429	11.7
16	9.33	2.562	12.51
18	10.9	3.695	13.32
20	12.47	4.828	14.13
22	14.04	5.961	14.93
24	15.61	7.094	15.74
26	17.18	8.227	16.55
28	18.75	9.36	17.35
30	20.32	10.49	18.16



T_o [C]
T_i [C]
T_0
T_r
25
20
15
10
5
0
-5
-10
0
5
10
15
20
25
30



Q [kJ]
T_i [C]
20
18
16
14
12
10
8
6
0
5
10
15
20
25
30

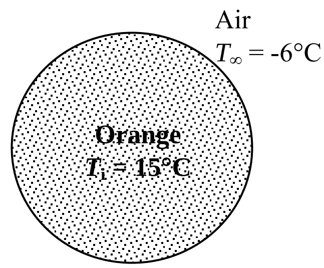
11-55 An orange is exposed to very cold ambient air. It is to be determined whether the orange will freeze in 4 h in subfreezing temperatures.

Assumptions **1** The orange is spherical in shape with a diameter of 8 cm. **2** Heat conduction in the orange is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the orange are constant, and are those of water. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the orange are approximated by those of water at the average temperature of about 5°C, $k = 0.571$ W/m.°C and $\alpha = k/\rho c_p = 0.571/(999.9\times 4205) = 0.136\times 10^{-6}\ \text{m}^2/\text{s}$ (Table A-15).

Analysis The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(15\ \text{W/m}^2.°\text{C})(0.04\ \text{m})}{(0.571\ \text{W/m.°C})} = 1.051 \approx 1.0$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.5708 \quad \text{and} \quad A_1 = 1.2732$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.136\times 10^{-6}\ \text{m}^2/\text{s})(4\ \text{h}\times 3600\ \text{s/h})}{(0.04\ \text{m})^2} = 1.224 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the temperature at the surface of the oranges becomes

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.2732)e^{-(1.5708)^2(1.224)}\frac{\sin(1.5708\ \text{rad})}{1.5708} = 0.0396$$

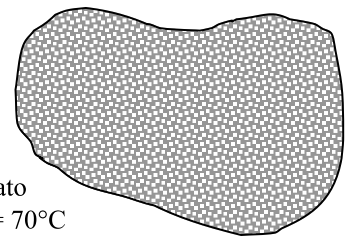
$$\frac{T(r_o,t)-(-6)}{15-(-6)} = 0.0396 \longrightarrow T(r_o,t) = \mathbf{-5.2\ °C}$$

which is less than 0°C. Therefore, the oranges will freeze.

11-56 A hot baked potato is taken out of the oven and wrapped so that no heat is lost from it. The time the potato is baked in the oven and the final equilibrium temperature of the potato after it is wrapped are to be determined.

Assumptions **1** The potato is spherical in shape with a diameter of 9 cm. **2** Heat conduction in the potato is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the potato are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the potato are given to be $k = 0.6$ W/m.°C, $\rho = 1100$ kg/m^3, $c_p = 3.9$ kJ/kg.°C, and $\alpha = 1.4\times10^{-7}$ m^2/s.



Analysis (*a*) The Biot number is

$$Bi = \frac{hr_o}{k} = \frac{(40\ \text{W/m}^2.°\text{C})(0.045\ \text{m})}{(0.6\ \text{W/m.°C})} = 3$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 2.2889 \quad \text{and} \quad A_1 = 1.6227$$

Then the Fourier number and the time period become

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{70-170}{25-170} = 0.69 = (1.6227)e^{-(2.2889)^2\tau} \longrightarrow \tau = 0.163$$

which is not greater than 0.2 but it is close. We may use one-term approximation knowing that the result may be somewhat in error. Then the baking time of the potatoes is determined to be

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.163)(0.045\ \text{m})^2}{1.4\times10^{-7}\ \text{m}^2/\text{s}} = 2358\ \text{s} = \mathbf{39.3\ min}$$

(*b*) The maximum amount of heat transfer is

$$m = \rho V = \rho \frac{4}{3}\pi r_o^3 = (1100\ \text{kg/m}^3)\left[\frac{4}{3}\pi(0.045\ \text{m})^3\right] = 0.420\ \text{kg}$$

$$Q_{\max} = mc_p(T_\infty - T_i) = (0.420\ \text{kg})(3.900\ \text{kJ/kg.°C})(170-25)°\text{C} = 237\ \text{kJ}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{\max}} = 1 - 3\theta_{o,sph}\frac{\sin(\lambda_1) - \lambda_1\cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.69)\frac{\sin(2.2889) - (2.2889)\cos(2.2889)}{(2.2889)^3} = 0.610$$

$$Q = 0.610 Q_{\max} = (0.610)(237\ \text{kJ}) = \mathbf{145\ kJ}$$

The final equilibrium temperature of the potato after it is wrapped is

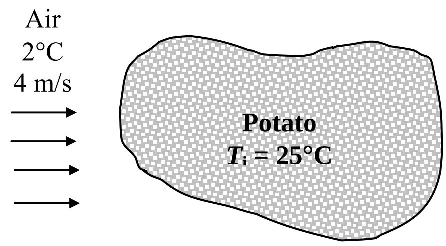
$$Q = mc_p(T_{eqv} - T_i) \longrightarrow T_{eqv} = T_i + \frac{Q}{mc_p} = 25°\text{C} + \frac{145\ \text{kJ}}{(0.420\ \text{kg})(3.9\ \text{kJ/kg.°C})} = \mathbf{114°C}$$

11-57 The center temperature of potatoes is to be lowered to 6°C during cooling. The cooling time and if any part of the potatoes will suffer chilling injury during this cooling process are to be determined.

Assumptions **1** The potatoes are spherical in shape with a radius of r_0 = 3 cm. **2** Heat conduction in the potato is one-dimensional in the radial direction because of the symmetry about the midpoint. **3** The thermal properties of the potato are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of potatoes are given to be k = 0.50 W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s.

Analysis First we find the Biot number:



$$\text{Bi} = \frac{hr_o}{k} = \frac{(19\text{ W/m}^2.°\text{C})(0.03\text{ m})}{0.5\text{ W/m.°C}} = 1.14$$

From Table 11-2 we read, for a sphere, λ_1 = 1.635 and A_1 = 1.302. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \rightarrow \frac{6-2}{25-2} = 1.302e^{-(1.635)^2\tau} \rightarrow \tau = 0.753$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_o^2} \longrightarrow t = \frac{\tau r_o^2}{\alpha} = \frac{(0.753)(0.03\text{ m})^2}{0.13\times10^{-6}\text{ m}^2/\text{s}} = 5213\text{ s} = \mathbf{1.45\ h}$$

The lowest temperature during cooling will occur on the surface (r/r_0 = 1), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \rightarrow \frac{T(r_o)-T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = \frac{T_o - T_\infty}{T_i - T_\infty}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o}$$

Substituting, $$\frac{T(r_o)-2}{25-2} = \left(\frac{6-2}{25-2}\right)\frac{\sin(1.635\text{ rad})}{1.635} \longrightarrow T(r_o) = 4.44°\text{C}$$

which is above the temperature range of 3 to 4 °C for chilling injury for potatoes. Therefore, **no part** of the potatoes will experience chilling injury during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{0.50\text{W/m.}^\text{o}\text{C}}{(19\text{W/m}^2.^\text{o}\text{C})(0.03\text{m})} = 0.877 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{6-2}{25-2} = 0.174 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.75 \quad \text{(Fig. 11-17a)}$$

Therefore, $$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.75)(0.03)^2}{0.13\times10^{-6}\text{ m}^2/s} = 5192\text{ s} = \mathbf{1.44\ h}$$

The surface temperature is determined from

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.877 \\ \frac{r}{r_o} &= 1 \end{aligned}\right\} \frac{T(r) - T_\infty}{T_o - T_\infty} = 0.6 \quad \text{(Fig.11-17b)}$$

which gives $T_{surface} = T_\infty + 0.6(T_o - T_\infty) = 2 + 0.6(6-2) = 4.4°\text{C}$

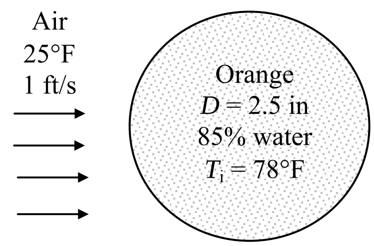
The slight difference between the two results is due to the reading error of the charts.

11-58E The center temperature of oranges is to be lowered to 40°F during cooling. The cooling time and if any part of the oranges will freeze during this cooling process are to be determined.

Assumptions **1** The oranges are spherical in shape with a radius of r_o =1.25 in = 0.1042 ft. **2** Heat conduction in the orange is one-dimensional in the radial direction because of the symmetry about the midpoint. **3** The thermal properties of the orange are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of oranges are given to be k = 0.26 Btu/h·ft·°F and $\alpha = 1.4\times10^{-6}$ ft²/s.

Analysis First we find the Biot number:



$$\text{Bi} = \frac{hr_o}{k} = \frac{(4.6\text{ Btu/h.ft}^2.°\text{F})(1.25/12\text{ ft})}{0.26\text{ Btu/h.ft.°F}} = 1.843$$

From Table 11-2 we read, for a sphere, λ_1 = 1.9569 and A_1 = 1.447. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{40-25}{78-25} = 1.447 e^{-(1.9569)^2 \tau} \rightarrow \tau = 0.426$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_o^2} \rightarrow t = \frac{\tau r_o^2}{\alpha} = \frac{(0.426)(1.25/12\text{ ft})^2}{1.4\times10^{-6}\text{ ft}^2/\text{s}} = 3302\text{ s} = \mathbf{55.0\ min}$$

The lowest temperature during cooling will occur on the surface (r/r_0 = 1), and is determined to be

$$\frac{T(r) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \rightarrow \frac{T(r_o) - T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = \frac{T_o - T_\infty}{T_i - T_\infty}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o}$$

Substituting,
$$\frac{T(r_o) - 25}{78-25} = \left(\frac{40-25}{78-25}\right)\frac{\sin(1.9569\text{ rad})}{1.9569} \longrightarrow T(r_o) = 32.1°\text{F}$$

which is above the freezing temperature of 31°F for oranges. Therefore, no part of the oranges will freeze during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = \frac{0.26\text{Btu/h.ft.}^\text{o}\text{ F}}{(4.6\text{ Btu/h.ft}^2.^\text{o}\text{ F})(1.25/12\text{ ft})} = 0.543 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{40-25}{78-25} = 0.283 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^{\ 2}} = 0.43 \quad \text{(Fig.11-17a)}$$

Therefore,
$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.43)(1.25/12\text{ft})^2}{1.4\times10^{-6}\text{ ft}^2/\text{s}} = 3333\text{ s} = 55.6\text{ min}$$

The lowest temperature during cooling will occur on the surface (r/r_o =1) of the oranges is determined to be

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.543 \\ \frac{r}{r_o} &= 1 \end{aligned}\right\} \frac{T(r) - T_\infty}{T_0 - T_\infty} = 0.45 \quad \text{(Fig.11–17b)}$$

which gives
$$T_{surface} = T_\infty + 0.45(T_0 - T_\infty) = 25 + 0.45(40-25) = 31.8°\text{F}$$

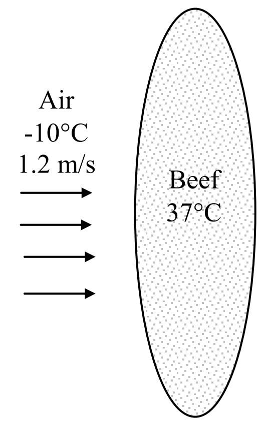
The slight difference between the two results is due to the reading error of the charts.

11-59 The center temperature of a beef carcass is to be lowered to 4°C during cooling. The cooling time and if any part of the carcass will suffer freezing injury during this cooling process are to be determined.

Assumptions **1** The beef carcass can be approximated as a cylinder with insulated top and base surfaces having a radius of r_o = 12 cm and a height of H = 1.4 m. **2** Heat conduction in the carcass is one-dimensional in the radial direction because of the symmetry about the centerline. **3** The thermal properties of the carcass are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of carcass are given to be k = 0.47 W/m·°C and $\alpha = 0.13\times10^{-6}$ m²/s.

Analysis First we find the Biot number:



$$\text{Bi} = \frac{hr_o}{k} = \frac{(22\ \text{W/m}^2.°\text{C})(0.12\ \text{m})}{0.47\ \text{W/m.°C}} = 5.62$$

From Table 11-2 we read, for a cylinder, $\lambda_1 = 2.027$ and $A_1 = 1.517$. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{4-(-10)}{37-(-10)} = 1.517 e^{-(2.027)^2 \tau} \rightarrow \tau = 0.396$$

which is greater than 0.2 and thus the one-term solution is applicable. Then the cooling time becomes

$$\tau = \frac{\alpha t}{r_o^2} \rightarrow t = \frac{\tau r_o^2}{\alpha} = \frac{(0.396)(0.12\ \text{m})^2}{0.13\times10^{-6}\ \text{m}^2/\text{s}} = 43{,}865\ \text{s} = \mathbf{12.2\ h}$$

The lowest temperature during cooling will occur on the surface (r/r_o = 1), and is determined to be

$$\frac{T(r)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r / r_o) \rightarrow \frac{T(r_o)-T_\infty}{T_i - T_\infty} = \theta_0 J_0(\lambda_1 r / r_o) = \frac{T_o - T_\infty}{T_i - T_\infty} J_0(\lambda_1 r_o / r_o)$$

Substituting,
$$\frac{T(r_0)-(-10)}{37-(-10)} = \left(\frac{4-(-10)}{37-(-10)}\right) J_0(\lambda_1) = 0.2979\times0.2084 = 0.0621 \longrightarrow T(r_o) = -7.1°\text{C}$$

which is below the freezing temperature of -1.7 °C. Therefore, the outer part of the beef carcass will freeze during this cooling process.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = \frac{0.47\text{W/m.°C}}{(22\ \text{W/m}^2.°\text{C})(0.12\ \text{m})} = 0.178 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{4-(-10)}{37-(-10)} = 0.298 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.4 \quad (\text{Fig.}11-16\text{a})$$

Therefore,
$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.4)(0.12\ \text{m})^2}{0.13\times10^{-6}\ \text{m}^2/\text{s}} = 44{,}308\text{s} \cong 12.3\text{h}$$

The surface temperature is determined from

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.178 \\ \frac{r}{r_o} &= 1 \end{aligned}\right\} \frac{T(r)-T_\infty}{T_0 - T_\infty} = 0.17 \quad (\text{Fig.}11-16\text{b})$$

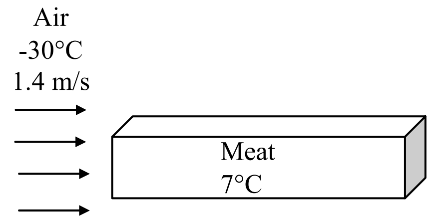
which gives $T_{surface} = T_\infty + 0.17(T_0 - T_\infty) = -10 + 0.17[4-(-10)] = -7.6°\text{C}$

The difference between the two results is due to the reading error of the charts.

11-60 The center temperature of meat slabs is to be lowered to -18°C during cooling. The cooling time and the surface temperature of the slabs at the end of the cooling process are to be determined.

Assumptions **1** The meat slabs can be approximated as very large plane walls of half-thickness L = 11.5 cm. **2** Heat conduction in the meat slabs is one-dimensional because of the symmetry about the centerplane. **3** The thermal properties of the meat slabs are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified). **6** The phase change effects are not considered, and thus the actual cooling time will be much longer than the value determined.

Properties The thermal conductivity and thermal diffusivity of meat slabs are given to be k = 0.47 W/m·°C and $\alpha = 0.13\times10^{-6}$ m^2/s. These properties will be used for both fresh and frozen meat.



Analysis First we find the Biot number:

$$\text{Bi} = \frac{hr_o}{k} = \frac{(20 \text{ W/m}^2.°\text{C})(0.115 \text{ m})}{0.47 \text{ W/m.°C}} = 4.89$$

From Table 11-2 we read, for a plane wall, $\lambda_1 = 1.308$ and A_1=1.239. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \rightarrow \frac{-18-(-30)}{7-(-30)} = 1.239 e^{-(1.308)^2 \tau} \rightarrow \tau = 0.783$$

which is greater than 0.2 and thus the one-term solution is applicable.
Then the cooling time becomes

$$\tau = \frac{\alpha t}{L^2} \rightarrow t = \frac{\tau L^2}{\alpha} = \frac{(0.783)(0.115 \text{ m})^2}{0.13\times10^{-6} \text{ m}^2/\text{s}} = 79{,}650 \text{ s} = \mathbf{22.1\,h}$$

The lowest temperature during cooling will occur on the surface (x/L = 1), and is determined to be

$$\frac{T(x)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}\cos(\lambda_1 x/L) \rightarrow \frac{T(L)-T_\infty}{T_i-T_\infty} = \theta_0 \cos(\lambda_1 L/L) = \frac{T_o - T_\infty}{T_i - T_\infty}\cos(\lambda_1)$$

Substituting,

$$\frac{T(L)-(-30)}{7-(-30)} = \left(\frac{-18-(-30)}{7-(-30)}\right)\cos(\lambda_1) = 0.3243\times0.2598 = 0.08425 \longrightarrow T(L) = \mathbf{-26.9°C}$$

which is close the temperature of the refrigerated air.

Alternative solution We could also solve this problem using transient temperature charts as follows:

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hL} = \frac{0.47 \text{W/m.°C}}{(20 \text{ W/m}^2.°\text{C})(0.115 \text{ m})} = 0.204 \\ \frac{T_o - T_\infty}{T_i - T_\infty} &= \frac{-18-(-30)}{7-(-30)} = 0.324 \end{aligned}\right\} \tau = \frac{\alpha t}{L^2} = 0.75 \quad (\text{Fig.11}-15\text{a})$$

$$\text{Therefore, } t = \frac{\tau r_o^2}{\alpha} = \frac{(0.75)(0.115 \text{ m})^2}{0.13\times10^{-6} \text{m}^2/\text{s}} = 76{,}300\text{s} \cong 21.2 \text{ h}$$

The surface temperature is determined from

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hL} = 0.204 \\ \frac{x}{L} &= 1 \end{aligned}\right\} \frac{T(x)-T_\infty}{T_o - T_\infty} = 0.22 \quad (\text{Fig.11}-15\text{b})$$

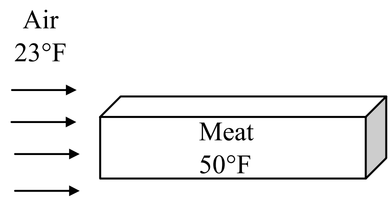
which gives $T_{surface} = T_\infty + 0.22(T_o - T_\infty) = -30 + 0.22[-18-(-30)] = -27.4°\text{C}$

The slight difference between the two results is due to the reading error of the charts.

11-61E The center temperature of meat slabs is to be lowered to 36°F during 12-h of cooling. The average heat transfer coefficient during this cooling process is to be determined.

Assumptions **1** The meat slabs can be approximated as very large plane walls of half-thickness L = 3-in. **2** Heat conduction in the meat slabs is one-dimensional because of symmetry about the centerplane. **3** The thermal properties of the meat slabs are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal conductivity and thermal diffusivity of meat slabs are given to be k = 0.26 Btu/h·ft·°F and $\alpha = 1.4\times10^{-6}$ ft²/s.



Analysis The average heat transfer coefficient during this cooling process is determined from the transient temperature charts for a flat plate as follows:

$$\left.\begin{aligned}\tau &= \frac{\alpha t}{L^2} = \frac{(1.4\times10^{-6}\ \text{ft}^2/\text{s})(12\times3600\ \text{s})}{(3/12\ \text{ft})^2} = 0.968\\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{36-23}{50-23} = 0.481\end{aligned}\right\}\ \frac{1}{Bi} = 0.7 \quad (\text{Fig.}11-15\text{a})$$

Therefore,

$$h = \frac{kBi}{L} = \frac{(0.26\text{Btu/h.ft.}^\circ\text{F})(1/0.7)}{(3/12)\ \text{ft}} = \mathbf{1.5\ Btu/h.ft^2.^\circ F}$$

Discussion We could avoid the uncertainty associated with the reading of the charts and obtain a more accurate result by using the one-term solution relation for an infinite plane wall, but it would require a trial and error approach since the Bi number is not known.

11-62 Chickens are to be chilled by holding them in agitated brine for 2.75 h. The center and surface temperatures of the chickens are to be determined, and if any part of the chickens will freeze during this cooling process is to be assessed.

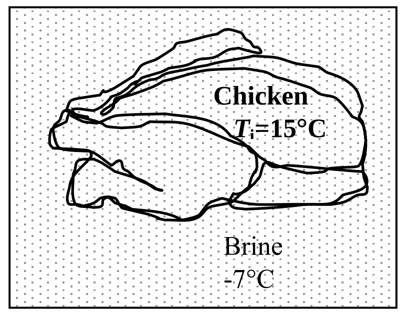
Assumptions **1** The chickens are spherical in shape. **2** Heat conduction in the chickens is one-dimensional in the radial direction because of symmetry about the midpoint. **3** The thermal properties of the chickens are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified). **6** The phase change effects are not considered, and thus the actual the temperatures will be much higher than the values determined since a considerable part of the cooling process will occur during phase change (freezing of chicken).

Properties The thermal conductivity, thermal diffusivity, and density of chickens are given to be $k = 0.45$ W/m·°C, $\alpha = 0.13\times10^{-6}$ m^2/s, and $\rho = 950$ kg/ m^3. These properties will be used for both fresh and frozen chicken.

Analysis We first find the volume and equivalent radius of the chickens:

$$V = m/\rho = 1700\text{g}/(0.95\text{g/cm}^3) = 1789\text{cm}^3$$

$$r_o = \left(\frac{3}{4\pi}V\right)^{1/3} = \left(\frac{3}{4\pi}1789\ \text{cm}^3\right)^{1/3} = 7.53\ \text{cm} = 0.0753\ \text{m}$$



Then the Biot and Fourier numbers become

$$\text{Bi} = \frac{hr_o}{k} = \frac{(440\ \text{W/m}^2.°\text{C})(0.0753\ \text{m})}{0.45\ \text{W/m.°C}} = 73.6$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.13\times10^{-6}\ \text{m}^2/\text{s})(2.75\times3600\ \text{s})}{(0.0753\ \text{m})^2} = 0.2270$$

Note that $\tau = 0.2270 > 0.2$, and thus the one-term solution is applicable. From Table 11-2 we read, for a sphere, $\lambda_1 = 3.094$ and $A_1 = 1.998$. Substituting these values into the one-term solution gives

$$\theta_0 = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \rightarrow \frac{T_0 - (-7)}{15 - (-7)} = 1.998e^{-(3.094)^2(0.2270)} = 0.2274 \longrightarrow T_0 = \mathbf{-2.0°C}$$

The lowest temperature during cooling will occur on the surface ($r/r_o = 1$), and is determined to be

$$\frac{T(r) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o} \rightarrow \frac{T(r_o) - T_\infty}{T_i - T_\infty} = \theta_0 \frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = \frac{T_o - T_\infty}{T_i - T_\infty}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o}$$

Substituting, $$\frac{T(r_o) - (-7)}{15 - (-7)} = 0.2274\frac{\sin(3.094\ \text{rad})}{3.094} \rightarrow T(r_o) = \mathbf{-6.9°C}$$

Most parts of chicken will freeze during this process since the freezing point of chicken is -2.8°C.

Discussion We could also solve this problem using transient temperature charts, but the data in this case falls at a point on the chart which is very difficult to read:

$$\left.\begin{aligned} \tau &= \frac{\alpha t}{r_o^2} = \frac{(0.13\times10^{-6}\ \text{m}^2/\text{s})(2.75\times3600\ \text{s})}{(0.0753\ \text{m})^2} = 0.227 \\ \frac{1}{Bi} &= \frac{k}{hr_o} = \frac{0.45\ \text{W/m.°C}}{(440\text{W/m}^2.°\text{C})(0.0753\text{m})} = 0.0136 \end{aligned}\right\} \frac{T_o - T_\infty}{T_i - T_\infty} = 0.15....0.30\ ?? \quad (\text{Fig. } 11-17)$$

Transient Heat Conduction in Semi-Infinite Solids

11-63C A semi-infinite medium is an idealized body which has a single exposed plane surface and extends to infinity in all directions. The earth and thick walls can be considered to be semi-infinite media.

11-64C A thick plane wall can be treated as a semi-infinite medium if all we are interested in is the variation of temperature in a region near one of the surfaces for a time period during which the temperature in the mid section of the wall does not experience any change.

11-65C The total amount of heat transfer from a semi-infinite solid up to a specified time t_0 can be determined by integration from

$$Q = \int_0^{t_o} Ah[T(0,t) - T_\infty]dt$$

where the surface temperature $T(0, t)$ is obtained from Eq. 11-47 by substituting $x = 0$.

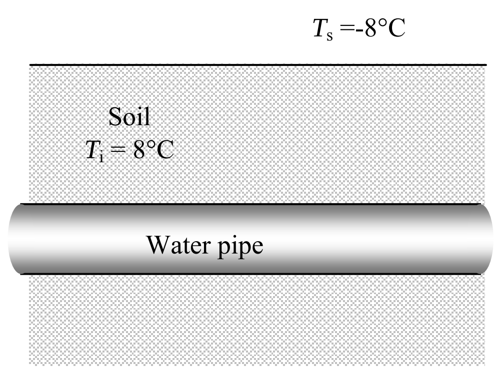
11-66 The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be $k = 0.35$ W/m.°C and $\alpha = 0.15\times10^{-6}$ m^2/s.

Analysis The length of time the snow pack stays on the ground is

$$t = (60\text{ days})(24\text{ hr/days})(3600\text{ s/hr}) = 5.184\times10^6\text{ s}$$



The surface is kept at -8°C at all times. The depth at which freezing at 0°C occurs can be determined from the analytical solution,

$$\frac{T(x,t)-T_i}{T_s - T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right)$$

$$\frac{0-8}{-8-8} = erfc\left(\frac{x}{2\sqrt{(0.15\times10^{-6}\text{ m}^2\text{/s})(5.184\times10^6\text{ s})}}\right)$$

$$0.5 = erfc\left(\frac{x}{1.7636}\right)$$

Then from Table 11-4 we get $\dfrac{x}{1.7636} = 0.4796 \longrightarrow x = \mathbf{0.846\ m}$

Discussion The solution could also be determined using the chart, but it would be subject to reading error.

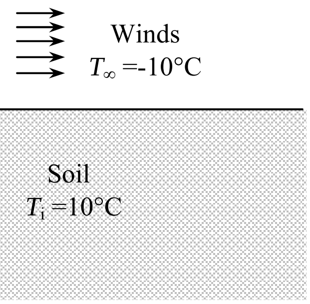
11-67 An area is subjected to cold air for a 10-h period. The soil temperatures at distances 0, 10, 20, and 50 cm from the earth's surface are to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be k = 0.9 W/m.°C and $\alpha = 1.6\times10^{-5}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the ground can be determined from

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$



where

$$\frac{h\sqrt{\alpha t}}{k} = \frac{(40\text{ W/m}^2.°\text{C})\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\times3600\text{ s})}}{0.9\text{ W/m.°C}} = 33.7$$

$$\frac{h^2\alpha t}{k^2} = \left(\frac{h\sqrt{\alpha t}}{k}\right)^2 = 33.7^2 = 1138$$

Then we conclude that the last term in the temperature distribution relation above must be zero regardless of x despite the exponential term tending to infinity since (1) $erfc(\eta)\to 0$ for $\eta > 4$ (see Table 11-4) and (2) the term has to remain less than 1 to have physically meaningful solutions. That is,

$$\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right] = \exp\left(\frac{hx}{k}+1138\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+33.7\right)\right] \cong 0$$

Therefore, the temperature distribution relation simplifies to

$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right) \to T(x,t) = T_i + (T_\infty - T_i)erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Then the temperatures at 0, 10, 20, and 50 cm depth from the ground surface become

$x = 0$: $$T(0{,}10\text{ h}) = T_i + (T_\infty - T_i)erfc\left(\frac{0}{2\sqrt{\alpha t}}\right) = T_i + (T_\infty - T_i)erfc(0) = T_i + (T_\infty - T_i)\times 1 = T_\infty = \mathbf{-10°C}$$

$x = 0.1$m: $$T(0.1\text{ m},10\text{ h}) = 10 + (-10-10)erfc\left(\frac{0.1\text{ m}}{2\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\text{ h}\times3600\text{ s/h})}}\right)$$

$$= 10 - 20erfc(0.066) = 10 - 20\times0.9257 = \mathbf{-8.5°C}$$

$x = 0.2$ m: $$T(0.2\text{ m},10\text{ h}) = 10 + (-10-10)erfc\left(\frac{0.2\text{ m}}{2\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\text{ h}\times3600\text{ s/h})}}\right)$$

$$= 10 - 20erfc(0.132) = 10 - 20\times0.8519 = \mathbf{-7.0°C}$$

$x = 0.5$ m: $$T(0.5\text{ m},10\text{ h}) = 10 + (-10-10)erfc\left(\frac{0.5\text{ m}}{2\sqrt{(1.6\times10^{-5}\text{ m}^2/\text{s})(10\text{ h}\times3600\text{ s/h})}}\right)$$

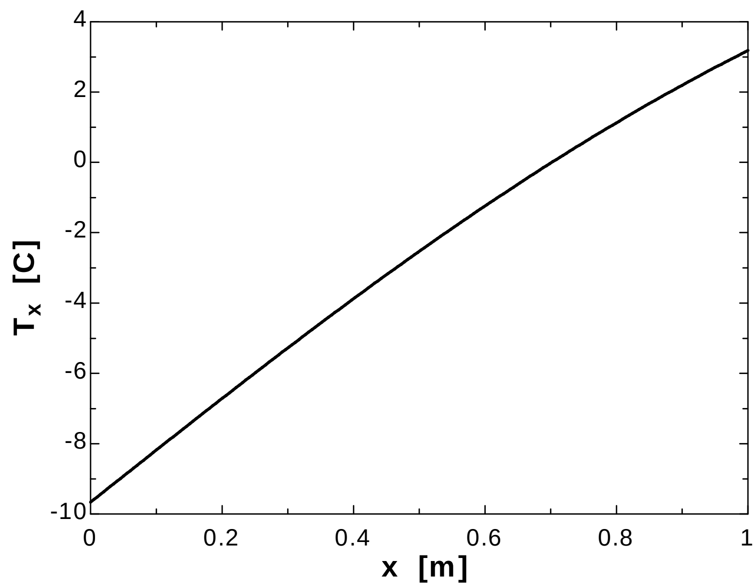
$$= 10 - 20erfc(0.329) = 10 - 20\times0.6418 = -\mathbf{2.8}°\text{C}$$

11-68 EES Prob. 11-67 is reconsidered. The soil temperature as a function of the distance from the earth's surface is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=10 [C]
T_infinity=-10 [C]
h=40 [W/m^2-C]
time=10*3600 [s]
x=0.1 [m]

"PROPERTIES"
k=0.9 [W/m-C]
alpha=1.6E-5 [m^2/s]

"ANALYSIS"
(T_x-T_i)/(T_infinity-T_i)=erfc(x/(2*sqrt(alpha*time)))-
exp((h*x)/k+(h^2*alpha*time)/k^2)*erfc(x/(2*sqrt(alpha*time))+(h*sqrt(alpha*time)/k))
```

x [m]	T_x [C]
0	-9.666
0.05	-8.923
0.1	-8.183
0.15	-7.447
0.2	-6.716
0.25	-5.993
0.3	-5.277
0.35	-4.572
0.4	-3.878
0.45	-3.197
0.5	-2.529
0.55	-1.877
0.6	-1.24
0.65	-0.6207
0.7	-0.01894
0.75	0.5643
0.8	1.128
0.85	1.672
0.9	2.196
0.95	2.7
1	3.183



11-69 An aluminum block is subjected to heat flux. The surface temperature of the block is to be determined.

Assumptions **1** All heat flux is absorbed by the block. **2** Heat loss from the block is disregarded (and thus the result obtained is the maximum temperature). **3** The block is sufficiently thick to be treated as a semi-infinite solid, and the properties of the block are constant.

Properties Thermal conductivity and diffusivity of aluminum at room temperature are k = 237 kg/m^3 and α = 97.1×10^{-6} m^2/s.

Analysis This is a transient conduction problem in a semi-infinite medium subjected to constant surface heat flux, and the surface temperature can be determined to be

$$T_s = T_i + \frac{\dot{q}_s}{k}\sqrt{\frac{4\alpha t}{\pi}} = 20°\text{C} + \frac{4000\ \text{W/m}^2}{237\ \text{W/m}\cdot°\text{C}}\sqrt{\frac{4(9.71\times10^{-5}\ \text{m}^2/\text{s})(30\times60\ \text{s})}{\pi}} = 28.0°\text{C}$$

Then the temperature rise of the surface becomes

$$\Delta T_s = 28 - 20 = \mathbf{8.0°C}$$

11-70 The contact surface temperatures when a bare footed person steps on aluminum and wood blocks are to be determined.

Assumptions **1** Both bodies can be treated as the semi-infinite solids. **2** Heat loss from the solids is disregarded. **3** The properties of the solids are constant.

Properties The $\sqrt{k\rho c_p}$ value is 24 kJ/m^2·°C for aluminum, 0.38 kJ/m^2·°C for wood, and 1.1 kJ/m^2·°C for the human flesh.

Analysis The surface temperature is determined from Eq. 11-49 to be

$$T_s = \frac{\sqrt{(k\rho c_p)_{\text{human}}}T_{\text{human}} + \sqrt{(k\rho c_p)_{\text{Al}}}T_{\text{Al}}}{\sqrt{(k\rho c_p)_{\text{human}}} + \sqrt{(k\rho c_p)_{\text{Al}}}} = \frac{(1.1\ \text{kJ/m}^2\cdot°\text{C})(32°\text{C}) + (24\ \text{kJ/m}^2\cdot°\text{C})(20°\text{C})}{(1.1\ \text{kJ/m}^2\cdot°\text{C}) + (24\ \text{kJ/m}^2\cdot°\text{C})} = \mathbf{20.5°C}$$

In the case of wood block, we obtain

$$\begin{aligned} T_s &= \frac{\sqrt{(k\rho c_p)_{\text{human}}}T_{\text{human}} + \sqrt{(k\rho c_p)_{\text{wood}}}T_{\text{wood}}}{\sqrt{(k\rho c_p)_{\text{human}}} + \sqrt{(k\rho c_p)_{\text{wood}}}} \\ &= \frac{(1.1\ \text{kJ/m}^2\cdot°\text{C})(32°\text{C}) + (0.38\ \text{kJ/m}^2\cdot°\text{C})(20°\text{C})}{(1.1\ \text{kJ/m}^2\cdot°\text{C}) + (0.38\ \text{kJ/m}^2\cdot°\text{C})} \\ &= \mathbf{28.9°C} \end{aligned}$$

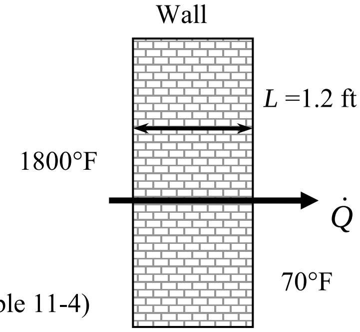
11-71E The walls of a furnace made of concrete are exposed to hot gases at the inner surfaces. The time it will take for the temperature of the outer surface of the furnace to change is to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at inner surfaces only and the convection heat transfer coefficient inside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature of 1800°F. **2** The thermal properties of the concrete wall are constant.

Properties The thermal properties of the concrete are given to be k = 0.64 Btu/h.ft.°F and α = 0.023 ft^2/h.

Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from

$$\frac{T(x,t)-T_i}{T_s-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$



But,

$$\frac{T(x,t)-T_i}{T_s-T_i}=\frac{70.1-70}{1800-70}=0.00006 \rightarrow 0.00006=erfc(2.85) \quad \text{(Table 11-4)}$$

Therefore,

$$\frac{x}{2\sqrt{\alpha t}}=2.85 \longrightarrow t=\frac{x^2}{4\times(2.85)^2\alpha}=\frac{(1.2\text{ ft})^2}{4\times(2.85)^2(0.023\text{ ft}^2/\text{h})}=1.93\text{ h}=\mathbf{116\text{ min}}$$

11-72 A thick wood slab is exposed to hot gases for a period of 5 minutes. It is to be determined whether the wood will ignite.

Assumptions **1** The wood slab is treated as a semi-infinite medium subjected to convection at the exposed surface. **2** The thermal properties of the wood slab are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The thermal properties of the wood are k = 0.17 W/m.°C and α = 1.28×10^{-7} m^2/s.

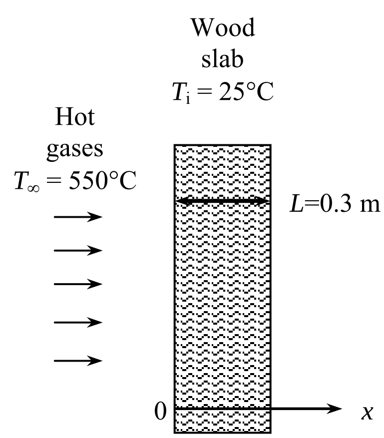
Analysis The one-dimensional transient temperature distribution in the wood can be determined from

$$\frac{T(x,t)-T_i}{T_\infty-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)-\exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

where

$$\frac{h\sqrt{\alpha t}}{k}=\frac{(35\text{ W/m}^2.°\text{C})\sqrt{(1.28\times10^{-7}\text{ m}^2/\text{s})(5\times60\text{ s})}}{0.17\text{ W/m.°C}}=1.276$$

$$\frac{h^2\alpha t}{k^2}=\left(\frac{h\sqrt{\alpha t}}{k}\right)^2=1.276^2=1.628$$



Noting that x = 0 at the surface and using Table 11-4 for *erfc* values,

$$\frac{T(x,t)-25}{550-25}=erfc(0)-\exp(0+1.628)erfc(0+1.276)$$
$$=1-(5.0937)(0.0712)$$
$$=0.637$$

Solving for $T(x, t)$ gives

$$T(x,t)=\mathbf{360°C}$$

which is less than the ignition temperature of 450°C. Therefore, the wood will not ignite.

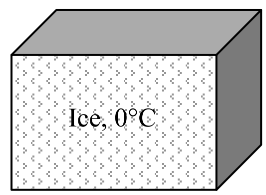
11-73 The outer surfaces of a large cast iron container filled with ice are exposed to hot water. The time before the ice starts melting and the rate of heat transfer to the ice are to be determined.

Assumptions **1** The temperature in the container walls is affected by the thermal conditions at outer surfaces only and the convection heat transfer coefficient outside is given to be very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature. **2** The thermal properties of the wall are constant.

Properties The thermal properties of the cast iron are given to be $k = 52$ W/m.°C and $\alpha = 1.70\times10^{-5}$ m^2/s.

Analysis The one-dimensional transient temperature distribution in the wall for that time period can be determined from

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$



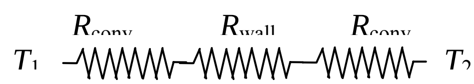
But,

$$\frac{T(x,t)-T_i}{T_s-T_i} = \frac{0.1-0}{60-0} = 0.00167 \rightarrow 0.00167 = erfc(2.226) \quad \text{(Table 11-4)}$$

Therefore,

$$\frac{x}{2\sqrt{\alpha t}} = 2.226 \longrightarrow t = \frac{x^2}{4\times(2.226)^2\alpha} = \frac{(0.05\text{ m})^2}{4(2.226)^2(1.7\times10^{-5}\text{ m}^2/\text{s})} = \mathbf{7.4\ s}$$

The rate of heat transfer to the ice when steady operation conditions are reached can be determined by applying the thermal resistance network concept as



$$R_{conv,i} = \frac{1}{h_i A} = \frac{1}{(250\text{ W/m}^2.°\text{C})(1.2\times2\text{ m}^2)} = 0.00167°\text{C/W}$$

$$R_{wall} = \frac{L}{kA} = \frac{0.05\text{ m}}{(52\text{ W/m.°C})(1.2\times2\text{ m}^2)} = 0.00040°\text{C/W}$$

$$R_{conv,o} = \frac{1}{h_o A} = \frac{1}{(\infty)(1.2\times2\text{ m}^2)} \cong 0°\text{C/W}$$

$$R_{total} = R_{conv,i} + R_{wall} + R_{conv,o} = 0.00167 + 0.00040 + 0 = 0.00207°\text{C/W}$$

$$\dot{Q} = \frac{T_2 - T_1}{R_{total}} = \frac{(60-0)°\text{C}}{0.00207°\text{ C/W}} = \mathbf{28{,}990\ W}$$

Transient Heat Conduction in Multidimensional Systems

11-74C The product solution enables us to determine the dimensionless temperature of two- or three-dimensional heat transfer problems as the product of dimensionless temperatures of one-dimensional heat transfer problems. The dimensionless temperature for a two-dimensional problem is determined by determining the dimensionless temperatures in both directions, and taking their product.

11-75C The dimensionless temperature for a three-dimensional heat transfer is determined by determining the dimensionless temperatures of one-dimensional geometries whose intersection is the three dimensional geometry, and taking their product.

11-76C This short cylinder is physically formed by the intersection of a long cylinder and a plane wall. The dimensionless temperatures at the center of plane wall and at the center of the cylinder are determined first. Their product yields the dimensionless temperature at the center of the short cylinder.

11-77C The heat transfer in this short cylinder is one-dimensional since there is no heat transfer in the axial direction. The temperature will vary in the radial direction only.

11-78 A short cylinder is allowed to cool in atmospheric air. The temperatures at the centers of the cylinder and the top surface as well as the total heat transfer from the cylinder for 15 min of cooling are to be determined.

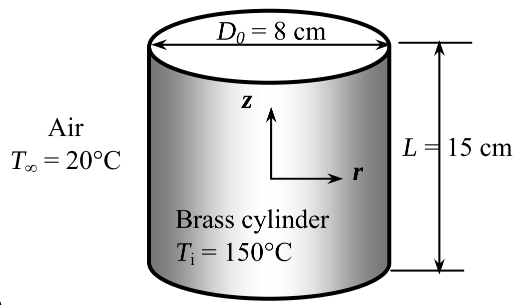
Assumptions **1** Heat conduction in the short cylinder is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the cylinder are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of brass are given to be $\rho = 8530 \text{ kg/m}^3$, $c_p = 0.389 \text{ kJ/kg}\cdot°\text{C}$, $k = 110 \text{ W/m}\cdot°\text{C}$, and $\alpha = 3.39\times10^{-5} \text{ m}^2\text{/s}$.

Analysis This short cylinder can physically be formed by the intersection of a long cylinder of radius $D/2$ = 4 cm and a plane wall of thickness $2L$ = 15 cm. We measure x from the midplane.

(*a*) The Biot number is calculated for the plane wall to be

$$Bi = \frac{hL}{k} = \frac{(40 \text{ W/m}^2.°\text{C})(0.075 \text{ m})}{(110 \text{ W/m}.°\text{C})} = 0.02727$$



The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 0.1620 \quad \text{and} \quad A_1 = 1.0045$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(3.39\times10^{-5} \text{ m}^2\text{/s})(15 \text{ min}\times 60 \text{ s/min})}{(0.075 \text{ m})^2} = 5.424 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0045)e^{-(0.1620)^2(5.424)} = 0.871$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(40 \text{ W/m}^2.°\text{C})(0.04 \text{ m})}{(110 \text{ W/m}.°\text{C})} = 0.01455$$

$$\lambda_1 = 0.1677 \quad \text{and} \quad A_1 = 1.0036$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(3.39\times10^{-5} \text{ m}^2\text{/s})(15\times 60 \text{ s})}{(0.04 \text{ m})^2} = 19.069 > 0.2$$

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.0036)e^{-(0.1677)^2(19.069)} = 0.587$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta_{o,wall} \times \theta_{o,cyl} = 0.871 \times 0.587 = 0.511$$

$$\frac{T(0,0,t) - 20}{150 - 20} = 0.511 \longrightarrow T(0,0,t) = \mathbf{86.4°C}$$

(*b*) The center of the top surface of the cylinder is still at the center of the long cylinder (r = 0), but at the outer surface of the plane wall (x = L). Therefore, we first need to determine the dimensionless temperature at the surface of the wall.

$$\theta(L,t)_{wall} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 L/L) = (1.0045)e^{-(0.1620)^2(5.424)} \cos(0.1620) = 0.860$$

Then the center temperature of the top surface of the cylinder becomes

$$\left[\frac{T(L,0,t)-T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta(L,t)_{wall} \times \theta_{o,cyl} = 0.860 \times 0.587 = 0.505$$

$$\frac{T(L,0,t)-20}{150-20} = 0.505 \longrightarrow T(L,0,t) = \mathbf{85.6°C}$$

(*c*) We first need to determine the maximum heat can be transferred from the cylinder

$$m = \rho V = \rho \pi r_o^2 L = (8530\ \text{kg/m}^3)\left[\pi (0.04\ \text{m})^2 (0.15\ \text{m})\right] = 6.43\ \text{kg}$$

$$Q_{\max} = mc_p(T_i - T_\infty) = (6.43\ \text{kg})(0.389\ \text{kJ/kg.°C})(150-20)°\text{C} = 325\ \text{kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1 - \theta_{o,wall} \frac{\sin(\lambda_1)}{\lambda_1} = 1 - (0.871)\frac{\sin(0.1620)}{0.1620} = 0.133$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl} \frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.587)\frac{0.0835}{0.1677} = 0.415$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{short\\cylinder}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{long\\cylinder}} \left[1 - \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}}\right] = 0.133 + (0.415)(1-0.133) = 0.493$$

Then the total heat transfer from the short cylinder during the first 15 minutes of cooling becomes

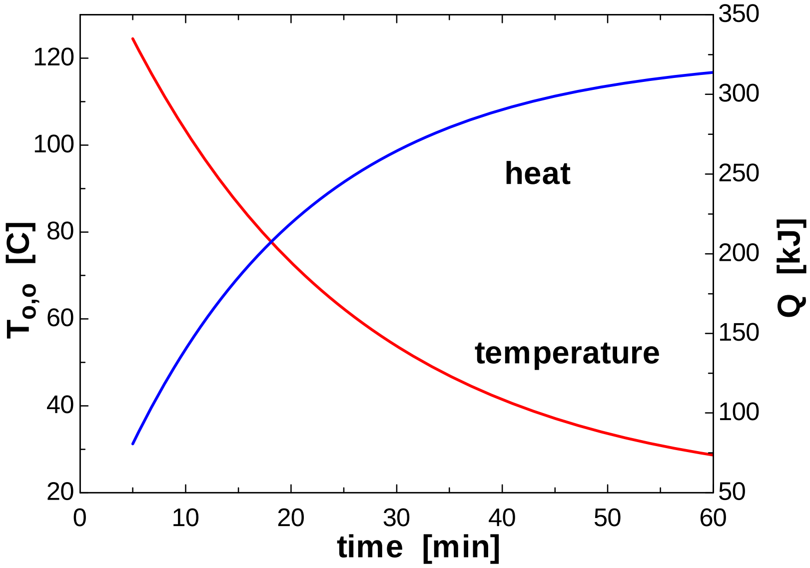
$$Q = 0.493 Q_{\max} = (0.493)(325\ \text{kJ}) = \mathbf{160\ kJ}$$

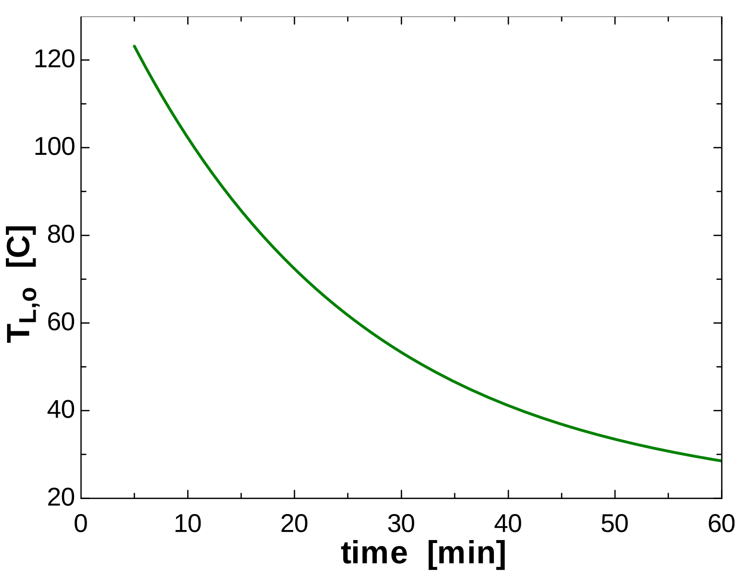
11-79 EES Prob. 11-78 is reconsidered. The effect of the cooling time on the center temperature of the cylinder, the center temperature of the top surface of the cylinder, and the total heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
D=0.08 [m]
r_o=D/2
height=0.15 [m]
L=height/2
T_i=150 [C]
T_infinity=20 [C]
h=40 [W/m^2-C]
time=15 [min]

"PROPERTIES"
k=110 [W/m-C]
rho=8530 [kg/m^3]
c_p=0.389 [kJ/kg-C]
alpha=3.39E-5 [m^2/s]

"ANALYSIS"
"(a)"
"This short cylinder can physically be formed by the intersection of a long cylinder of radius r_o
and a plane wall of thickness 2L"
"For plane wall"
Bi_w=(h*L)/k
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1_w=0.1620 "w stands for wall"
A_1_w=1.0045
tau_w=(alpha*time*Convert(min, s))/L^2
theta_o_w=A_1_w*exp(-lambda_1_w^2*tau_w) "theta_o_w=(T_o_w-T_infinity)/(T_i-T_infinity)"
"For long cylinder"
Bi_c=(h*r_o)/k "c stands for cylinder"
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1_c=0.1677
A_1_c=1.0036
tau_c=(alpha*time*Convert(min, s))/r_o^2
theta_o_c=A_1_c*exp(-lambda_1_c^2*tau_c) "theta_o_c=(T_o_c-T_infinity)/(T_i-T_infinity)"
(T_o_o-T_infinity)/(T_i-T_infinity)=theta_o_w*theta_o_c "center temperature of short cylinder"
"(b)"
theta_L_w=A_1_w*exp(-lambda_1_w^2*tau_w)*Cos(lambda_1_w*L/L) "theta_L_w=(T_L_w-
T_infinity)/(T_i-T_infinity)"
(T_L_o-T_infinity)/(T_i-T_infinity)=theta_L_w*theta_o_c "center temperature of the top surface"
"(c)"
V=pi*r_o^2*(2*L)
m=rho*V
Q_max=m*c_p*(T_i-T_infinity)
Q_w=1-theta_o_w*Sin(lambda_1_w)/lambda_1_w "Q_w=(Q/Q_max)_w"
Q_c=1-2*theta_o_c*J_1/lambda_1_c "Q_c=(Q/Q_max)_c"
J_1=0.0835 "From Table 11-3, at lambda_1_c"
Q/Q_max=Q_w+Q_c*(1-Q_w) "total heat transfer"
```

time [min]	$T_{o,o}$ [C]	$T_{L,o}$ [C]	Q [kJ]
5	124.5	123.2	65.97
10	103.4	102.3	118.5
15	86.49	85.62	160.3
20	73.03	72.33	193.7
25	62.29	61.74	220.3
30	53.73	53.29	241.6
35	46.9	46.55	258.5
40	41.45	41.17	272
45	37.11	36.89	282.8
50	33.65	33.47	291.4
55	30.88	30.74	298.2
60	28.68	28.57	303.7





11-80 A semi-infinite aluminum cylinder is cooled by water. The temperature at the center of the cylinder 5 cm from the end surface is to be determined.

Assumptions **1** Heat conduction in the semi-infinite cylinder is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the cylinder are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of aluminum are given to be $k = 237$ W/m.°C and $\alpha = 9.71\times10^{-5}$m^2/s.

Analysis This semi-infinite cylinder can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 7.5$ cm and a semi-infinite medium. The dimensionless temperature 5 cm from the surface of a semi-infinite medium is first determined from

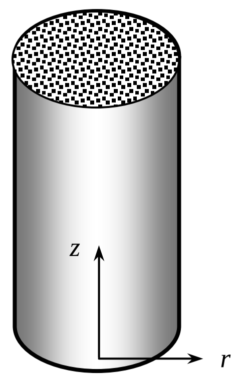
$$\frac{T(x,t)-T_i}{T_\infty - T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k}+\frac{h^2\alpha t}{k^2}\right)\left[erfc\left(\frac{x}{2\sqrt{\alpha t}}+\frac{h\sqrt{\alpha t}}{k}\right)\right]$$

$$= erfc\left(\frac{0.05}{2\sqrt{(9.71\times10^{-5})(8\times60)}}\right) - \exp\left(\frac{(140)(0.05)}{237}+\frac{(140)^2(9.71\times10^{-5})(8\times60)}{(237)^2}\right)$$

$$\times\left[erfc\left(\frac{0.05}{2\sqrt{(9.71\times10^{-5})(8\times60)}}+\frac{(140)\sqrt{(9.71\times10^{-5})(8\times60)}}{237}\right)\right]$$

$$= erfc(0.1158) - \exp(0.0458)erfc(0.2433) = 0.8699 - (1.0468)(0.7308) = 0.1049$$

$$\theta_{semi-\inf} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = 1 - 0.1049 = 0.8951$$



The Biot number is calculated for the long cylinder to be

$$Bi = \frac{hr_o}{k} = \frac{(140\text{ W/m}^2.°\text{C})(0.075\text{ m})}{237\text{ W/m.°C}} = 0.0443$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 0.2948 \quad \text{and} \quad A_1 = 1.0110$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(9.71\times10^{-5}\text{ m}^2/\text{s})(8\times60\text{ s})}{(0.075\text{ m})^2} = 8.286 > 0.2$$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0110)e^{-(0.2948)^2(8.286)} = 0.4921$$

The center temperature of the semi-infinite cylinder then becomes

$$\left[\frac{T(x,0,t)-T_\infty}{T_i - T_\infty}\right]_{\substack{\text{semi-infinite}\\\text{cylinder}}} = \theta_{\text{semi-inf}}(x,t)\times\theta_{o,cyl} = 0.8951\times0.4921 = 0.4405$$

$$\left[\frac{T(x,0,t)-10}{115-10}\right]_{\substack{\text{semi-infinite}\\\text{cylinder}}} = 0.4405 \longrightarrow T(x,0,t) = \mathbf{56.3°C}$$

11-81E A hot dog is dropped into boiling water. The center temperature of the hot dog is do be determined by treating hot dog as a finite cylinder and also as an infinitely long cylinder.

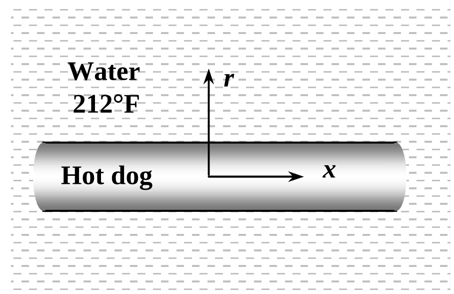
Assumptions **1** When treating hot dog as a finite cylinder, heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. When treating hot dog as an infinitely long cylinder, heat conduction is one-dimensional in the radial *r*- direction. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be k = 0.44 Btu/h.ft.°F, ρ = 61.2 lbm/ft^3 c_p = 0.93 Btu/lbm.°F, and α = 0.0077 ft^2/h.

Analysis (a) This hot dog can physically be formed by the intersection of a long cylinder of radius $r_o = D/2$ = (0.4/12) ft and a plane wall of thickness $2L$ = (5/12) ft. The distance x is measured from the midplane.

<u>After 5 minutes</u>

First the Biot number is calculated for the plane wall to be



$$Bi = \frac{hL}{k} = \frac{(120\ \text{Btu/h.ft}^2.°\text{F})(2.5/12\ \text{ft})}{(0.44\ \text{Btu/h.ft.°F})} = 56.8$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.5421 \quad \text{and} \quad A_1 = 1.2728$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(5/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.015 < 0.2 \quad \text{(Be cautious!)}$$

Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.015)} = 1.228$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(120\ \text{Btu/h.ft}^2.°\text{F})(0.4/12\ \text{ft})}{(0.44\ \text{Btu/h.ft.°F})} = 9.1$$

$$\lambda_1 = 2.1589 \quad \text{and} \quad A_1 = 1.5618$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(5/60\ \text{h})}{(0.4/12\ \text{ft})^2} = 0.578 > 0.2$$

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(0.578)} = 0.106$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.228 \times 0.106 = 0.130$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.130 \longrightarrow T(0,0,t) = \mathbf{190°F}$$

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(10/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.03 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.03)} = 1.185$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(10/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 1.156 > 0.2$$

$$\theta_{o,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.156)} = 0.0071$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.185 \times 0.0071 = 0.0084$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.0084 \longrightarrow T(0,0,t) = \mathbf{211°F}$$

After 15 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(15/60 \text{ h})}{(2.5/12 \text{ ft})^2} = 0.045 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.045)} = 1.143$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077 \text{ ft}^2/\text{h})(15/60 \text{ h})}{(0.4/12 \text{ ft})^2} = 1.734 > 0.2$$

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.734)} = 0.00048$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.143 \times 0.00048 = 0.00055$$

$$\frac{T(0,0,t) - 212}{40 - 212} = 0.00055 \longrightarrow T(0,0,t) = \mathbf{212\ °F}$$

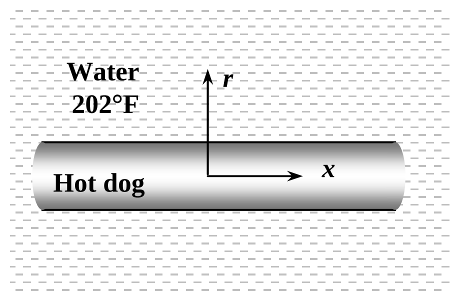
(*b*) Treating the hot dog as an infinitely long cylinder will not change the results obtained in the part (a) since dimensionless temperatures for the plane wall is 1 for all cases.

11-82E A hot dog is dropped into boiling water. The center temperature of the hot dog is do be determined by treating hot dog as a finite cylinder and an infinitely long cylinder.

Assumptions **1** When treating hot dog as a finite cylinder, heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. When treating hot dog as an infinitely long cylinder, heat conduction is one-dimensional in the radial *r*- direction. **2** The thermal properties of the hot dog are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be k = 0.44 Btu/h.ft.°F, ρ = 61.2 lbm/ft^3 c_p = 0.93 Btu/lbm.°F, and α = 0.0077 ft^2/h.

Analysis (a) This hot dog can physically be formed by the intersection of a long cylinder of radius $r_o = D/2$ = (0.4/12) ft and a plane wall of thickness $2L$ = (5/12) ft. The distance x is measured from the midplane.



After 5 minutes

First the Biot number is calculated for the plane wall to be

$$Bi = \frac{hL}{k} = \frac{(120\ \text{Btu/h.ft}^2.°\text{F})(2.5/12\ \text{ft})}{(0.44\ \text{Btu/h.ft.°F})} = 56.8$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.5421 \quad \text{and} \quad A_1 = 1.2728$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(5/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.015 < 0.2 \quad \text{(Be cautious!)}$$

Then the dimensionless temperature at the center of the plane wall is determined from

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.015)} = 1.228$$

We repeat the same calculations for the long cylinder,

$$Bi = \frac{hr_o}{k} = \frac{(120\ \text{Btu/h.ft}^2.°\text{F})(0.4/12\ \text{ft})}{(0.44\ \text{Btu/h.ft.°F})} = 9.1$$

$$\lambda_1 = 2.1589 \quad \text{and} \quad A_1 = 1.5618$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(5/60\ \text{h})}{(0.4/12\ \text{ft})^2} = 0.578 > 0.2$$

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(0.578)} = 0.106$$

Then the center temperature of the short cylinder becomes

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.228 \times 0.106 = 0.130$$

$$\frac{T(0,0,t) - 202}{40 - 202} = 0.130 \longrightarrow T(0,0,t) = \mathbf{181°F}$$

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(10/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.03 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.03)} = 1.185$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(10/60\ \text{h})}{(0.4/12\ \text{ft})^2} = 1.156 > 0.2$$

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.156)} = 0.007$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.185 \times 0.0071 = 0.0084$$

$$\frac{T(0,0,t) - 202}{40 - 202} = 0.0084 \longrightarrow T(0,0,t) = \mathbf{201°F}$$

After 15 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(15/60\ \text{h})}{(2.5/12\ \text{ft})^2} = 0.045 < 0.2 \quad \text{(Be cautious!)}$$

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.2728)e^{-(1.5421)^2(0.045)} = 1.143$$

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.0077\ \text{ft}^2/\text{h})(15/60\ \text{h})}{(0.4/12\ \text{ft})^2} = 1.734 > 0.2$$

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} = (1.5618)e^{-(2.1589)^2(1.734)} = 0.00048$$

$$\left[\frac{T(0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\ \text{cylinder}}} = \theta_{o,wall} \times \theta_{o,cyl} = 1.143 \times 0.00048 = 0.00055$$

$$\frac{T(0,0,t) - 202}{40 - 202} = 0.00055 \longrightarrow T(0,0,t) = \mathbf{202\ °F}$$

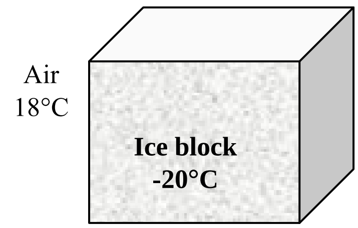
(b) Treating the hot dog as an infinitely long cylinder will not change the results obtained in the part (a) since dimensionless temperatures for the plane wall is 1 for all cases.

11-83 A rectangular ice block is placed on a table. The time the ice block starts melting is to be determined.

Assumptions **1** Heat conduction in the ice block is two-dimensional, and thus the temperature varies in both *x*- and *y*- directions. **2** The thermal properties of the ice block are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the ice are given to be $k = 2.22$ W/m.°C and $\alpha = 0.124\times10^{-7}$ m^2/s.

Analysis This rectangular ice block can be treated as a short rectangular block that can physically be formed by the intersection of two infinite plane wall of thickness $2L = 4$ cm and an infinite plane wall of thickness $2L = 10$ cm. We measure x from the bottom surface of the block since this surface represents the adiabatic center surface of the plane wall of thickness $2L = 10$ cm. Since the melting starts at the corner of the top surface, we need to determine the time required to melt ice block which will happen when the temperature drops below 0°C at this location. The Biot numbers and the corresponding constants are first determined to be



$$Bi_{\text{wall},1} = \frac{hL_1}{k} = \frac{(12 \text{ W/m}^2.°\text{C})(0.02 \text{ m})}{(2.22 \text{ W/m.°C})} = 0.1081 \longrightarrow \lambda_1 = 0.3208 \text{ and } A_1 = 1.0173$$

$$Bi_{\text{wall},3} = \frac{hL_3}{k} = \frac{(12 \text{ W/m}^2.°\text{C})(0.05 \text{ m})}{(2.22 \text{ W/m.°C})} = 0.2703 \longrightarrow \lambda_1 = 0.4951 \text{ and } A_1 = 1.0408$$

The ice will start melting at the corners because of the maximum exposed surface area there. Noting that $\tau = \alpha t / L^2$ and assuming that $\tau > 0.2$ in all dimensions so that the one-term approximate solution for transient heat conduction is applicable, the product solution method can be written for this problem as

$$\theta(L_1, L_2, L_3, t)_{block} = \theta(L_1, t)_{\text{wall},1}{}^2\,\theta(L_3, t)_{\text{wall},2}$$

$$\frac{0-18}{-20-18} = \left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L_1 / L_1)\right]^2\left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L_3 / L_3)\right]$$

$$0.4737 = \left\{(1.0173)\exp\left[-(0.3208)^2\frac{(0.124\times10^{-7})t}{(0.02)^2}\right]\cos(0.3208)\right\}^2$$

$$\times\left\{(1.0408)\exp\left[-(0.4951)^2\frac{(0.124\times10^{-7})t}{(0.05)^2}\right]\cos(0.4951)\right\}$$

$$\longrightarrow t = 77{,}500 \text{ s} = 1292 \text{ min} = \mathbf{21.5\ hours}$$

Therefore, the ice will start melting in about 21 hours.

Discussion Note that

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.124\times10^{-7} \text{ m}^2\text{/s})(77{,}500 \text{ s/h})}{(0.05 \text{ m})^2} = 0.384 > 0.2$$

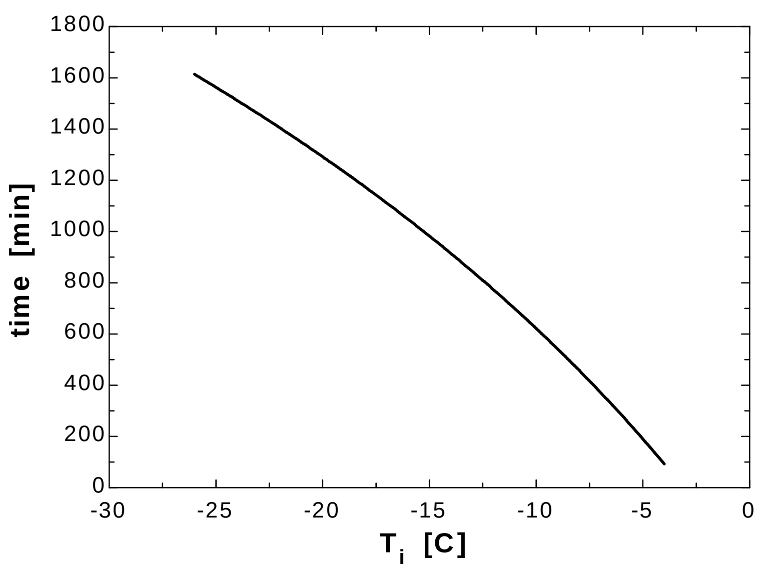
and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified.

11-84 EES Prob. 11-83 is reconsidered. The effect of the initial temperature of the ice block on the time period before the ice block starts melting is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
2*L_1=0.04 [m]
L_2=L_1
2*L_3=0.10 [m]
T_i=-20 [C]
T_infinity=18 [C]
h=12 [W/m^2-C]
T_L1_L2_L3=0 [C]
"PROPERTIES"
k=2.22 [W/m-C]
alpha=0.124E-7 [m^2/s]
"ANALYSIS"
"This block can physically be formed by the intersection of two infinite plane wall of thickness 2L=4 cm and an infinite plane wall of thickness 2L=10 cm"
"For the two plane walls"
Bi_w1=(h*L_1)/k
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1_w1=0.3208 "w stands for wall"
A_1_w1=1.0173
time*Convert(min, s)=tau_w1*L_1^2/alpha
"For the third plane wall"
Bi_w3=(h*L_3)/k
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1_w3=0.4951
A_1_w3=1.0408
time*Convert(min, s)=tau_w3*L_3^2/alpha
theta_L_w1=A_1_w1*exp(-lambda_1_w1^2*tau_w1)*Cos(lambda_1_w1*L_1/L_1)
"theta_L_w1=(T_L_w1-T_infinity)/(T_i-T_infinity)"
theta_L_w3=A_1_w3*exp(-lambda_1_w3^2*tau_w3)*Cos(lambda_1_w3*L_3/L_3)
"theta_L_w3=(T_L_w3-T_infinity)/(T_i-T_infinity)"
(T_L1_L2_L3-T_infinity)/(T_i-T_infinity)=theta_L_w1^2*theta_L_w3 "corner temperature"
```

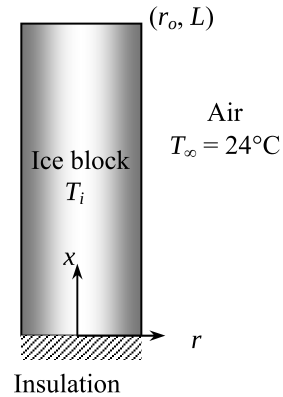
T_i [C]	time [min]
-26	1614
-24	1512
-22	1405
-20	1292
-18	1173
-16	1048
-14	914.9
-12	773.3
-10	621.9
-8	459.4
-6	283.7
-4	92.84



11-85 A cylindrical ice block is placed on a table. The initial temperature of the ice block to avoid melting for 2 h is to be determined.

Assumptions **1** Heat conduction in the ice block is two-dimensional, and thus the temperature varies in both *x*- and *r*- directions. **2** Heat transfer from the base of the ice block to the table is negligible. **3** The thermal properties of the ice block are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the ice are given to be k = 2.22 W/m.°C and $\alpha = 0.124\times10^{-7}$ m^2/s.



Analysis This cylindrical ice block can be treated as a short cylinder that can physically be formed by the intersection of a long cylinder of diameter D = 2 cm and an infinite plane wall of thickness $2L$ = 4 cm. We measure x from the bottom surface of the block since this surface represents the adiabatic center surface of the plane wall of thickness $2L$ = 4 cm. The melting starts at the outer surfaces of the top surface when the temperature drops below 0°C at this location. The Biot numbers, the corresponding constants, and the Fourier numbers are

$$Bi_{\text{wall}} = \frac{hL}{k} = \frac{(13\text{ W/m}^2.^\circ\text{C})(0.02\text{ m})}{(2.22\text{ W/m.}^\circ\text{C})} = 0.1171 \longrightarrow \lambda_1 = 0.3319 \text{ and } A_1 = 1.0187$$

$$Bi_{\text{cyl}} = \frac{hr_o}{k} = \frac{(13\text{ W/m}^2.^\circ\text{C})(0.01\text{ m})}{(2.22\text{ W/m.}^\circ\text{C})} = 0.05856 \longrightarrow \lambda_1 = 0.3393 \text{ and } A_1 = 1.0144$$

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(0.124\times10^{-7}\text{ m}^2\text{/s})(3\text{ h}\times3600\text{ s/h})}{(0.02\text{ m})^2} = 0.3348 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^2} = \frac{(0.124\times10^{-7}\text{ m}^2\text{/s})(3\text{ h}\times3600\text{ s/h})}{(0.01\text{ m})^2} = 1.3392 > 0.2$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable. The product solution for this problem can be written as

$$\theta(L, r_o, t)_{\text{block}} = \theta(L,t)_{\text{wall}}\,\theta(r_o,t)_{\text{cyl}}$$

$$\frac{0-24}{T_i-24} = \left[A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L)\right]\left[A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o/r_o)\right]$$

$$\frac{0-24}{T_i-24} = \left[(1.0187)e^{-(0.3319)^2(0.3348)}\cos(0.3319)\right]\left[(1.0146)e^{-(0.3393)^2(1.3392)}(0.9708)\right]$$

which gives $T_i = \mathbf{-6.6^\circ C}$

Therefore, the ice will not start melting for at least 3 hours if its initial temperature is -6.6°C or below.

11-86 A cubic block and a cylindrical block are exposed to hot gases on all of their surfaces. The center temperatures of each geometry in 10, 20, and 60 min are to be determined.

Assumptions **1** Heat conduction in the cubic block is three-dimensional, and thus the temperature varies in all *x*-, *y*, and *z*- directions. **2** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **3** The thermal properties of the granite are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the granite are given to be $k = 2.5$ W/m.°C and $\alpha = 1.15\times10^{-6}$ m²/s.

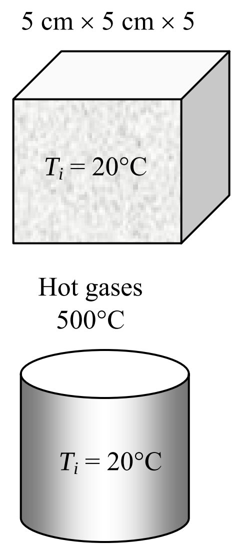
***Analysis*:**

Cubic block: This cubic block can physically be formed by the intersection of three infinite plane walls of thickness $2L = 5$ cm.

After 10 minutes: The Biot number, the corresponding constants, and the Fourier number are

$$Bi = \frac{hL}{k} = \frac{(40\text{ W/m}^2.°\text{C})(0.025\text{ m})}{(2.5\text{ W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.5932 \text{ and } A_1 = 1.0580$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\text{ m}^2/\text{s})(10\text{ min}\times 60\text{ s/min})}{(0.025\text{ m})^2} = 1.104 > 0.2$$



To determine the center temperature, the product solution can be written as

$$\theta(0,0,0,t)_{\text{block}} = \left[\theta(0,t)_{\text{wall}}\right]^3$$

$$\frac{T(0,0,0,t)-T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2\tau}\right)^3$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}^3 = 0.369$$

$$T(0,0,0,t) = \mathbf{323°C}$$

After 20 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\text{ m}^2/\text{s})(20\text{ min}\times 60\text{ s/min})}{(0.025\text{ m})^2} = 2.208 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}^3 = 0.115 \longrightarrow T(0,0,0,t) = \mathbf{445°C}$$

After 60 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\text{ m}^2/\text{s})(60\text{ min}\times 60\text{ s/min})}{(0.025\text{ m})^2} = 6.624 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}^3 = 0.00109 \longrightarrow T(0,0,0,t) = \mathbf{500°C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

Cylinder: This cylindrical block can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 2.5$ cm and a plane wall of thickness $2L = 5$ cm.

After 10 minutes: The Biot number and the corresponding constants for the long cylinder are

$$Bi = \frac{hr_o}{k} = \frac{(40\text{ W/m}^2.°\text{C})(0.025\text{ m})}{(2.5\text{ W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.8516 \quad \text{and} \quad A_1 = 1.0931$$

To determine the center temperature, the product solution can be written as

$$\theta(0,0,t)_{block} = \left[\theta(0,t)_{wall}\right]\left[\theta(0,t)_{cyl}\right]$$

$$\frac{T(0,0,t)-T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2 \tau}\right)_{wall}\left(A_1 e^{-\lambda_1^2 \tau}\right)_{cyl}$$

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}\left\{(1.0931)e^{-(0.8516)^2(1.104)}\right\} = 0.352 \longrightarrow T(0,0,t) = \mathbf{331°C}$$

After 20 minutes

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}\left\{(1.0931)e^{-(0.8516)^2(2.208)}\right\} = 0.107 \longrightarrow T(0,0,t) = \mathbf{449°C}$$

After 60 minutes

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}\left\{(1.0931)e^{-(0.8516)^2(6.624)}\right\} = 0.00092 \longrightarrow T(0,0,t) = \mathbf{500°C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

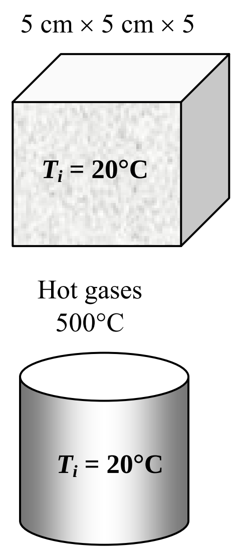
11-87 A cubic block and a cylindrical block are exposed to hot gases on all of their surfaces. The center temperatures of each geometry in 10, 20, and 60 min are to be determined.

Assumptions **1** Heat conduction in the cubic block is three-dimensional, and thus the temperature varies in all *x*-, *y*, and *z*- directions. **2** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **3** The thermal properties of the granite are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the granite are $k = 2.5$ W/m.°C and $\alpha = 1.15\times10^{-6}$ m²/s.

***Analysis*:**

Cubic block: This cubic block can physically be formed by the intersection of three infinite plane wall of thickness $2L = 5$ cm. Two infinite plane walls are exposed to the hot gases with a heat transfer coefficient of $h = 40\text{ W/m}^2.°\text{C}$ and one with $h = 80\text{ W/m}^2.°\text{C}$.



After 10 minutes: The Biot number and the corresponding constants for $h = 40\text{ W/m}^2.°\text{C}$ are

$$Bi = \frac{hL}{k} = \frac{(40\text{ W/m}^2.°\text{C})(0.025\text{ m})}{(2.5\text{ W/m.°C})} = 0.400 \longrightarrow \lambda_1 = 0.5932 \text{ and } A_1 = 1.0580$$

The Biot number and the corresponding constants for $h = 80\text{ W/m}^2.°\text{C}$ are

$$Bi = \frac{hL}{k} = \frac{(80\text{ W/m}^2.°\text{C})(0.025\text{ m})}{(2.5\text{ W/m.°C})} = 0.800$$

$$\longrightarrow \lambda_1 = 0.7910 \text{ and } A_1 = 1.1016$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\text{ m}^2\text{/s})(10\text{ min}\times60\text{ s/min})}{(0.025\text{ m})^2} = 1.104 > 0.2$$

To determine the center temperature, the product solution method can be written as

$$\theta(0,0,0,t)_{\text{block}} = [\theta(0,t)_{\text{wall}}]^2[\theta(0,t)_{\text{wall}}]$$

$$\frac{T(0,0,0,t)-T_\infty}{T_i-T_\infty} = \left(A_1 e^{-\lambda_1^2\tau}\right)^2\left(A_1 e^{-\lambda_1^2\tau}\right)$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(1.104)}\right\}^2\left\{(1.1016)e^{-(0.7910)^2(1.104)}\right\} = 0.284$$

$$T(0,0,0,t) = \mathbf{364°C}$$

After 20 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\text{ m}^2\text{/s})(20\text{ min}\times60\text{ s/min})}{(0.025\text{ m})^2} = 2.208 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(2.208)}\right\}^2\left\{(1.1016)e^{-(0.7910)^2(2.208)}\right\} = 0.0654$$

$$\longrightarrow T(0,0,0,t) = \mathbf{469°C}$$

After 60 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(1.15\times10^{-6}\ \text{m}^2/\text{s})(60\ \text{min}\times 60\ \text{s/min})}{(0.025\ \text{m})^2} = 6.624 > 0.2$$

$$\frac{T(0,0,0,t)-500}{20-500} = \left\{(1.0580)e^{-(0.5932)^2(6.624)}\right\}^2\left\{(1.1016)e^{-(0.7910)^2(6.624)}\right\} = 0.000186$$

$$\longrightarrow T(0,0,0,t) = \mathbf{500^\circ C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

Cylinder: This cylindrical block can physically be formed by the intersection of a long cylinder of radius $r_o = D/2 = 2.5$ cm exposed to the hot gases with a heat transfer coefficient of $h = 40\ \text{W/m}^2.^\circ\text{C}$ and a plane wall of thickness $2L = 5$ cm exposed to the hot gases with $h = 80\ \text{W/m}^2.^\circ\text{C}$.

After 10 minutes: The Biot number and the corresponding constants for the long cylinder are

$$Bi = \frac{hr_o}{k} = \frac{(40\ \text{W/m}^2.^\circ\text{C})(0.025\ \text{m})}{(2.5\ \text{W/m.}^\circ\text{C})} = 0.400 \longrightarrow \lambda_1 = 0.8516 \text{ and } A_1 = 1.0931$$

To determine the center temperature, the product solution method can be written as

$$\theta(0,0,t)_{\text{block}} = \left[\theta(0,t)_{\text{wall}}\right]\left[\theta(0,t)_{cyl}\right]$$

$$\frac{T(0,0,t)-T_\infty}{T_i - T_\infty} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}}\left(A_1 e^{-\lambda_1^2\tau}\right)_{cyl}$$

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(1.104)}\right\}\left\{(1.0931)e^{-(0.8516)^2(1.104)}\right\} = 0.271$$

$$T(0,0,t) = \mathbf{370^\circ C}$$

After 20 minutes

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(2.208)}\right\}\left\{(1.0931)e^{-(0.8516)^2(2.208)}\right\} = 0.06094 \longrightarrow T(0,0,t) = \mathbf{471^\circ C}$$

After 60 minutes

$$\frac{T(0,0,t)-500}{20-500} = \left\{(1.1016)e^{-(0.7910)^2(6.624)}\right\}\left\{(1.0931)e^{-(0.8516)^2(6.624)}\right\} = 0.0001568 \longrightarrow T(0,0,t) = \mathbf{500^\circ C}$$

Note that $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable.

11-88 A cylindrical aluminum block is heated in a furnace. The length of time the block should be kept in the furnace and the amount of heat transfer to the block are to be determined.

Assumptions **1** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **2** The thermal properties of the aluminum are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (it will be verified).

Properties The thermal properties of the aluminum block are given to be k = 236 W/m.°C, ρ = 2702 kg/m^3, c_p = 0.896 kJ/kg.°C, and $\alpha = 9.75\times10^{-5}$ m^2/s.

Analysis This cylindrical aluminum block can physically be formed by the intersection of an infinite plane wall of thickness 2L = 20 cm, and a long cylinder of radius $r_o = D/2$ = 7.5 cm. The Biot numbers and the corresponding constants are first determined to be

$$Bi = \frac{hL}{k} = \frac{(80\text{ W/m}^2.°\text{C})(0.1\text{ m})}{(236\text{ W/m.°C})} = 0.0339 \longrightarrow \lambda_1 = 0.1811 \text{ and } A_1 = 1.0056$$

$$Bi = \frac{hr_0}{k} = \frac{(80\text{ W/m}^2.°\text{C})(0.075\text{ m})}{236\text{ W/m.°C}} = 0.0254 \longrightarrow \lambda_1 = 0.2217 \text{ and } A_1 = 1.0063$$

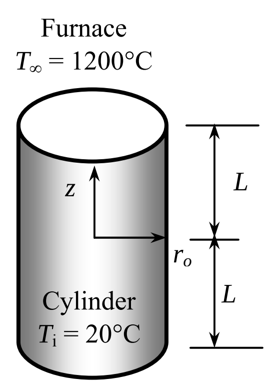
Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{\text{block}} = \theta(0,t)_{\text{wall}}\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}}\left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}}$$

$$\frac{300-1200}{20-1200} = \left\{(1.0056)\exp\left[-(0.1811)^2\frac{(9.75\times10^{-5})t}{(0.1)^2}\right]\right\}\times\left\{(1.0063)\exp\left[-(0.2217)^2\frac{(9.75\times10^{-5})t}{(0.075)^2}\right]\right\}$$
$$= 0.7627$$

Solving for the time t gives

t = 241 s = **4.0 min**.



We note that

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(9.75\times10^{-5}\text{ m}^2\text{/s})(241\text{ s})}{(0.1\text{ m})^2} = 2.350 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^2} = \frac{(9.75\times10^{-5}\text{ m}^2\text{/s})(241\text{ s})}{(0.075\text{ m})^2} = 4.177 > 0.2$$

and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified. The dimensionless temperatures at the center are

$$\theta(0,t)_{\text{wall}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}} = (1.0056)\exp\left[-(0.1811)^2(2.350)\right] = 0.9310$$

$$\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}} = (1.0063)\exp\left[-(0.2217)^2(4.177)\right] = 0.8195$$

The maximum amount of heat transfer is

$$m = \rho V = \rho\pi r_o^2 L = (2702\text{ kg/m}^3)\left[\pi(0.075\text{ m})^2(0.2\text{ m})\right] = 9.550\text{ kg}$$
$$Q_{\max} = mc_p(T_i - T_\infty) = (9.550\text{ kg})(0.896\text{ kJ/kg.°C})(20-1200)°\text{C} = 10{,}100\text{ kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1-\theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1-(0.9310)\frac{\sin(0.1811)}{0.1811} = 0.07408$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1-2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1-2(0.8195)\frac{0.1101}{0.2217} = 0.1860$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{short\\cylinder}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{long\\cylinder}}\left[1-\left(\frac{Q}{Q_{\max}}\right)_{\substack{plane\\wall}}\right]$$

$$= 0.07408 + (0.1860)(1-0.07408) = 0.2463$$

Then the total heat transfer from the short cylinder as it is cooled from 300°C at the center to 20°C becomes

$$Q = 0.2463Q_{\max} = (0.2463)(10{,}100\text{ kJ}) = \mathbf{2490\ kJ}$$

which is identical to the heat transfer to the cylinder as the cylinder at 20°C is heated to 300°C at the center.

11-89 A cylindrical aluminum block is heated in a furnace. The length of time the block should be kept in the furnace and the amount of heat transferred to the block are to be determined.

Assumptions **1** Heat conduction in the cylindrical block is two-dimensional, and thus the temperature varies in both axial *x*- and radial *r*- directions. **2** Heat transfer from the bottom surface of the block is negligible. **3** The thermal properties of the aluminum are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the aluminum block are given to be $k = 236$ W/m.°C, $\rho = 2702$ kg/m^3, $c_p = 0.896$ kJ/kg.°C, and $\alpha = 9.75\times10^{-5}$ m^2/s.

Analysis This cylindrical aluminum block can physically be formed by the intersection of an infinite plane wall of thickness $2L = 40$ cm and a long cylinder of radius $r_o = D/2 = 7.5$ cm. Note that the height of the short cylinder represents the half thickness of the infinite plane wall where the bottom surface of the short cylinder is adiabatic. The Biot numbers and corresponding constants are first determined to be

$$Bi = \frac{hL}{k} = \frac{(80\ \text{W/m}^2.°\text{C})(0.2\ \text{m})}{(236\ \text{W/m.°C})} = 0.0678 \rightarrow \lambda_1 = 0.2568 \quad \text{and} \quad A_1 = 1.0110$$

$$Bi = \frac{hr_o}{k} = \frac{(80\ \text{W/m}^2.°\text{C})(0.075\ \text{m})}{(236\ \text{W/m.°C})} = 0.0254 \rightarrow \lambda_1 = 0.2217 \text{ and } A_1 = 1.0063$$

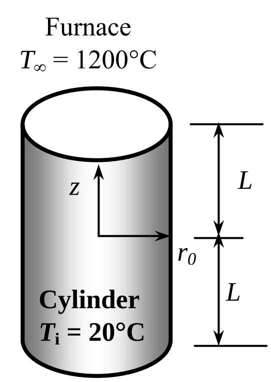
Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{\text{block}} = \theta(0,t)_{\text{wall}}\,\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}}\left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}}$$

$$\frac{300-1200}{20-1200} = \left\{(1.0110)\exp\left[-(0.2568)^2\frac{(9.75\times10^{-5})t}{(0.2)^2}\right]\right\}\left\{(1.0063)\exp\left[-(0.2217)^2\frac{(9.75\times10^{-5})t}{(0.075)^2}\right]\right\}$$
$$= 0.7627$$

Solving for the time *t* gives

$t = 285$ s = **4.7 min**.



We note that

$$\tau_{\text{wall}} = \frac{\alpha t}{L^2} = \frac{(9.75\times10^{-5}\ \text{m}^2/\text{s})(285\ \text{s})}{(0.2\ \text{m})^2} = 0.6947 > 0.2$$

$$\tau_{\text{cyl}} = \frac{\alpha t}{r_o^2} = \frac{(9.75\times10^{-5}\ \text{m}^2/\text{s})(285\ \text{s})}{(0.075\ \text{m})^2} = 4.940 > 0.2$$

and thus the assumption of $\tau > 0.2$ for the applicability of the one-term approximate solution is verified. The dimensionless temperatures at the center are

$$\theta(0,t)_{\text{wall}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{wall}} = (1.0110)\exp\left[-(0.2568)^2(0.6947)\right] = 0.9658$$

$$\theta(0,t)_{\text{cyl}} = \left(A_1 e^{-\lambda_1^2\tau}\right)_{\text{cyl}} = (1.0063)\exp\left[-(0.2217)^2(4.940)\right] = 0.7897$$

The maximum amount of heat transfer is

$$m = \rho \boldsymbol{V} = \rho\pi r_o L = (2702\ \text{kg/m}^3)\left[\pi(0.075\ \text{m})^2(0.2\ \text{m})\right] = 9.55\ \text{kg}$$
$$Q_{\max} = mc_p(T_i - T_\infty) = (9.55\ \text{kg})(0.896\ \text{kJ/kg.°C})(20-1200)°\text{C} = 10{,}100\ \text{kJ}$$

Then we determine the dimensionless heat transfer ratios for both geometries as

$$\left(\frac{Q}{Q_{\max}}\right)_{wall} = 1 - \theta_{o,wall}\frac{\sin(\lambda_1)}{\lambda_1} = 1 - (0.9658)\frac{\sin(0.2568)}{0.2568} = 0.04477$$

$$\left(\frac{Q}{Q_{\max}}\right)_{cyl} = 1 - 2\theta_{o,cyl}\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2(0.7897)\frac{0.1101}{0.2217} = 0.2156$$

The heat transfer ratio for the short cylinder is

$$\left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{short}\\\text{cylinder}}} = \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{plane}\\\text{wall}}} + \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{long}\\\text{cylinder}}}\left[1 - \left(\frac{Q}{Q_{\max}}\right)_{\substack{\text{plane}\\\text{wall}}}\right]$$

$$= 0.04477 + (0.2156)(1 - 0.04477) = 0.2507$$

Then the total heat transfer from the short cylinder as it is cooled from 300°C at the center to 20°C becomes

$$Q = 0.2507Q_{\max} = (0.2507)(10{,}100\ \text{kJ}) = \mathbf{2530\ kJ}$$

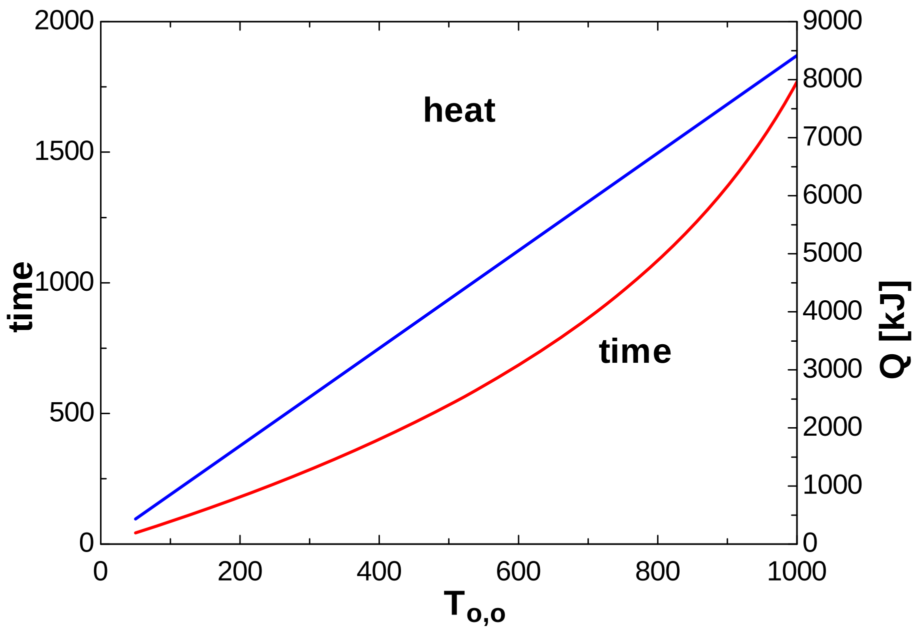
which is identical to the heat transfer to the cylinder as the cylinder at 20°C is heated to 300°C at the center.

11-90 EES Prob. 11-88 is reconsidered. The effect of the final center temperature of the block on the heating time and the amount of heat transfer is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.20 [m]
2*r_o=0.15 [m]
T_i=20 [C]
T_infinity=1200 [C]
T_o_o=300 [C]
h=80 [W/m^2-C]

"PROPERTIES"
k=236 [W/m-C]
rho=2702 [kg/m^3]
c_p=0.896 [kJ/kg-C]
alpha=9.75E-5 [m^2/s]

"ANALYSIS"
"This short cylinder can physically be formed by the intersection of a long cylinder of radius r_o
and a plane wall of thickness 2L"
"For plane wall"
Bi_w=(h*L)/k
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1_w=0.2568 "w stands for wall"
A_1_w=1.0110
tau_w=(alpha*time)/L^2
theta_o_w=A_1_w*exp(-lambda_1_w^2*tau_w) "theta_o_w=(T_o_w-T_infinity)/(T_i-T_infinity)"
"For long cylinder"
Bi_c=(h*r_o)/k "c stands for cylinder"
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1_c=0.2217
A_1_c=1.0063
tau_c=(alpha*time)/r_o^2
theta_o_c=A_1_c*exp(-lambda_1_c^2*tau_c) "theta_o_c=(T_o_c-T_infinity)/(T_i-T_infinity)"
(T_o_o-T_infinity)/(T_i-T_infinity)=theta_o_w*theta_o_c "center temperature of cylinder"
V=pi*r_o^2*L
m=rho*V
Q_max=m*c_p*(T_infinity-T_i)
Q_w=1-theta_o_w*Sin(lambda_1_w)/lambda_1_w "Q_w=(Q/Q_max)_w"
Q_c=1-2*theta_o_c*J_1/lambda_1_c "Q_c=(Q/Q_max)_c"
J_1=0.1101 "From Table 11-3, at lambda_1_c"
Q/Q_max=Q_w+Q_c*(1-Q_w) "total heat transfer"
```

$T_{o,o}$ [C]	time [s]	Q [kJ]
50	42.43	430.3
100	86.33	850.6
150	132.3	1271
200	180.4	1691
250	231.1	2111
300	284.5	2532
350	340.9	2952
400	400.8	3372
450	464.5	3793
500	532.6	4213
550	605.8	4633
600	684.9	5053
650	770.8	5474
700	864.9	5894
750	968.9	6314
800	1085	6734
850	1217	7155
900	1369	7575
950	1549	7995
1000	1770	8416



Review Problems

11-91 Two large steel plates are stuck together because of the freezing of the water between the two plates. Hot air is blown over the exposed surface of the plate on the top to melt the ice. The length of time the hot air should be blown is to be determined.

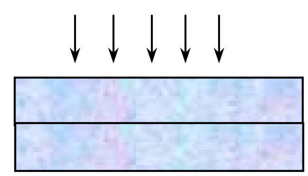
Assumptions **1** Heat conduction in the plates is one-dimensional since the plate is large relative to its thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the steel plates are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of steel plates are given to be $k = 43$ W/m.°C and $\alpha = 1.17\times10^{-5}$ m²/s

Analysis The characteristic length of the plates and the Biot number are

$$L_c = \frac{V}{A_s} = L = 0.02 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(40 \text{ W/m}^2.°\text{C})(0.02 \text{ m})}{(43 \text{ W/m.°C})} = 0.019 < 0.1$$



Since $Bi < 0.1$, the lumped system analysis is applicable. Therefore,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{40 \text{ W/m}^2.°\text{C}}{(3.675\times10^6 \text{ J/m}^3.°\text{C})(0.02 \text{ m})} = 0.000544 \text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{0-50}{-15-50} = e^{-(0.000544 \text{ s}^{-1})t} \longrightarrow t = \mathbf{482\ s = 8.0\ min}$$

where $\rho c_p = \dfrac{k}{\alpha} = \dfrac{43 \text{ W/m.°C}}{1.17\times10^{-5} \text{ m}^2/\text{s}} = 3.675\times10^6 \text{ J/m}^3.°\text{C}$

Alternative solution: This problem can also be solved using the transient chart Fig. 11-15a,

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{1}{0.019} = 52.6 \\ \frac{T_0 - T_\infty}{T_i - T_\infty} &= \frac{0-50}{-15-50} = 0.769 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 15 > 0.2$$

Then,

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(15)(0.02 \text{ m})^2}{(1.17\times10^{-5} \text{ m}^2/\text{s})} = \mathbf{513\ s}$$

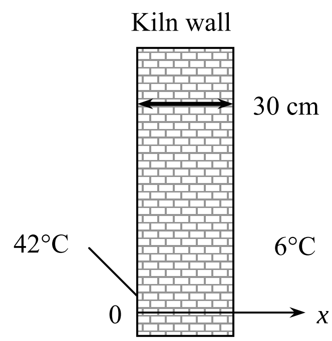
The difference is due to the reading error of the chart.

11-92 A curing kiln is heated by injecting steam into it and raising its inner surface temperature to a specified value. It is to be determined whether the temperature at the outer surfaces of the kiln changes during the curing period.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at inner surfaces only and the convection heat transfer coefficient inside is very large. Therefore, the wall can be considered to be a semi-infinite medium with a specified surface temperature of 45°C. **2** The thermal properties of the concrete wall are constant.

Properties The thermal properties of the concrete wall are given to be k = 0.9 W/m.°C and $\alpha = 0.23\times10^{-5}$ m^2/s.

Analysis We determine the temperature at a depth of x = 0.3 m in 2.5 h using the analytical solution,



$$\frac{T(x,t)-T_i}{T_s-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Substituting,

$$\frac{T(x,t)-6}{42-6}=erfc\left(\frac{0.3\text{ m}}{2\sqrt{(0.23\times10^{-5}\text{ m}^2/\text{s})(2.5\text{ h}\times3600\text{ s/h})}}\right)$$

$$=erfc(1.043)=0.1402$$

$$T(x,t)=\mathbf{11.0\ °C}$$

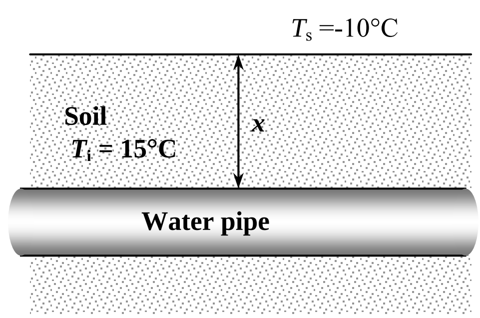
which is greater than the initial temperature of 6°C. Therefore, heat will propagate through the 0.3 m thick wall in 2.5 h, and thus it may be desirable to insulate the outer surface of the wall to save energy.

11-93 The water pipes are buried in the ground to prevent freezing. The minimum burial depth at a particular location is to be determined.

Assumptions **1** The temperature in the soil is affected by the thermal conditions at one surface only, and thus the soil can be considered to be a semi-infinite medium with a specified surface temperature of -10°C. **2** The thermal properties of the soil are constant.

Properties The thermal properties of the soil are given to be k = 0.7 W/m.°C and $\alpha = 1.4\times10^{-5}$ m^2/s.

Analysis The depth at which the temperature drops to 0°C in 75 days is determined using the analytical solution,



$$\frac{T(x,t)-T_i}{T_s-T_i}=erfc\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Substituting and using Table 11-4, we obtain

$$\frac{0-15}{-10-15}=erfc\left(\frac{x}{2\sqrt{(1.4\times10^{-5}\text{ m}^2/\text{s})(75\text{ day}\times24\text{ h/day}\times3600\text{ s/h})}}\right)$$

$$\longrightarrow x=\mathbf{7.05\ m}$$

Therefore, the pipes must be buried at a depth of at least 7.05 m.

11-94 A hot dog is to be cooked by dropping it into boiling water. The time of cooking is to be determined.

Assumptions **1** Heat conduction in the hot dog is two-dimensional, and thus the temperature varies in both the axial *x*- and the radial *r*- directions. **2** The thermal properties of the hot dog are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of the hot dog are given to be $k = 0.76$ W/m.°C, $\rho = 980$ kg/m^3, $c_p = 3.9$ kJ/kg.°C, and $\alpha = 2\times10^{-7}$ m^2/s.

Analysis This hot dog can physically be formed by the intersection of an infinite plane wall of thickness $2L = 12$ cm, and a long cylinder of radius $r_o = D/2 = 1$ cm. The Biot numbers and corresponding constants are first determined to be

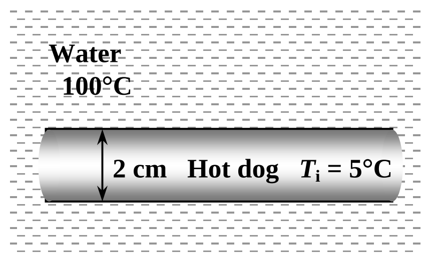
$$Bi = \frac{hL}{k} = \frac{(600\ \text{W/m}^2.°\text{C})(0.06\ \text{m})}{(0.76\ \text{W/m.°C})} = 47.37 \longrightarrow \lambda_1 = 1.5380 \quad \text{and} \quad A_1 = 1.2726$$

$$Bi = \frac{hr_o}{k} = \frac{(600\ \text{W/m}^2.°\text{C})(0.01\ \text{m})}{(0.76\ \text{W/m.°C})} = 7.895 \longrightarrow \lambda_1 = 2.1249 \quad \text{and} \quad A_1 = 1.5514$$

Noting that $\tau = \alpha t / L^2$ and assuming $\tau > 0.2$ in all dimensions and thus the one-term approximate solution for transient heat conduction is applicable, the product solution for this problem can be written as

$$\theta(0,0,t)_{block} = \theta(0,t)_{wall}\,\theta(0,t)_{cyl} = \left(A_1 e^{-\lambda_1^2\tau}\right)\left(A_1 e^{-\lambda_1^2\tau}\right)$$

$$\frac{80-100}{5-100} = \left\{(1.2726)\exp\left[-(1.5380)^2\frac{(2\times10^{-7})t}{(0.06)^2}\right]\right\}$$
$$\times\left\{(1.5514)\exp\left[-(2.1249)^2\frac{(2\times10^{-7})t}{(0.01)^2}\right]\right\} = 0.2105$$



which gives

$$t = \mathbf{244\ s = 4.1\ min}$$

Therefore, it will take about 4.1 min for the hot dog to cook. Note that

$$\tau_{cyl} = \frac{\alpha t}{r_o^2} = \frac{(2\times10^{-7}\ \text{m}^2/\text{s})(244\ \text{s})}{(0.01\ \text{m})^2} = 0.49 > 0.2$$

and thus the assumption $\tau > 0.2$ for the applicability of the one-term approximate solution is verified.

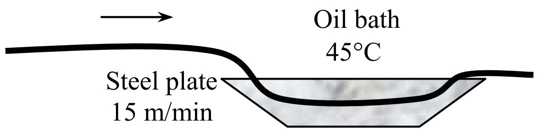
Discussion This problem could also be solved by treating the hot dog as an infinite cylinder since heat transfer through the end surfaces will have little effect on the mid section temperature because of the large distance.

11-95 A long roll of large 1-Mn manganese steel plate is to be quenched in an oil bath at a specified rate. The temperature of the sheet metal after quenching and the rate at which heat needs to be removed from the oil in order to keep its temperature constant are to be determined.

Assumptions **1** The thermal properties of the steel plate are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface. **3** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be checked).

Properties The properties of the steel plate are k = 60.5 W/m.°C, ρ = 7854 kg/m^3, and c_p = 434 J/kg.°C (Table A-24).

Analysis The characteristic length of the steel plate and the Biot number are



$$L_c = \frac{V}{A_s} = L = 0.0025 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(860 \text{ W/m}^2.°\text{C})(0.0025 \text{ m})}{60.5 \text{ W/m.°C}} = 0.036 < 0.1$$

Since Bi < 0.1 , the lumped system analysis is applicable. Therefore,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{860 \text{ W/m}^2.°\text{C}}{(7854 \text{ kg/m}^3)(434 \text{ J/kg.°C})(0.0025 \text{ m})} = 0.10092 \text{ s}^{-1}$$

$$\text{time} = \frac{\text{length}}{\text{velocity}} = \frac{9 \text{ m}}{15 \text{ m/min}} = 0.6 \text{ min} = 36 \text{ s}$$

Then the temperature of the sheet metal when it leaves the oil bath is determined to be

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{T(t) - 45}{820 - 45} = e^{-(0.10092 \text{ s}^{-1})(36 \text{ s})} \longrightarrow T(t) = \mathbf{65.5°C}$$

The mass flow rate of the sheet metal through the oil bath is

$$\dot{m} = \rho \dot{V} = \rho wtV = (7854 \text{ kg/m}^3)(2 \text{ m})(0.005 \text{ m})(15 \text{ m/min}) = 1178 \text{ kg/min}$$

Then the rate of heat transfer from the sheet metal to the oil bath and thus the rate at which heat needs to be removed from the oil in order to keep its temperature constant at 45°C becomes

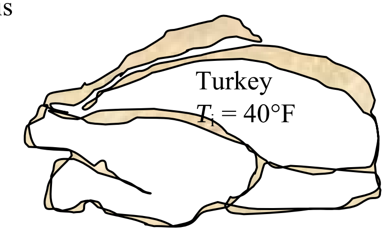
$$\dot{Q} = \dot{m}c_p[T_i - T(t)] = (1178 \text{ kg/min})(0.434 \text{ kJ/kg.°C})(820 - 65.5)°\text{C} = 385{,}740 \text{ kJ/min} = \mathbf{6429 \text{ kW}}$$

11-96E A stuffed turkey is cooked in an oven. The average heat transfer coefficient at the surface of the turkey, the temperature of the skin of the turkey in the oven and the total amount of heat transferred to the turkey in the oven are to be determined.

Assumptions **1** The turkey is a homogeneous spherical object. **2** Heat conduction in the turkey is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the turkey are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable (this assumption will be verified).

Properties The properties of the turkey are given to be $k = 0.26$ Btu/h.ft.°F, $\rho = 75$ lbm/ft^3, $c_p = 0.98$ Btu/lbm.°F, and $\alpha = 0.0035$ ft^2/h.

Analysis (*a*) Assuming the turkey to be spherical in shape, its radius is determined to be



$$m = \rho V \longrightarrow V = \frac{m}{\rho} = \frac{14 \text{ lbm}}{75 \text{ lbm/ft}^3} = 0.1867 \text{ ft}^3$$

$$V = \frac{4}{3}\pi r_o^3 \longrightarrow r_o = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.1867 \text{ ft}^3)}{4\pi}} = 0.3545 \text{ ft}$$

The Fourier number is $$\tau = \frac{\alpha t}{r_o^2} = \frac{(3.5\times10^{-3} \text{ ft}^2\text{/h})(5 \text{ h})}{(0.3545 \text{ ft})^2} = 0.1392$$

which is close to 0.2 but a little below it. Therefore, assuming the one-term approximate solution for transient heat conduction to be applicable, the one-term solution formulation at one-third the radius from the center of the turkey can be expressed as

$$\theta(x,t)_{sph} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r / r_o)}{\lambda_1 r / r_o}$$

$$\frac{185-325}{40-325} = 0.491 = A_1 e^{-\lambda_1^2 (0.14)} \frac{\sin(0.333\lambda_1)}{0.333\lambda_1}$$

By trial and error, it is determined from Table 11-2 that the equation above is satisfied when $Bi = 20$ corresponding to $\lambda_1 = 2.9857$ and $A_1 = 1.9781$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.26 \text{ Btu/h.ft.°F})(20)}{(0.3545 \text{ ft})} = \mathbf{14.7 \text{ Btu/h.ft}^2\text{.°F}}$$

(*b*) The temperature at the surface of the turkey is

$$\frac{T(r_o,t)-325}{40-325} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r_o / r_o)}{\lambda_1 r_o / r_o} = (1.9781)e^{-(2.9857)^2(0.14)} \frac{\sin(2.9857)}{2.9857} = 0.02953$$

$$\longrightarrow T(r_o,t) = \mathbf{317 \text{ °F}}$$

(*c*) The maximum possible heat transfer is

$$Q_{max} = mc_p(T_\infty - T_i) = (14 \text{ lbm})(0.98 \text{ Btu/lbm.°F})(325-40)\text{°F} = 3910 \text{ Btu}$$

Then the actual amount of heat transfer becomes

$$\frac{Q}{Q_{max}} = 1 - 3\theta_{o,sph} \frac{\sin(\lambda_1) - \lambda_1 \cos(\lambda_1)}{\lambda_1^3} = 1 - 3(0.491)\frac{\sin(2.9857) - (2.9857)\cos(2.9857)}{(2.9857)^3} = 0.828$$

$$Q = 0.828 Q_{max} = (0.828)(3910 \text{ Btu}) = \mathbf{3240 \text{ Btu}}$$

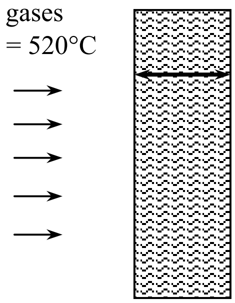
Discussion The temperature of the outer parts of the turkey will be greater than that of the inner parts when the turkey is taken out of the oven. Then heat will continue to be transferred from the outer parts of the turkey to the inner as a result of temperature difference. Therefore, after 5 minutes, the thermometer reading will probably be more than 185°F.

11-97 CD EES The trunks of some dry oak trees are exposed to hot gases. The time for the ignition of the trunks is to be determined.

Assumptions **1** Heat conduction in the trunks is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the trunks are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the trunks are given to be $k = 0.17$ W/m.°C and $\alpha = 1.28\times10^{-7}$ m²/s.

Analysis We treat the trunks of the trees as an infinite cylinder since heat transfer is primarily in the radial direction. Then the Biot number becomes



$$Bi = \frac{hr_o}{k} = \frac{(65\ \text{W/m}^2.°\text{C})(0.1\ \text{m})}{(0.17\ \text{W/m.°C})} = 38.24$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 2.3420 \quad \text{and} \quad A_1 = 1.5989$$

The Fourier number is

$$\tau = \frac{\alpha t}{r_o^2} = \frac{(1.28\times10^{-7}\ \text{m}^2/\text{s})(4\ \text{h}\times3600\ \text{s/h})}{(0.1\ \text{m})^2} = 0.184$$

which is slightly below 0.2 but close to it. Therefore, assuming the one-term approximate solution for transient heat conduction to be applicable, the temperature at the surface of the trees in 4 h becomes

$$\theta(r_o,t)_{cyl} = \frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r/r_o)$$

$$\frac{T(r_o,t)-520}{30-520} = (1.5989)e^{-(2.3420)^2(0.184)}(0.0332) = 0.01935 \longrightarrow T(r_o,t) = \mathbf{511\,°C} \ > \ 410°\text{C}$$

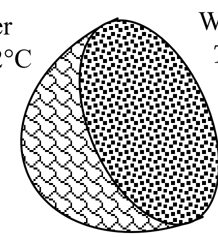
Therefore, the trees will ignite. (Note: J_0 is read from Table 11-3).

11-98 A spherical watermelon that is cut into two equal parts is put into a freezer. The time it will take for the center of the exposed cut surface to cool from 25 to 3°C is to be determined.

Assumptions **1** The temperature of the exposed surfaces of the watermelon is affected by the convection heat transfer at those surfaces only. Therefore, the watermelon can be considered to be a semi-infinite medium **2** The thermal properties of the watermelon are constant.

Properties The thermal properties of the water is closely approximated by those of water at room temperature, $k = 0.607$ W/m.°C and $\alpha = k/\rho c_p = 0.146\times10^{-6}$ m²/s (Table A-15).

Analysis We use the transient chart in Fig. 11-29 in this case for convenience (instead of the analytic solution),



$$\left.\begin{aligned} 1-\frac{T(x,t)-T_\infty}{T_i-T_\infty} &= 1-\frac{3-(-12)}{25-(-12)} = 0.595 \\ \xi &= \frac{x}{2\sqrt{\alpha t}} = 0 \end{aligned}\right\} \frac{h\sqrt{\alpha t}}{k} = 1$$

Therefore, $t = \dfrac{(1)^2 k^2}{h^2\alpha} = \dfrac{(0.607\ \text{W/m.°C})^2}{(22\ \text{W/m}^2.°\text{C})^2(0.146\times10^{-6}\ \text{m}^2/\text{s})} = 5214\ \text{s} = \mathbf{86.9\ min}$

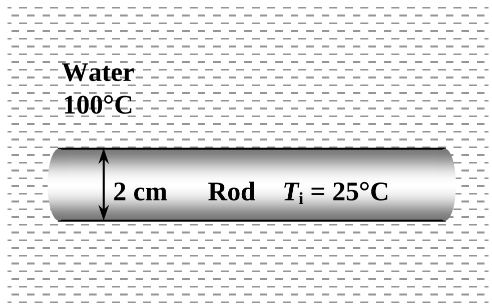
11-99 A cylindrical rod is dropped into boiling water. The thermal diffusivity and the thermal conductivity of the rod are to be determined.

Assumptions **1** Heat conduction in the rod is one-dimensional since the rod is sufficiently long, and thus temperature varies in the radial direction only. **2** The thermal properties of the rod are constant.

Properties The thermal properties of the rod available are given to be ρ = 3700 kg/m^3 and C_p = 920 J/kg.°C.

Analysis From Fig. 11-16b we have

$$\left.\begin{aligned} \frac{T-T_\infty}{T_0-T_\infty} &= \frac{93-100}{75-100} = 0.28 \\ \frac{x}{r_o} &= \frac{r_o}{r_o} = 1 \end{aligned}\right\} \frac{1}{Bi} = \frac{k}{hr_o} = 0.25$$



From Fig. 11-16a we have

$$\left.\begin{aligned} \frac{1}{Bi} &= \frac{k}{hr_o} = 0.25 \\ \frac{T_o-T_\infty}{T_i-T_\infty} &= \frac{75-100}{25-100} = 0.33 \end{aligned}\right\} \tau = \frac{\alpha t}{r_o^2} = 0.40$$

Then the thermal diffusivity and the thermal conductivity of the material become

$$\alpha = \frac{0.40 r_o^2}{t} = \frac{(0.40)(0.01\text{ m})^2}{3\text{ min}\times 60\text{ s/min}} = \mathbf{2.22\times 10^{-7}\ m^2/s}$$

$$\alpha = \frac{k}{\alpha c_p} \longrightarrow k = \alpha\rho c_p = (2.22\times 10^{-7}\text{ m}^2\text{/s})(3700\text{ kg/m}^3)(920\text{ J/kg.°C}) = \mathbf{0.756\ W/m.°C}$$

11-100 The time it will take for the diameter of a raindrop to reduce to a certain value as it falls through ambient air is to be determined.

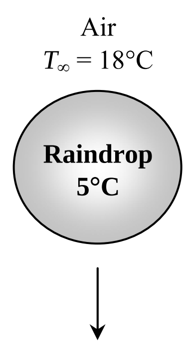
Assumptions **1** The water temperature remains constant. **2** The thermal properties of the water are constant.

Properties The density and heat of vaporization of the water are ρ = 1000 kg/m^3 and h_{fg} = 2490 kJ/kg (Table A-15).

Analysis The initial and final masses of the raindrop are

$$m_i = \rho \boldsymbol{V}_i = \rho\frac{4}{3}\pi r_i^3 = (1000\text{ kg/m}^3)\frac{4}{3}\pi(0.0025\text{ m})^3 = 0.0000654\text{ kg}$$

$$m_f = \rho \boldsymbol{V}_f = \rho\frac{4}{3}\pi r_f^3 = (1000\text{ kg/m}^3)\frac{4}{3}\pi(0.0015\text{ m})^3 = 0.0000141\text{ kg}$$



whose difference is

$$m = m_i - m_f = 0.0000654 - 0.0000141 = 0.0000513\text{ kg}$$

The amount of heat transfer required to cause this much evaporation is

$$Q = (0.0000513\text{ kg})(2490\text{ kJ/kg}) = 0.1278\text{ kJ}$$

The average heat transfer surface area and the rate of heat transfer are

$$A_s = \frac{4\pi(r_i^2 + r_f^2)}{2} = \frac{4\pi[(0.0025\text{ m})^2 + (0.0015\text{ m})^2}{2} = 5.341\times 10^{-5}\text{ m}^2$$

$$\dot{Q} = hA_s(T_i - T_\infty) = (400\text{ W/m}^2.\text{°C})(5.341\times 10^{-5}\text{ m}^2)(18-5)\text{°C} = 0.2777\text{ J/s}$$

Then the time required for the raindrop to experience this reduction in size becomes

$$\dot{Q} = \frac{Q}{\Delta t} \longrightarrow \Delta t = \frac{Q}{\dot{Q}} = \frac{127.8\text{ J}}{0.2777\text{ J/s}} = 460\text{ s} = \mathbf{7.7\ min}$$

11-101E A plate, a long cylinder, and a sphere are exposed to cool air. The center temperature of each geometry is to be determined.

Assumptions **1** Heat conduction in each geometry is one-dimensional. **2** The thermal properties of the bodies are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of bronze are given to be $k = 15$ Btu/h.ft.°F and $\alpha = 0.333$ ft^2/h.

Analysis **After 5 minutes**

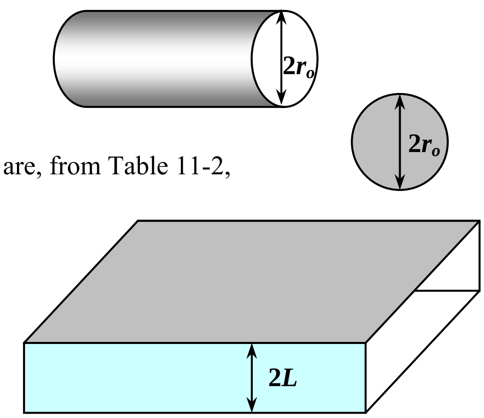


Plate: First the Biot number is calculated to be

$$Bi = \frac{hL}{k} = \frac{(7\ \text{Btu/h.ft}^2.°\text{F})(0.5/12\ \text{ft})}{(15\ \text{Btu/h.ft.°F})} = 0.01944$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 0.1387 \quad \text{and} \quad A_1 = 1.0032$$

The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.333\ \text{ft}^2/\text{h})(5\ \text{min/60 min/h})}{(0.5/12\ \text{ft})^2} = 15.98 > 0.2$$

Then the center temperature of the plate becomes

$$\theta_{o,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0032)e^{-(0.1387)^2(15.98)} = 0.738 \longrightarrow T_0 = \mathbf{315°F}$$

Cylinder:

$$Bi = 0.01944 \xrightarrow{\text{Table 4-2}} \lambda_1 = 0.1962 \quad \text{and} \quad A_1 = 1.0049$$

$$\theta_{0,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0049)e^{-(0.1962)^2(15.98)} = 0.543 \longrightarrow T_0 = \mathbf{252°F}$$

Sphere:

$$Bi = 0.01944 \xrightarrow{\text{Table 4-2}} \lambda_1 = 0.2405 \quad \text{and} \quad A_1 = 1.0058$$

$$\theta_{0,sph} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0058)e^{-(0.2405)^2(15.98)} = 0.399 \longrightarrow T_0 = \mathbf{205°F}$$

After 10 minutes

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.333\ \text{ft}^2/\text{h})(10\ \text{min/60 min/h})}{(0.5/12\ \text{ft})^2} = 31.97 > 0.2$$

Plate:

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0032)e^{-(0.1387)^2(31.97)} = 0.542 \longrightarrow T_0 = \mathbf{251°F}$$

Cylinder:

$$\theta_{0,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0049)e^{-(0.1962)^2(31.97)} = 0.293 \longrightarrow T_0 = \mathbf{170°F}$$

Sphere:

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0058)e^{-(0.2405)^2(31.97)} = 0.158 \longrightarrow T_0 = \mathbf{126°F}$$

<u>After 30 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.333\ \text{ft}^2/\text{h})(30\ \text{min}/60\ \text{min/h})}{(0.5/12\ \text{ft})^2} = 95.9 > 0.2$$

Plate:

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0032)e^{-(0.1387)^2(95.9)} = 0.159 \longrightarrow T_0 = \mathbf{127°F}$$

Cylinder:

$$\theta_{0,cyl} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0049)e^{-(0.1962)^2(95.9)} = 0.025 \longrightarrow T_0 = \mathbf{83°F}$$

Sphere:

$$\theta_{0,sph} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0058)e^{-(0.2405)^2(95.9)} = 0.00392 \longrightarrow T_0 = \mathbf{76°F}$$

The sphere has the largest surface area through which heat is transferred per unit volume, and thus the highest rate of heat transfer. Consequently, the center temperature of the sphere is always the lowest.

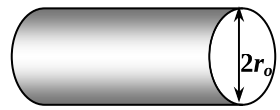
11-102E A plate, a long cylinder, and a sphere are exposed to cool air. The center temperature of each geometry is to be determined.

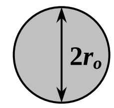
Assumptions **1** Heat conduction in each geometry is one-dimensional. **2** The thermal properties of the geometries are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of cast iron are given to be $k = 29$ Btu/h.ft.°F and $\alpha = 0.61$ ft^2/h.

Analysis **<u>After 5 minutes</u>**

Plate: First the Biot number is calculated to be

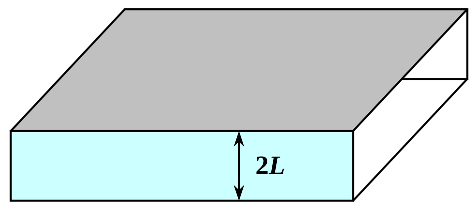




$$Bi = \frac{hL}{k} = \frac{(7\text{ Btu/h.ft}^2.°\text{F})(0.5/12\text{ ft})}{(29\text{ Btu/h.ft.°F})} = 0.01006 \cong 0.01$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 0.0998 \quad \text{and} \quad A_1 = 1.0017$$



The Fourier number is

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.61\text{ ft}^2\text{/h})(5\text{ min/60 min/h})}{(0.5/12\text{ ft})^2} = 29.28 > 0.2$$

Then the center temperature of the plate becomes

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0017)e^{-(0.0998)^2(29.28)} = 0.748 \longrightarrow T_0 = \mathbf{318°F}$$

Cylinder:

$$Bi = 0.01 \xrightarrow{\text{Table 4-2}} \lambda_1 = 0.1412 \quad \text{and} \quad A_1 = 1.0025$$

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0025)e^{-(0.1412)^2(29.28)} = 0.559 \longrightarrow T_0 = \mathbf{257°F}$$

Sphere:

$$Bi = 0.01 \xrightarrow{\text{Table 4-2}} \lambda_1 = 0.1730 \quad \text{and} \quad A_1 = 1.0030$$

$$\theta_{0,sph} = \frac{T_o - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0030)e^{-(0.1730)^2(29.28)} = 0.418 \longrightarrow T_0 = \mathbf{211°F}$$

<u>After 10 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.61\text{ ft}^2\text{/h})(10\text{ min/60 min/h})}{(0.5/12\text{ ft})^2} = 58.56 > 0.2$$

Plate:

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0017)e^{-(0.0998)^2(58.56)} = 0.559 \longrightarrow T_0 = \mathbf{257°F}$$

Cylinder:

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0025)e^{-(0.1412)^2(58.56)} = 0.312 \longrightarrow T_0 = \mathbf{176°F}$$

Sphere:

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0030)e^{-(0.1730)^2(58.56)} = 0.174 \longrightarrow T_0 = \mathbf{132°F}$$

<u>After 30 minutes</u>

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.61\ \text{ft}^2/\text{h})(30\ \text{min}/60\ \text{min/h})}{(0.5/12\ \text{ft})^2} = 175.68 > 0.2$$

Plate:

$$\theta_{0,wall} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0017)e^{-(0.0998)^2(175.68)} = 0.174 \longrightarrow T_0 = \mathbf{132°F}$$

Cylinder:

$$\theta_{0,cyl} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0025)e^{-(0.1412)^2(175.68)} = 0.030 \longrightarrow T_0 = \mathbf{84.8°F}$$

Sphere:

$$\theta_{0,sph} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \longrightarrow \frac{T_0 - 75}{400 - 75} = (1.0030)e^{-(0.1730)^2(175.68)} = 0.0052 \longrightarrow T_0 = \mathbf{76.7°F}$$

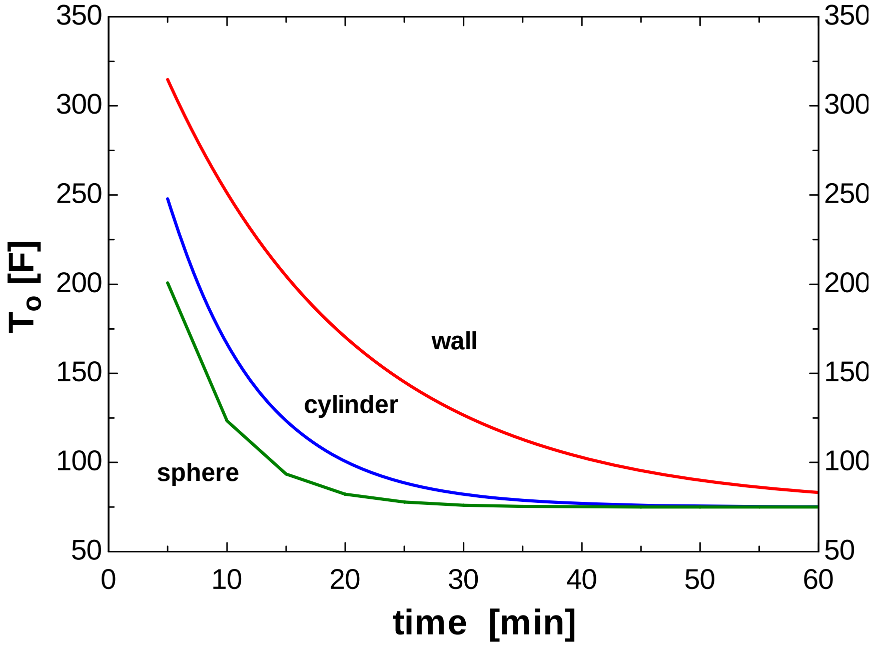
The sphere has the largest surface area through which heat is transferred per unit volume, and thus the highest rate of heat transfer. Consequently, the center temperature of the sphere is always the lowest.

11-103E EES Prob. 11-101E is reconsidered. The center temperature of each geometry as a function of the cooling time is to be plotted.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
2*L=(1/12) [ft]
2*r_o_c=(1/12) [ft] "c stands for cylinder"
2*r_o_s=(1/12) [ft] "s stands for sphere"
T_i=400 [F]
T_infinity=75 [F]
h=7 [Btu/h-ft^2-F]
time=5 [min]

"PROPERTIES"
k=15 [Btu/h-ft-F]
alpha=0.333 [ft^2/h]*Convert(ft^2/h, ft^2/min)

"ANALYSIS"
"For plane wall"
Bi_w=(h*L)/k
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1_w=0.1387
A_1_w=1.0032
tau_w=(alpha*time)/L^2
(T_o_w-T_infinity)/(T_i-T_infinity)=A_1_w*exp(-lambda_1_w^2*tau_w)
"For long cylinder"
Bi_c=(h*r_o_c)/k
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1_c=0.1962
A_1_c=1.0049
tau_c=(alpha*time)/r_o_c^2
(T_o_c-T_infinity)/(T_i-T_infinity)=A_1_c*exp(-lambda_1_c^2*tau_c)
"For sphere"
Bi_s=(h*r_o_s)/k
"From Table 11-2 corresponding to this Bi number, we read"
lambda_1_s=0.2405
A_1_s=1.0058
tau_s=(alpha*time)/r_o_s^2
(T_o_s-T_infinity)/(T_i-T_infinity)=A_1_s*exp(-lambda_1_s^2*tau_s)
```

time [min]	$T_{o,w}$ [F]	$T_{o,c}$ [F]	$T_{o,s}$ [F]
5	314.7	251.5	204.7
10	251.3	170.4	126.4
15	204.6	126.6	95.41
20	170.3	102.9	83.1
25	145.1	90.06	78.21
30	126.5	83.14	76.27
35	112.9	79.4	75.51
40	102.9	77.38	75.2
45	95.48	76.29	75.08
50	90.06	75.69	75.03
55	86.07	75.38	75.01
60	83.14	75.2	75

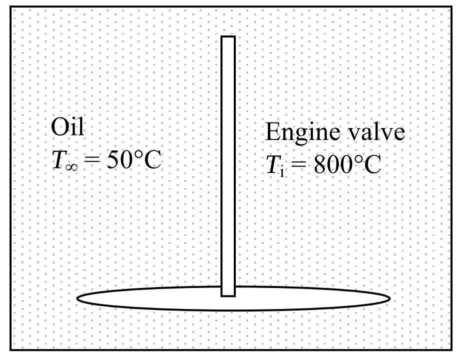


350
300
250
200
150
100
50
0
10
20
30
40
50
60
T o [F]
time [min]
wall
cylinder
sphere

11-104 Internal combustion engine valves are quenched in a large oil bath. The time it takes for the valve temperature to drop to specified temperatures and the maximum heat transfer are to be determined.

Assumptions **1** The thermal properties of the valves are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface. **3** Depending on the size of the oil bath, the oil bath temperature will increase during quenching. However, an average canstant temperature as specified in the problem will be used. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The thermal conductivity, density, and specific heat of the balls are given to be k = 48 W/m.°C, ρ = 7840 kg/m^3, and c_p = 440 J/kg.°C.



Analysis (*a*) The characteristic length of the balls and the Biot number are

$$L_c = \frac{V}{A_s} = \frac{1.8(\pi D^2 L/4)}{2\pi DL} = \frac{1.8D}{8} = \frac{1.8(0.008\text{ m})}{8} = 0.0018\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(800\text{ W/m}^2.\text{°C})(0.0018\text{ m})}{48\text{ W/m.°C}} = 0.03 < 0.1$$

Therefore, we can use lumped system analysis. Then the time for a final valve temperature of 400°C becomes

$$b = \frac{hA_s}{\rho c_p V} = \frac{8h}{1.8\rho c_p D} = \frac{8(800\text{ W/m}^2.\text{°C})}{1.8(7840\text{ kg/m}^3)(440\text{ J/kg.°C})(0.008\text{ m})} = 0.1288\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{400-50}{800-50} = e^{-(0.1288\text{ s}^{-1})t} \longrightarrow t = \mathbf{5.9\text{ s}}$$

(*b*) The time for a final valve temperature of 200°C is

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{200-50}{800-50} = e^{-(0.1288\text{ s}^{-1})t} \longrightarrow t = \mathbf{12.5\text{ s}}$$

(*c*) The time for a final valve temperature of 51°C is

$$\frac{T(t)-T_\infty}{T_i-T_\infty} = e^{-bt} \longrightarrow \frac{51-50}{800-50} = e^{-(0.1288\text{ s}^{-1})t} \longrightarrow t = \mathbf{51.4\text{ s}}$$

(*d*) The maximum amount of heat transfer from a single valve is determined from

$$m = \rho V = \rho\frac{1.8\pi D^2 L}{4} = (7840\text{ kg/m}^3)\frac{1.8\pi(0.008\text{ m})^2(0.10\text{ m})}{4} = 0.0709\text{ kg}$$

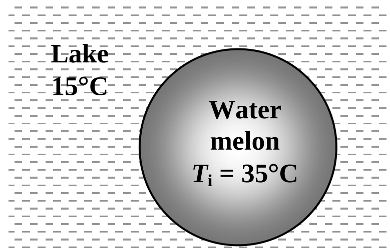
$$Q = mc_p[T_f - T_i] = (0.0709\text{ kg})(440\text{ J/kg.°C})(800-50)\text{°C} = 23{,}400\text{ J} = \mathbf{23.4\text{ kJ}}\ \text{(per valve)}$$

11-105 A watermelon is placed into a lake to cool it. The heat transfer coefficient at the surface of the watermelon and the temperature of the outer surface of the watermelon are to be determined.

Assumptions **1** The watermelon is a homogeneous spherical object. **2** Heat conduction in the watermelon is one-dimensional because of symmetry about the midpoint. **3** The thermal properties of the watermelon are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of the watermelon are given to be $k = 0.618$ W/m.°C, $\alpha = 0.15\times10^{-6}$ m^2/s, ρ = 995 kg/m^3 and $c_p = 4.18$ kJ/kg.°C.

Analysis The Fourier number is



$$\tau = \frac{\alpha t}{r_o^2} = \frac{(0.15\times10^{-6}\ \text{m}^2/\text{s})\left[(4\times60+40\ \text{min})\times60\ \text{s/min}\right]}{(0.10\ \text{m})^2} = 0.252$$

which is greater than 0.2. Then the one-term solution can be written in the form

$$\theta_{0,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} \longrightarrow \frac{20-15}{35-15} = 0.25 = A_1 e^{-\lambda_1^2(0.252)}$$

It is determined from Table 11-2 by trial and error that this equation is satisfied when Bi = 10, which corresponds to $\lambda_1 = 2.8363$ and $A_1 = 1.9249$. Then the heat transfer coefficient can be determined from

$$Bi = \frac{hr_o}{k} \longrightarrow h = \frac{kBi}{r_o} = \frac{(0.618\ \text{W/m.°C})(10)}{(0.10\ \text{m})} = \mathbf{61.8\ W/m^2.°C}$$

The temperature at the surface of the watermelon is

$$\theta(r_o,t)_{sph} = \frac{T(r_o,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\frac{\sin(\lambda_1 r_o/r_o)}{\lambda_1 r_o/r_o} = (1.9249)e^{-(2.8363)^2(0.252)}\frac{\sin(2.8363\ \text{rad})}{2.8363}$$

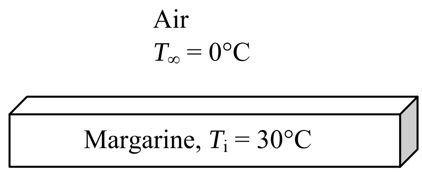
$$\frac{T(r_o,t)-15}{35-15} = 0.0269 \longrightarrow T(r_o,t) = \mathbf{15.5\ °C}$$

11-106 Large food slabs are cooled in a refrigeration room. Center temperatures are to be determined for different foods.

Assumptions **1** Heat conduction in the slabs is one-dimensional since the slab is large relative to its thickness and there is thermal symmetry about the center plane. **3** The thermal properties of the slabs are constant. **4** The heat transfer coefficient is constant and uniform over the entire surface. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of foods are given to be $k = 0.233$ W/m.°C and $\alpha = 0.11\times10^{-6}$ m^2/s for margarine, $k = 0.082$ W/m.°C and $\alpha = 0.10\times10^{-6}$ m^2/s for white cake, and $k = 0.106$ W/m.°C and $\alpha = 0.12\times10^{-6}$ m^2/s for chocolate cake.

Analysis (*a*) In the case of margarine, the Biot number is



$$Bi = \frac{hL}{k} = \frac{(25\text{ W/m}^2.°\text{C})(0.05\text{ m})}{(0.233\text{ W/m.°C})} = 5.365$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.3269 \quad \text{and} \quad A_1 = 1.2431$$

The Fourier number is $\tau = \dfrac{\alpha t}{L^2} = \dfrac{(0.11\times10^{-6}\text{ m}^2/\text{s})(6\text{ h}\times3600\text{ s/h})}{(0.05\text{ m})^2} = 0.9504 > 0.2$

Therefore, the one-term approximate solution (or the transient temperature charts) is applicable. Then the temperature at the center of the box if the box contains margarine becomes

$$\theta(0,t)_{wall} = \frac{T(0,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.2431)e^{-(1.3269)^2(0.9504)}$$

$$\frac{T(0,t)-0}{30-0} = 0.233 \longrightarrow T(0,t) = \mathbf{7.0\ °C}$$

(*b*) Repeating the calculations for white cake,

$$Bi = \frac{hL}{k} = \frac{(25\text{ W/m}^2.°\text{C})(0.05\text{ m})}{(0.082\text{ W/m.°C})} = 15.24 \longrightarrow \lambda_1 = 1.4641 \quad \text{and} \quad A_1 = 1.2661$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.10\times10^{-6}\text{ m}^2/\text{s})(6\text{ h}\times3600\text{ s/h})}{(0.05\text{ m})^2} = 0.864 > 0.2$$

$$\theta(0,t)_{wall} = \frac{T(0,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.2661)e^{-(1.4641)^2(0.864)}$$

$$\frac{T(0,t)-0}{30-0} = 0.199 \longrightarrow T(0,t) = \mathbf{6.0\ °C}$$

(*c*) Repeating the calculations for chocolate cake,

$$Bi = \frac{hL}{k} = \frac{(25\text{ W/m}^2.°\text{C})(0.05\text{ m})}{(0.106\text{ W/m.°C})} = 11.79 \longrightarrow \lambda_1 = 1.4409 \quad \text{and} \quad A_1 = 1.2634$$

$$\tau = \frac{\alpha t}{L^2} = \frac{(0.12\times10^{-6}\text{ m}^2/\text{s})(6\text{ h}\times3600\text{ s/h})}{(0.05\text{ m})^2} = 1.0368 > 0.2$$

$$\theta(0,t)_{wall} = \frac{T(0,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.2634)e^{-(1.4409)^2(1.0368)}$$

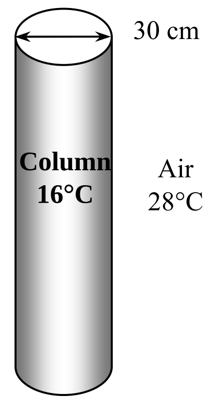
$$\frac{T(0,t)-0}{30-0} = 0.147 \longrightarrow T(0,t) = \mathbf{4.4\ °C}$$

11-107 A cold cylindrical concrete column is exposed to warm ambient air during the day. The time it will take for the surface temperature to rise to a specified value, the amounts of heat transfer for specified values of center and surface temperatures are to be determined.

Assumptions **1** Heat conduction in the column is one-dimensional since it is long and it has thermal symmetry about the center line. **2** The thermal properties of the column are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The properties of concrete are given to be $k = 0.79$ W/m.°C, $\alpha = 5.94\times10^{-7}$ m^2/s, $\rho = 1600$ kg/m^3 and $c_p = 0.84$ kJ/kg.°C

Analysis (*a*) The Biot number is



$$Bi = \frac{hr_o}{k} = \frac{(14\text{ W/m}^2.\text{°C})(0.15\text{ m})}{(0.79\text{ W/m.°C})} = 2.658$$

The constants λ_1 and A_1 corresponding to this Biot number are, from Table 11-2,

$$\lambda_1 = 1.7240 \quad\text{and}\quad A_1 = 1.3915$$

Once the constant J_0=0.3841 is determined from Table 11-3 corresponding to the constant λ_1, the Fourier number is determined to be

$$\frac{T(r_o,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} J_0(\lambda_1 r_o / r_o) \longrightarrow \frac{27-28}{14-28} = (1.3915)e^{-(1.7240)^2\tau}(0.3841) \longrightarrow \tau = 0.6771$$

which is above the value of 0.2. Therefore, the one-term approximate solution (or the transient temperature charts) can be used. Then the time it will take for the column surface temperature to rise to 27°C becomes

$$t = \frac{\tau r_o^2}{\alpha} = \frac{(0.6771)(0.15\text{ m})^2}{5.94\times10^{-7}\text{ m}^2\text{/s}} = 25{,}650\text{ s} = \mathbf{7.1\ hours}$$

(*b*) The heat transfer to the column will stop when the center temperature of column reaches to the ambient temperature, which is 28°C. That is, we are asked to determine the maximum heat transfer between the ambient air and the column.

$$m = \rho V = \rho\pi r_o^2 L = (1600\text{ kg/m}^3)[\pi(0.15\text{ m})^2(4\text{ m})] = 452.4\text{ kg}$$

$$Q_{\max} = mc_p[T_\infty - T_i] = (452.4\text{ kg})(0.84\text{ kJ/kg.°C})(28-14)\text{°C} = \mathbf{5320\ kJ}$$

(*c*) To determine the amount of heat transfer until the surface temperature reaches to 27°C, we first determine

$$\frac{T(0,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.3915)e^{-(1.7240)^2(0.6771)} = 0.1860$$

Once the constant J_1 = 0.5787 is determined from Table 11-3 corresponding to the constant λ_1, the amount of heat transfer becomes

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\left(\frac{T_0 - T_\infty}{T_i - T_\infty}\right)\frac{J_1(\lambda_1)}{\lambda_1} = 1 - 2\times0.1860\times\frac{0.5787}{1.7240} = 0.875$$

$$Q = 0875 Q_{\max}$$

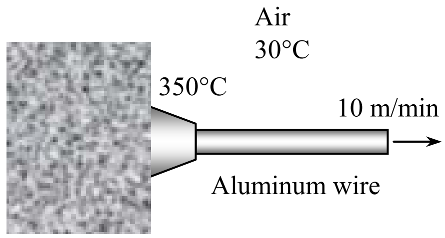
$$Q = 0.875(5320\text{ kJ}) = \mathbf{4660\ kJ}$$

11-108 Long aluminum wires are extruded and exposed to atmospheric air. The time it will take for the wire to cool, the distance the wire travels, and the rate of heat transfer from the wire are to be determined.

Assumptions **1** Heat conduction in the wires is one-dimensional in the radial direction. **2** The thermal properties of the aluminum are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of aluminum are given to be k = 236 W/m.°C, ρ = 2702 kg/m^3, c_p = 0.896 kJ/kg.°C, and α = 9.75×10^{-5} m^2/s.

Analysis (*a*) The characteristic length of the wire and the Biot number are



$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L} = \frac{r_o}{2} = \frac{0.0015 \text{ m}}{2} = 0.00075 \text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(35 \text{ W/m}^2.°\text{C})(0.00075 \text{ m})}{236 \text{ W/m.°C}} = 0.00011 < 0.1$$

Since Bi < 0.1, the lumped system analysis is applicable. Then,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{35 \text{ W/m}^2.°\text{C}}{(2702 \text{ kg/m}^3)(896 \text{ J/kg.°C})(0.00075 \text{ m})} = 0.0193 \text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-30}{350-30} = e^{-(0.0193 \text{ s}^{-1})t} \longrightarrow t = \mathbf{144 \text{ s}}$$

(*b*) The wire travels a distance of

$$\text{velocity} = \frac{\text{length}}{\text{time}} \to \text{length} = (10/60 \text{ m/s})(144 \text{ s}) = \mathbf{24 \text{ m}}$$

This distance can be reduced by cooling the wire in a water or oil bath.

(*c*) The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho \dot{V} = \rho(\pi r_o^2)V = (2702 \text{ kg/m}^3)\pi(0.0015 \text{ m})^2(10 \text{ m/min}) = 0.191 \text{ kg/min}$$

Then the rate of heat transfer from the wire to the air becomes

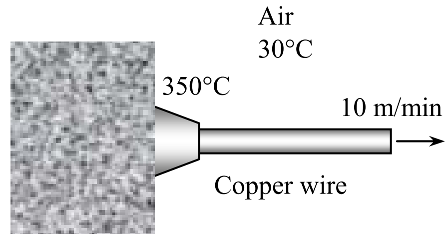
$$\dot{Q} = \dot{m} c_p [T(t) - T_\infty] = (0.191 \text{ kg/min})(0.896 \text{ kJ/kg.°C})(350-50)°\text{C} = 51.3 \text{ kJ/min} = \mathbf{856 \text{ W}}$$

11-109 Long copper wires are extruded and exposed to atmospheric air. The time it will take for the wire to cool, the distance the wire travels, and the rate of heat transfer from the wire are to be determined.

Assumptions **1** Heat conduction in the wires is one-dimensional in the radial direction. **2** The thermal properties of the copper are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Biot number is Bi < 0.1 so that the lumped system analysis is applicable (this assumption will be verified).

Properties The properties of copper are given to be k = 386 W/m.°C, ρ = 8950 kg/m^3, c_p = 0.383 kJ/kg.°C, and α = 1.13×10^{-4} m^2/s.

Analysis (*a*) The characteristic length of the wire and the Biot number are



$$L_c = \frac{V}{A_s} = \frac{\pi r_o^2 L}{2\pi r_o L} = \frac{r_o}{2} = \frac{0.0015\text{ m}}{2} = 0.00075\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(35\text{ W/m}^2.°\text{C})(0.00075\text{ m})}{386\text{ W/m.°C}} = 0.000068 < 0.1$$

Since Bi < 0.1 the lumped system analysis is applicable. Then,

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{35\text{ W/m}^2.°\text{C}}{(8950\text{ kg/m}^3)(383\text{ J/kg.°C})(0.00075\text{ m})} = 0.0136\text{ s}^{-1}$$

$$\frac{T(t)-T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50-30}{350-30} = e^{-(0.0136\text{ s}^{-1})t} \longrightarrow t = \mathbf{204\text{ s}}$$

(*b*) The wire travels a distance of

$$\text{velocity} = \frac{\text{length}}{\text{time}} \longrightarrow \text{length} = \left(\frac{10\text{ m/min}}{60\text{ s/min}}\right)(204\text{ s}) = \mathbf{34\text{ m}}$$

This distance can be reduced by cooling the wire in a water or oil bath.

(*c*) The mass flow rate of the extruded wire through the air is

$$\dot{m} = \rho \dot{V} = \rho(\pi r_o^2)V = (8950\text{ kg/m}^3)\pi(0.0015\text{ m})^2(10\text{ m/min}) = 0.633\text{ kg/min}$$

Then the rate of heat transfer from the wire to the air becomes

$$\dot{Q} = \dot{m}c_p[T(t) - T_\infty] = (0.633\text{ kg/min})(0.383\text{ kJ/kg.°C})(350-50)°\text{C} = 72.7\text{ kJ/min} = \mathbf{1212\text{ W}}$$

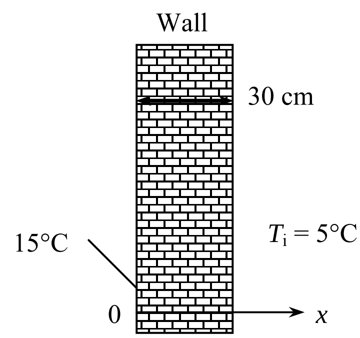
11-110 A brick house made of brick that was initially cold is exposed to warm atmospheric air at the outer surfaces. The time it will take for the temperature of the inner surfaces of the house to start changing is to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at outer surfaces only, and thus the wall can be considered to be a semi-infinite medium with a specified outer surface temperature of 18°C. **2** The thermal properties of the brick wall are constant.

Properties The thermal properties of the brick are given to be k = 0.72 W/m.°C and $\alpha = 0.45\times10^{-6}$ m²/s.

Analysis The exact analytical solution to this problem is

$$\frac{T(x,t)-T_i}{T_s-T_i} = erfc\left(\frac{x}{\sqrt{\alpha t}}\right)$$



Substituting,

$$\frac{5.1-5}{15-5} = 0.01 = erfc\left(\frac{0.3\text{ m}}{2\sqrt{(0.45\times10^{-6}\text{ m}^2/\text{s})t}}\right)$$

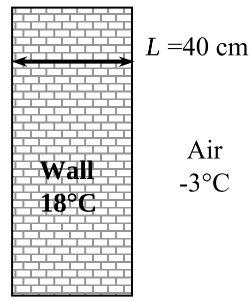
Noting from Table 11-4 that 0.01 = erfc(1.8215), the time is determined to be

$$\left(\frac{0.3\text{ m}}{2\sqrt{(0.45\times10^{-6}\text{ m}^2/\text{s})t}}\right) = 1.8215 \longrightarrow t = 15{,}070\text{ s} = \mathbf{251\ min}$$

11-111 A thick wall is exposed to cold outside air. The wall temperatures at distances 15, 30, and 40 cm from the outer surface at the end of 2-hour cooling period are to be determined.

Assumptions **1** The temperature in the wall is affected by the thermal conditions at outer surfaces only. Therefore, the wall can be considered to be a semi-infinite medium **2** The thermal properties of the wall are constant.

Properties The thermal properties of the brick are given to be $k = 0.72$ W/m.°C and $\alpha = 1.6\times10^{-7}$ m²/s.



Analysis For a 15 cm distance from the outer surface, from Fig. 11-29 we have

$$\left.\begin{array}{l} \dfrac{h\sqrt{\alpha t}}{k} = \dfrac{(20\text{ W/m}^2.°\text{C})\sqrt{(1.6\times10^{-6}\text{ m}^2/\text{s})(2\times3600\text{ s})}}{0.72\text{ W/m.°C}} = 2.98 \\ \eta = \dfrac{x}{2\sqrt{\alpha t}} = \dfrac{0.15\text{ m}}{2\sqrt{(1.6\times10^{-6}\text{ m}^2/\text{s})(2\times3600\text{ s})}} = 0.70 \end{array}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0.25$$

$$1-\frac{T-(-3)}{18-(-3)} = 0.25 \longrightarrow T = \mathbf{12.8°C}$$

For a 30 cm distance from the outer surface, from Fig. 11-29 we have

$$\left.\begin{array}{l} \dfrac{h\sqrt{\alpha t}}{k} = \dfrac{(20\text{ W/m}^2.°\text{C})\sqrt{(1.6\times10^{-6}\text{ m}^2/\text{s})(2\times3600\text{ s})}}{0.72\text{ W/m.°C}} = 2.98 \\ \eta = \dfrac{x}{2\sqrt{\alpha t}} = \dfrac{0.3\text{ m}}{2\sqrt{(1.6\times10^{-6}\text{ m}^2/\text{s})(2\times3600\text{ s})}} = 1.40 \end{array}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0.038$$

$$1-\frac{T-(-3)}{18-(-3)} = 0.038 \longrightarrow T = \mathbf{17.2°C}$$

For a 40 cm distance from the outer surface, that is for the inner surface, from Fig. 11-29 we have

$$\left.\begin{array}{l} \dfrac{h\sqrt{\alpha t}}{k} = \dfrac{(20\text{ W/m}^2.°\text{C})\sqrt{(1.6\times10^{-6}\text{ m}^2/\text{s})(2\times3600\text{ s})}}{0.72\text{ W/m.°C}} = 2.98 \\ \eta = \dfrac{x}{2\sqrt{\alpha t}} = \dfrac{0.4\text{ m}}{2\sqrt{(1.6\times10^{-6}\text{ m}^2/\text{s})(2\times3600\text{ s})}} = 1.87 \end{array}\right\} 1-\frac{T-T_\infty}{T_i-T_\infty} = 0$$

$$1-\frac{T-(-3)}{18-(-3)} = 0 \longrightarrow T = \mathbf{18.0°C}$$

Discussion This last result shows that the semi-infinite medium assumption is a valid one.

11-112 The engine block of a car is allowed to cool in atmospheric air. The temperatures at the center of the top surface and at the corner after a specified period of cooling are to be determined.

Assumptions **1** Heat conduction in the block is three-dimensional, and thus the temperature varies in all three directions. **2** The thermal properties of the block are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

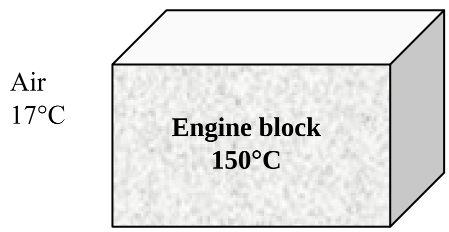
Properties The thermal properties of cast iron are given to be $k = 52$ W/m.°C and $\alpha = 1.7\times10^{-5}$ m^2/s.

Analysis This rectangular block can physically be formed by the intersection of two infinite plane walls of thickness $2L = 40$ cm (call planes A and B) and an infinite plane wall of thickness $2L = 80$ cm (call plane C). We measure x from the center of the block.

(*a*) The Biot number is calculated for each of the plane wall to be

$$Bi_A = Bi_B = \frac{hL}{k} = \frac{(6 \text{ W/m}^2.°\text{C})(0.2 \text{ m})}{(52 \text{ W/m.°C})} = 0.0231$$

$$Bi_C = \frac{hL}{k} = \frac{(6 \text{ W/m}^2.°\text{C})(0.4 \text{ m})}{(52 \text{ W/m.°C})} = 0.0462$$



The constants λ_1 and A_1 corresponding to these Biot numbers are, from Table 11-2,

$$\lambda_{1(A,B)} = 0.150 \quad \text{and} \quad A_{1(A,B)} = 1.0038$$

$$\lambda_{1(C)} = 0.212 \quad \text{and} \quad A_{1(C)} = 1.0076$$

The Fourier numbers are

$$\tau_{A,B} = \frac{\alpha t}{L^2} = \frac{(1.70\times10^{-5} \text{ m}^2/\text{s})(45 \text{ min}\times 60 \text{ s/min})}{(0.2 \text{ m})^2} = 1.1475 > 0.2$$

$$\tau_C = \frac{\alpha t}{L^2} = \frac{(1.70\times10^{-5} \text{ m}^2/\text{s})(45 \text{ min}\times 60 \text{ s/min})}{(0.4 \text{ m})^2} = 0.2869 > 0.2$$

The center of the top surface of the block (whose sides are 80 cm and 40 cm) is at the center of the plane wall with $2L = 80$ cm, at the center of the plane wall with $2L = 40$ cm, and at the surface of the plane wall with $2L = 40$ cm. The dimensionless temperatures are

$$\theta_{o,\text{wall(A)}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0038)e^{-(0.150)^2(1.1475)} = 0.9782$$

$$\theta(L,t)_{\text{wall(B)}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau}\cos(\lambda_1 L/L) = (1.0038)e^{-(0.150)^2(1.1475)}\cos(0.150) = 0.9672$$

$$\theta_{o,\text{wall(C)}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2\tau} = (1.0076)e^{-(0.212)^2(0.2869)} = 0.9947$$

Then the center temperature of the top surface of the cylinder becomes

$$\left[\frac{T(L,0,0,t) - T_\infty}{T_i - T_\infty}\right]_{\substack{\text{short}\\\text{cylinder}}} = \theta(L,t)_{\text{wall(B)}} \times \theta_{o,\text{wall(A)}} \times \theta_{o,\text{wall(C)}} = 0.9672\times0.9782\times0.9947 = 0.9411$$

$$\frac{T(L,0,0,t) - 17}{150 - 17} = 0.9411 \longrightarrow T(L,0,0,t) = \mathbf{142.2°C}$$

(*b*) The corner of the block is at the surface of each plane wall. The dimensionless temperature for the surface of the plane walls with 2*L* = 40 cm is determined in part (*a*). The dimensionless temperature for the surface of the plane wall with 2*L* = 80 cm is determined from

$$\theta(L,t)_{\text{wall (C)}} = \frac{T(x,t)-T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 L/L) = (1.0076)e^{-(0.212)^2(0.2869)} \cos(0.212) = 0.9724$$

Then the corner temperature of the block becomes

$$\left[\frac{T(L,L,L,t)-T_\infty}{T_i - T_\infty}\right]_{\substack{short\\cylinder}} = \theta(L,t)_{\text{wall,C}} \times \theta(L,t)_{\text{wall,B}} \times \theta(L,t)_{\text{wall,A}} = 0.9724 \times 0.9672 \times 0.9672 = 0.9097$$

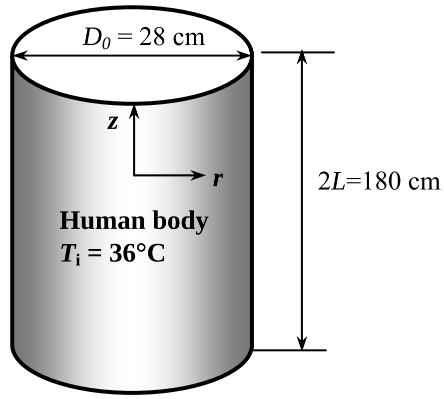
$$\frac{T(L,L,L,t)-17}{150-17} = 0.9097 \longrightarrow T(L,L,L,t) = \mathbf{138.0°C}$$

11-113 A man is found dead in a room. The time passed since his death is to be estimated.

Assumptions **1** Heat conduction in the body is two-dimensional, and thus the temperature varies in both radial *r*- and *x*- directions. **2** The thermal properties of the body are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface. **4** The human body is modeled as a cylinder. **5** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions (or the transient temperature charts) are applicable (this assumption will be verified).

Properties The thermal properties of body are given to be $k = 0.62$ W/m.°C and $\alpha = 0.15\times10^{-6}$ m^2/s.

Analysis A short cylinder can be formed by the intersection of a long cylinder of radius $D/2 = 14$ cm and a plane wall of thickness $2L = 180$ cm. We measure *x* from the midplane. The temperature of the body is specified at a point that is at the center of the plane wall but at the surface of the cylinder. The Biot numbers and the corresponding constants are first determined to be



$$Bi_{\text{wall}} = \frac{hL}{k} = \frac{(9\ \text{W/m}^2.°\text{C})(0.90\ \text{m})}{(0.62\ \text{W/m.°C})} = 13.06$$

$$\longrightarrow \lambda_1 = 1.4495 \quad \text{and} \quad A_1 = 1.2644$$

$$Bi_{\text{cyl}} = \frac{hr_o}{k} = \frac{(9\ \text{W/m}^2.°\text{C})(0.14\ \text{m})}{(0.62\ \text{W/m.°C})} = 2.03$$

$$\longrightarrow \lambda_1 = 1.6052 \quad \text{and} \quad A_1 = 1.3408$$

Noting that $\tau = \alpha t / L^2$ for the plane wall and $\tau = \alpha t / r_o^2$ for cylinder and $J_0(1.6052)=0.4524$ from Table 11-3, and assuming that $\tau > 0.2$ in all dimensions so that the one-term approximate solution for transient heat conduction is applicable, the product solution method can be written for this problem as

$$\theta(0, r_0, t)_{block} = \theta(0,t)_{\text{wall}}\, \theta(r_0, t)_{\text{cyl}}$$

$$\frac{23-16}{36-16} = \left(A_1 e^{-\lambda_1^2 \tau}\right)\left[A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r / r_0)\right]$$

$$0.40 = \left\{(1.2644)\exp\left[-(1.4495)^2 \frac{(0.15\times10^{-6})t}{(0.90)^2}\right]\right\}$$

$$\times\left\{(1.3408)\exp\left[-(1.6052)^2 \frac{(0.15\times10^{-6})t}{(0.14)^2}\right](0.4524)\right\}$$

$$\longrightarrow t = 32{,}404\ \text{s} = \mathbf{9.0\ hours}$$

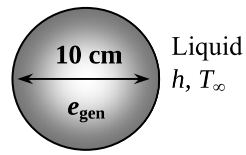
11-114 An exothermic process occurs uniformly throughout a sphere. The variation of temperature with time is to be obtained. The steady-state temperature of the sphere and the time needed for the sphere to reach the average of its initial and final (steady) temperatures are to be determined.

Assumptions **1** The sphere may be approximated as a lumped system. **2** The thermal properties of the sphere are constant. **3** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of sphere are given to be k = 300 W/m·K, c_p = 400 J/kg·K, ρ = 7500 kg/m^3.

Analysis (*a*) First, we check the applicability of lumped system as follows:

$$L_c = \frac{V}{A_{\text{surface}}} = \frac{\pi D^3/6}{\pi D^2} = \frac{D}{6} = \frac{0.10\text{ m}}{6} = 0.0167\text{ m}$$



$$Bi = \frac{hL_c}{k} = \frac{(250\text{ W/m}^2.°\text{C})(0.0167\text{ m})}{300\text{ W/m.°C}} = 0.014 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. An energy balance on the system may be written to give

$$\dot{e}_{\text{gen}} V = hA(T - T_\infty) + mc\frac{dT}{dt}$$

$$\dot{e}_{\text{gen}}(\pi D^3/6) = h\pi D^2(T - T_\infty) + \rho(\pi D^3/6)\frac{dT}{dt}$$

$$(1.2\times10^6)\pi(0.10)^3/6 = (250)\pi(0.10)^2(T-20) + (7500)[\pi(0.10)^3/6](400)\frac{dT}{dt}$$

$$20{,}000 = 250T - 5000 + 50{,}000\frac{dT}{dt}$$

$$\frac{dT}{dt} = 0.5 - 0.005T$$

(*b*) Now, we use integration to get the variation of sphere temperature with time

$$\frac{dT}{dt} = 0.5 - 0.005T$$

$$\frac{dT}{0.5-0.005T} = dt \longrightarrow \int_{20}^{T}\frac{dT}{0.5-0.005T} = \int_0^t dt$$

$$-\frac{1}{0.005}\ln(0.5-0.005T)\Big]_{20}^{T} = t\Big]_0^t = t$$

$$\ln\left(\frac{0.5-0.005T}{0.5-0.005\times20}\right) = -0.005t \longrightarrow \frac{0.5-0.005T}{0.4} = e^{-0.005t}$$

$$0.005T = 0.5 - 0.4e^{-0.005t} \longrightarrow T = 100 - 80e^{-0.005t}$$

We obtain the steady-state temperature by setting time to infinity:

$$T = 100 - 80e^{-0.005t} = 100 - e^{-\infty} = \mathbf{100°C}$$

or $\quad \frac{dT}{dt} = 0 \longrightarrow 0.5 - 0.005T = 0 \longrightarrow T = 100°\text{C}$

(*c*) The time needed for the sphere to reach the average of its initial and final (steady) temperatures is determined from

$$T = 100 - 80e^{-0.005t}$$

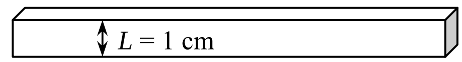
$$\frac{20+100}{2} = 100 - 80e^{-0.005t} \longrightarrow t = \mathbf{139\ s}$$

11-115 Large steel plates are quenched in an oil reservoir. The quench time is to be determined.

Assumptions **1** The thermal properties of the plates are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of steel plates are given to be k = 45 W/m·K, ρ = 7800 kg/m^3, and c_p = 470 J/kg·K.

Analysis For sphere, the characteristic length and the Biot number are



$$L_c = \frac{V}{A_{\text{surface}}} = \frac{L}{2} = \frac{0.01\text{ m}}{2} = 0.005\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(400\text{ W/m}^2.°\text{C})(0.005\text{ m})}{45\text{ W/m.°C}} = 0.044 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the cooling time is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{400\text{ W/m}^2.°\text{C}}{(7800\text{ kg/m}^3)(470\text{ J/kg.°C})(0.005\text{ m})} = 0.02182\text{ s}^{-1}$$

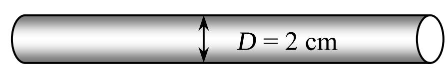
$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{100 - 30}{600 - 30} = e^{-(0.02182\text{ s}^{-1})t} \longrightarrow t = 96\text{ s} = \mathbf{1.6\ min}$$

11-116 Aluminum wires leaving the extruder at a specified rate are cooled in air. The necessary length of the wire is to be determined.

Assumptions **1** The thermal properties of the geometry are constant. **2** The heat transfer coefficient is constant and uniform over the entire surface.

Properties The properties of aluminum are k = 237 W/m·ºC, ρ = 2702 kg/m^3, and c_p = 0.903 kJ/kg·ºC (Table A-24).

Analysis For a long cylinder, the characteristic length and the Biot number are



$$L_c = \frac{V}{A_{\text{surface}}} = \frac{(\pi D^2/4)L}{\pi DL} = \frac{D}{4} = \frac{0.003\text{ m}}{4} = 0.00075\text{ m}$$

$$Bi = \frac{hL_c}{k} = \frac{(50\text{ W/m}^2.°\text{C})(0.00075\text{ m})}{237\text{ W/m.°C}} = 0.00016 < 0.1$$

Since $Bi < 0.1$, the lumped system analysis is applicable. Then the cooling time is determined from

$$b = \frac{hA}{\rho c_p V} = \frac{h}{\rho c_p L_c} = \frac{50\text{ W/m}^2.°\text{C}}{(2702\text{ kg/m}^3)(903\text{ J/kg.°C})(0.00075\text{ m})} = 0.02732\text{ s}^{-1}$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt} \longrightarrow \frac{50 - 25}{350 - 25} = e^{-(0.02732\text{ s}^{-1})t} \longrightarrow t = 93.9\text{ s}$$

Then the necessary length of the wire in the cooling section is determined to be

$$\text{Length} = \frac{t}{V} = \frac{(93.9/60)\text{ min}}{10\text{ m/min}} = \mathbf{0.157\ m}$$

11-117 ··· 11-120 Design and Essay Problems