Solutions Manual

for

Introduction to Thermodynamics and Heat Transfer

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Chapter 13

INTERNAL FORCED CONVECTION

General Flow Analysis

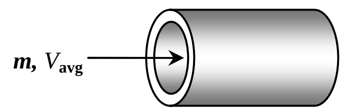
13-1C Liquids are usually transported in circular pipes because pipes with a circular cross-section can withstand large pressure differences between the inside and the outside without undergoing any distortion.

13-2C Reynolds number for flow in a circular tube of diameter D is expressed as

$$\mathrm{Re} = \frac{V_{\mathrm{avg}} D}{\nu} \quad \text{where} \quad V_\infty = \frac{\dot{m}}{\rho A_c} = \frac{\dot{m}}{\rho(\pi D^2 / 4)} = \frac{4\dot{m}}{\rho \pi D^2} \quad \text{and} \quad \nu = \frac{\mu}{\rho}$$

Substituting,

$$\mathrm{Re} = \frac{V_{\mathrm{avg}} D}{\nu} = \frac{4\dot{m} D}{\rho \pi D^2 (\mu / \rho)} = \frac{4\dot{m}}{\pi D \mu}$$



13-3C Engine oil requires a larger pump because of its much larger density.

13-4C The generally accepted value of the Reynolds number above which the flow in a smooth pipe is turbulent is 4000.

13-5C For flow through non-circular tubes, the Reynolds number as well as the Nusselt number and the friction factor are based on the hydraulic diameter D_h defined as $D_h = \frac{4A_c}{p}$ where A_c is the cross-sectional area of the tube and p is its perimeter. The hydraulic diameter is defined such that it reduces to ordinary diameter D for circular tubes since $D_h = \frac{4A_c}{p} = \frac{4\pi D^2 / 4}{\pi D} = D$.

13-6C The region from the tube inlet to the point at which the boundary layer merges at the centerline is called the *hydrodynamic entry region*, and the length of this region is called *hydrodynamic entry length*. The entry length is much longer in laminar flow than it is in turbulent flow. But at very low Reynolds numbers, L_h is very small ($L_h = 1.2D$ at Re = 20).

13-7C The friction factor is highest at the tube inlet where the thickness of the boundary layer is zero, and decreases gradually to the fully developed value. The same is true for turbulent flow.

13-8C In turbulent flow, the tubes with rough surfaces have much higher friction factors than the tubes with smooth surfaces. In the case of laminar flow, the effect of surface roughness on the friction factor is negligible.

13-9C The friction factor f remains constant along the flow direction in the fully developed region in both laminar and turbulent flow.

13-10C The fluid viscosity is responsible for the development of the velocity boundary layer. For the idealized inviscid fluids (fluids with zero viscosity), there will be no velocity boundary layer.

13-11C The number of transfer units NTU is a measure of the heat transfer area and effectiveness of a heat transfer system. A small value of NTU (NTU < 5) indicates more opportunities for heat transfer whereas a large NTU value (NTU >5) indicates that heat transfer will not increase no matter how much we extend the length of the tube.

13-12C The logarithmic mean temperature difference $\Delta T_{\ln}$ is an exact representation of the average temperature difference between the fluid and the surface for the entire tube. It truly reflects the exponential decay of the local temperature difference. The error in using the arithmetic mean temperature increases to undesirable levels when ΔT_e differs from ΔT_i by great amounts. Therefore we should always use the logarithmic mean temperature.

13-13C The region of flow over which the thermal boundary layer develops and reaches the tube center is called the thermal entry region, and the length of this region is called the thermal entry length. The region in which the flow is both hydrodynamically (the velocity profile is fully developed and remains unchanged) and thermally (the dimensionless temperature profile remains unchanged) developed is called the fully developed region.

13-14C The heat flux will be higher near the inlet because the heat transfer coefficient is highest at the tube inlet where the thickness of thermal boundary layer is zero, and decreases gradually to the fully developed value.

13-15C The heat flux will be higher near the inlet because the heat transfer coefficient is highest at the tube inlet where the thickness of thermal boundary layer is zero, and decreases gradually to the fully developed value.

13-16C In the fully developed region of flow in a circular tube, the velocity profile will not change in the flow direction but the temperature profile may.

13-17C The hydrodynamic and thermal entry lengths are given as $L_h = 0.05\,\text{Re}\,D$ and $L_t = 0.05\,\text{Re}\,\text{Pr}\,D$ for laminar flow, and $L_h \approx L_t \approx 10D$ in turbulent flow. Noting that $\text{Pr} >> 1$ for oils, the thermal entry length is larger than the hydrodynamic entry length in laminar flow. In turbulent, the hydrodynamic and thermal entry lengths are independent of Re or Pr numbers, and are comparable in magnitude.

13-18C The hydrodynamic and thermal entry lengths are given as $L_h = 0.05\,\text{Re}\,D$ and $L_t = 0.05\,\text{Re}\,\text{Pr}\,D$ for laminar flow, and $L_h \approx L_t \approx 10D$ in turbulent flow. Noting that $\text{Pr} << 1$ for liquid metals, the thermal entry length is smaller than the hydrodynamic entry length in laminar flow. In turbulent, the hydrodynamic and thermal entry lengths are independent of Re or Pr numbers, and are comparable in magnitude.

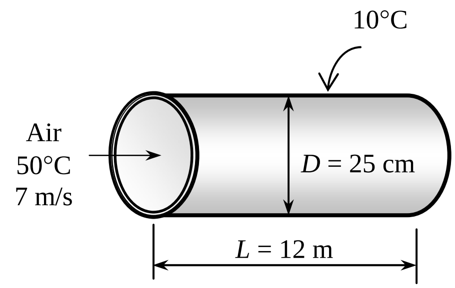
13-19C In fluid flow, it is convenient to work with an average or mean velocity V_{avg} and an average or mean temperature T_m which remain constant in incompressible flow when the cross-sectional area of the tube is constant. The V_{avg} and T_m represent the velocity and temperature, respectively, at a cross section if all the particles were at the same velocity and temperature.

13-20C When the surface temperature of tube is constant, the appropriate temperature difference for use in the Newton's law of cooling is logarithmic mean temperature difference that can be expressed as

$$\Delta T_{\ln} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$

13-21 Air flows inside a duct and it is cooled by water outside. The exit temperature of air and the rate of heat transfer are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the duct is constant. **3** The thermal resistance of the duct is negligible.



Properties The properties of air at the anticipated average temperature of 30°C are (Table A-22)

$$\rho = 1.164\ \text{kg/m}^3$$
$$c_p = 1007\ \text{J/kg.°C}$$

Analysis The mass flow rate of water is

$$\dot{m} = \rho A_c V_{\text{avg}} = \rho\left(\frac{\pi D^2}{4}\right)V_{\text{avg}} = (1.164\ \text{kg/m}^3)\frac{\pi(0.25\ \text{m})^2}{4}(7\ \text{m/s}) = 0.400\ \text{kg/s}$$

$$A_s = \pi DL = \pi(0.25\ \text{m})(12\ \text{m}) = 9.425\ \text{m}^2$$

The exit temperature of air is determined from

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}c_p)} = 10 - (10-50)e^{-\frac{(85)(9.425)}{(0.400)(1007)}} = \mathbf{15.47\ °C}$$

The logarithmic mean temperature difference and the rate of heat transfer are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{15.47-50}{\ln\left(\frac{10-15.47}{10-50}\right)} = 17.36°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (85\ \text{W/m}^2.°\text{C})(9.425\ \text{m}^2)(17.36°\text{C}) = 13{,}900\ \text{W} = \mathbf{13.9\ kW}$$

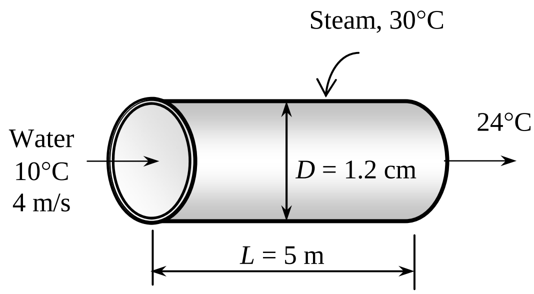
13-22 Steam is condensed by cooling water flowing inside copper tubes. The average heat transfer coefficient and the number of tubes needed are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is constant. **3** The thermal resistance of the pipe is negligible.

Properties The properties of water at the average temperature of (10+24)/2=17°C are (Table A-15)

$$\rho = 998.7\ \text{kg/m}^3$$
$$c_p = 4183.8\ \text{J/kg.°C}$$

Also, the heat of vaporization of water at 30°C is $h_{fg} = 2431\ \text{kJ/kg}$.



Analysis The mass flow rate of water and the surface area are

$$\dot{m} = \rho A_c V_{avg} = \rho\left(\frac{\pi D^2}{4}\right)V_{avg} = (998.7\ \text{kg/m}^3)\frac{\pi(0.012\ \text{m})^2}{4}(4\ \text{m/s}) = 0.4518\ \text{kg/s}$$

The rate of heat transfer for one tube is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.4518\ \text{kg/s})(4183.8\ \text{J/kg.°C})(24-10°\text{C}) = 26{,}460\ \text{W}$$

The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{24-10}{\ln\left(\frac{30-24}{30-10}\right)} = 11.63°\text{C}$$

$$A_s = \pi DL = \pi(0.012\ \text{m})(5\ \text{m}) = 0.1885\ \text{m}^2$$

The average heat transfer coefficient is determined from

$$\dot{Q} = hA_s\Delta T_{\ln} \longrightarrow h = \frac{\dot{Q}}{A_s\Delta T_{\ln}} = \frac{26{,}460\ \text{W}}{(0.1885\ \text{m}^2)(11.63°\text{C})}\left(\frac{1\ \text{kW}}{1000\ \text{W}}\right) = \mathbf{12.1\ kW/m^2.°C}$$

The total rate of heat transfer is determined from

$$\dot{Q}_{total} = \dot{m}_{cond}h_{fg} = (0.15\ \text{kg/s})(2431\ \text{kJ/kg}) = 364.65\ \text{kW}$$

Then the number of tubes becomes

$$N_{tube} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{364{,}650\ \text{W}}{26{,}460\ \text{W}} = \mathbf{13.8}$$

13-23 Steam is condensed by cooling water flowing inside copper tubes. The average heat transfer coefficient and the number of tubes needed are to be determined.

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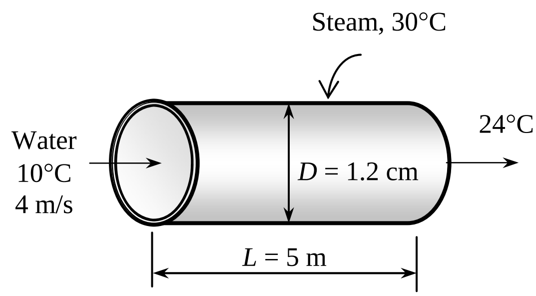

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The logarithmic mean temperature difference and the surface area are

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The total rate of heat transfer is determined from

$$\dot{Q}_{total} = \dot{m}_{cond} h_{fg} = (0.60 \text{ kg/s})(2431 \text{ kJ/kg}) = 1458.6 \text{ kW}$$

Then the number of tubes becomes

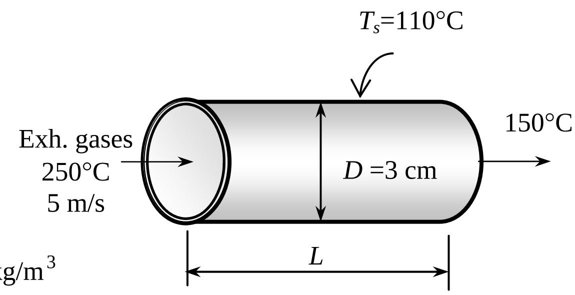
$$N_{tube} = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{1{,}458{,}600 \text{ W}}{26{,}460 \text{ W}} = \mathbf{55.1}$$

13-24 Combustion gases passing through a tube are used to vaporize waste water. The tube length and the rate of evaporation of water are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is constant. **3** The thermal resistance of the pipe is negligible. **4** Air properties are to be used for exhaust gases.

Properties The properties of air at the average temperature of (250+150)/2=200°C are (Table A-22)

$$c_p = 1023 \text{ J/kg.°C}$$
$$R = 0.287 \text{ kJ/kg.K}$$



Also, the heat of vaporization of water at 1 atm or 100°C is $h_{fg} = 2257 \text{ kJ/kg}$ (Table A-15).

Analysis The density of air at the inlet and the mass flow rate of exhaust gases are

$$\rho = \frac{P}{RT} = \frac{115 \text{ kPa}}{(0.287 \text{ kJ/kg.K})(250+273 \text{ K})} = 0.7662 \text{ kg/m}^3$$

$$\dot{m} = \rho A_c V_{\text{avg}} = \rho\left(\frac{\pi D^2}{4}\right)V_{\text{avg}} = (0.7662 \text{ kg/m}^3)\frac{\pi(0.03 \text{ m})^2}{4}(5 \text{ m/s}) = 0.002708 \text{ kg/s}$$

The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_i - T_e) = (0.002708 \text{ kg/s})(1023 \text{ J/kg.°C})(250-150\text{°C}) = 277.0 \text{ W}$$

The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{150-250}{\ln\left(\frac{110-150}{110-250}\right)} = 79.82\text{°C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\ln}} = \frac{277.0 \text{ W}}{(120 \text{ W/m}^2.\text{°C})(79.82\text{°C})} = 0.02891 \text{ m}^2$$

Then the tube length becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{0.02891 \text{ m}^2}{\pi(0.03 \text{ m})} = 0.3067 \text{ m} = \mathbf{30.7 \text{ cm}}$$

The rate of evaporation of water is determined from

$$\dot{Q} = \dot{m}_{evap}h_{fg} \longrightarrow \dot{m}_{evap} = \frac{\dot{Q}}{h_{fg}} = \frac{(0.2770 \text{ kW})}{(2257 \text{ kJ/kg})} = 0.0001227 \text{ kg/s} = \mathbf{0.442 \text{ kg/h}}$$

13-25 Combustion gases passing through a tube are used to vaporize waste water. The tube length and the rate of evaporation of water are to be determined.

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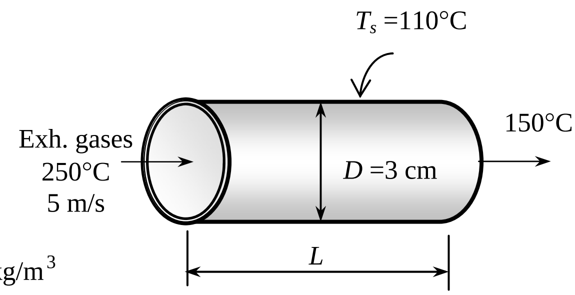

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$$\dot{m} = \rho A_c V_{\text{avg}} = \rho\left(\frac{\pi D^2}{4}\right)V_{\text{avg}} = (0.7662 \text{ kg/m}^3)\frac{\pi(0.03 \text{ m})^2}{4}(5 \text{ m/s}) = 0.002708 \text{ kg/s}$$

The rate of heat transfer is

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The logarithmic mean temperature difference and the surface area are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{150-250}{\ln\left(\frac{110-150}{110-250}\right)} = 79.82\text{°C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} \longrightarrow A_s = \frac{\dot{Q}}{h\Delta T_{\ln}} = \frac{277.0 \text{ W}}{(40 \text{ W/m}^2.\text{°C})(79.82\text{°C})} = 0.08673 \text{ m}^2$$

Then the tube length becomes

$$A_s = \pi DL \longrightarrow L = \frac{A_s}{\pi D} = \frac{0.08673 \text{ m}^2}{\pi(0.03 \text{ m})} = 0.920 \text{ m} = \mathbf{92.0 \text{ cm}}$$

The rate of evaporation of water is determined from

$$\dot{Q} = \dot{m}_{evap}h_{fg} \longrightarrow \dot{m}_{evap} = \frac{\dot{Q}}{h_{fg}} = \frac{(0.2770 \text{ kW})}{(2257 \text{ kJ/kg})} = 0.0001227 \text{ kg/s} = \mathbf{0.442 \text{ kg/h}}$$

Laminar and Turbulent Flow in Tubes

13-26C The friction factor for flow in a tube is proportional to the pressure drop. Since the pressure drop along the flow is directly related to the power requirements of the pump to maintain flow, the friction factor is also proportional to the power requirements. The applicable relations are

$$\Delta P = f\,\frac{L}{D}\frac{\rho V^2}{2} \quad \text{and} \quad \dot{W}_{\text{pump}} = \frac{\dot{m}\Delta P}{\rho}$$

13-27C The shear stress at the center of a circular tube during fully developed laminar flow is zero since the shear stress is proportional to the velocity gradient, which is zero at the tube center.

13-28C Yes, the shear stress at the surface of a tube during fully developed turbulent flow is maximum since the shear stress is proportional to the velocity gradient, which is maximum at the tube surface.

13-29C In fully developed flow in a circular pipe with negligible entrance effects, if the length of the pipe is doubled, the pressure drop will also *double* (the pressure drop is proportional to length).

13-30C Yes, the volume flow rate in a circular pipe with laminar flow can be determined by measuring the velocity at the centerline in the fully developed region, multiplying it by the cross-sectional area, and dividing the result by 2 since $\dot{V} = V_{\text{avg}} A_c = (V_{\max}/2) A_c$.

13-31C No, the average velocity in a circular pipe in fully developed laminar flow **cannot** be determined by simply measuring the velocity at $R/2$ (midway between the wall surface and the centerline). The mean velocity is $V_{\max}/2$, but the velocity at $R/2$ is

$$V(R/2) = V_{\max}\left(1 - \frac{r^2}{R^2}\right)_{r=R/2} = \frac{3V_{\max}}{4}$$

13-32C In fully developed laminar flow in a circular pipe, the pressure drop is given by

$$\Delta P = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2}$$

The mean velocity can be expressed in terms of the flow rate as $V_{\text{avg}} = \dfrac{\dot{V}}{A_c} = \dfrac{\dot{V}}{\pi D^2/4}$. Substituting,

$$\Delta P = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2} = \frac{32\mu L}{D^2}\frac{\dot{V}}{\pi D^2/4} = \frac{128\mu L \dot{V}}{\pi D^4}$$

Therefore, at constant flow rate and pipe length, the pressure drop is inversely proportional to the 4$^{\text{th}}$ power of diameter, and thus reducing the pipe diameter by half will increase the pressure drop **by a factor of 16** .

13-33C In fully developed laminar flow in a circular pipe, the pressure drop is given by

$$\Delta P = \frac{8\mu L V_{\text{avg}}}{R^2} = \frac{32\mu L V_{\text{avg}}}{D^2}$$

When the flow rate and thus mean velocity are held constant, the pressure drop becomes proportional to viscosity. Therefore, pressure drop will be **reduced by half** when the viscosity is reduced by half.

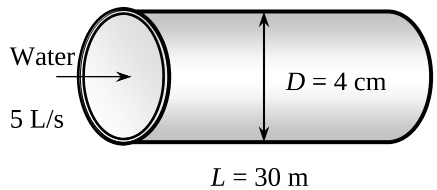
13-34C The tubes with rough surfaces have much higher heat transfer coefficients than the tubes with smooth surfaces. In the case of laminar flow, the effect of surface roughness on the heat transfer coefficient is negligible.

13-35 The flow rate through a specified water pipe is given. The pressure drop and the pumping power requirements are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water are given to be ρ = 999.1 kg/m^3 and μ = 1.138×10^{-3} kg/m·s, respectively. The roughness of stainless steel is 0.002 mm (Table 13-3).

Analysis First, we calculate the mean velocity and the Reynolds number to determine the flow regime:



$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.005\,\text{m}^3/s}{\pi(0.04\,\text{m})^2/4} = 3.98\,\text{m}/s$$

$$\text{Re} = \frac{\rho V_{avg} D}{\mu} = \frac{(999.1\,\text{kg/m}^3)(3.98\,\text{m/s})(0.04\,\text{m})}{1.138\times10^{-3}\,\text{kg/m}\cdot\text{s}} = 1.40\times10^5$$

which is greater than 10,000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{2\times10^{-6}\,\text{m}}{0.04\,\text{m}} = 5\times10^{-5}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \;\rightarrow\; \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{5\times10^{-5}}{3.7} + \frac{2.51}{1.40\times10^5\sqrt{f}}\right)$$

It gives f = 0.0171. Then the pressure drop and the required power input become

$$\Delta P = f\,\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.0171\frac{30\,\text{m}}{0.04\,\text{m}}\frac{(999.1\,\text{kg/m}^3)(3.98\,\text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right) = 101.5\ \text{kPa}$$

$$\dot{W}_{pump,u} = \dot{V}\Delta P = (0.005\,\text{m}^3/s)(101.5\,\text{kPa})\left(\frac{1\,\text{kW}}{1\,\text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.508\,kW}$$

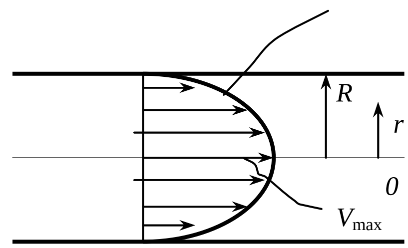
Therefore, useful power input in the amount of 0.508 kW is needed to overcome the frictional losses in the pipe.

Discussion The friction factor could also be determined easily from the explicit Haaland relation. It would give f = 0.0169, which is sufficiently close to 0.0171. Also, the friction factor corresponding to ε = 0 in this case is 0.0168, which indicates that stainless steel pipes can be assumed to be smooth with an error of about 2%. Also, the power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

13-36 In fully developed laminar flow in a circular pipe, the velocity at $r = R/2$ is measured. The velocity at the center of the pipe ($r = 0$) is to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by



$$V(r) = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$

where V_{max} is the maximum velocity which occurs at pipe center, $r = 0$. At $r = R/2$,

$$V(R/2) = V_{max}\left(1 - \frac{(R/2)^2}{R^2}\right) = V_{max}\left(1 - \frac{1}{4}\right) = \frac{3V_{max}}{4}$$

Solving for V_{max} and substituting,

$$V_{max} = \frac{4V(R/2)}{3} = \frac{4(6 \text{ m/s})}{3} = \mathbf{8\ m/s}$$

which is the velocity at the pipe center.

13-37 The velocity profile in fully developed laminar flow in a circular pipe is given. The mean and maximum velocities are to be determined.

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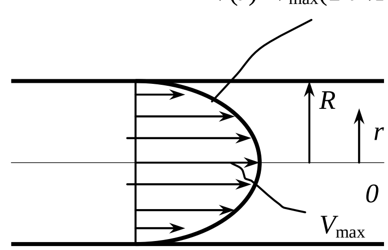
Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by



$$V(r) = V_{max}\left(1 - \frac{r^2}{R^2}\right)$$

The velocity profile in this case is given by

$$V(r) = 4(1 - r^2/R^2)$$

Comparing the two relations above gives the maximum velocity to be $V_{max} = 4$ m/s. Then the mean velocity and volume flow rate become

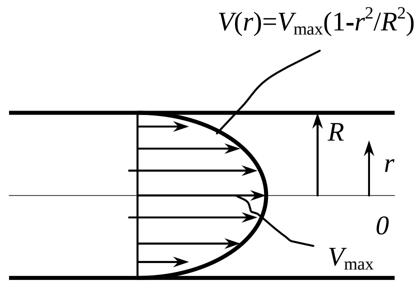
$$V_{avg} = \frac{V_{max}}{2} = \frac{4 \text{ m/s}}{2} = \mathbf{2\ m/s}$$

$$\dot{V} = V_{avg}A_c = V_{avg}(\pi R^2) = (2 \text{ m/s})[\pi(0.10 \text{ m})^2] = \mathbf{0.0628\ m^3/s}$$

13-38 The velocity profile in fully developed laminar flow in a circular pipe is given. The mean and maximum velocities are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is given by



$$V(r)=V_{max}\left(1-\frac{r^2}{R^2}\right)$$

The velocity profile in this case is given by

$$V(r)=4(1-r^2/R^2)$$

Comparing the two relations above gives the maximum velocity to be V_{max} = 4 m/s. Then the mean velocity and volume flow rate become

$$V_{avg}=\frac{V_{max}}{2}=\frac{4\text{ m/s}}{2}=\mathbf{2\ m/s}$$

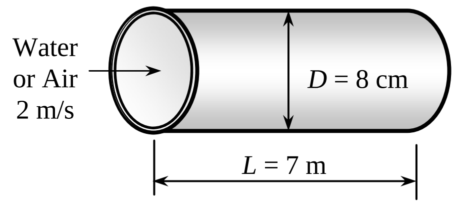
$$\dot{V}=V_{avg}A_c=V_{avg}(\pi R^2)=(2\text{ m/s})[\pi(0.05\text{ m})^2]=\mathbf{0.0157\ m^3/s}$$

13-39 The convection heat transfer coefficients for the flow of air and water are to be determined under similar conditions.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth.

Properties The properties of air at 25°C are (Table A-22)

$$k = 0.02551 \text{ W/m.°C}$$
$$\nu = 1.562\times10^{-5} \text{ m}^2\text{/s}$$
$$\text{Pr} = 0.7296$$



The properties of water at 25°C are (Table A-15)

$$\rho = 997 \text{ kg/m}^3$$
$$k = 0.607 \text{ W/m.°C}$$
$$\nu = \mu/\rho = 0.891\times10^{-3}/997 = 8.937\times10^{-7} \text{ m}^2\text{/s}$$
$$\text{Pr} = 6.14$$

Analysis The Reynolds number is

$$\text{Re} = \frac{VD}{\nu} = \frac{(2 \text{ m/s})(0.08 \text{ m})}{1.562\times10^{-5} \text{ m}^2\text{/s}} = 10{,}243$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.08 \text{ m}) = 0.8 \text{ m}$$

which is much shorter than the total length of the tube. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(10{,}243)^{0.8}(0.7296)^{0.4} = 32.76$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02551 \text{ W/m.°C}}{0.08 \text{ m}}(32.76) = \mathbf{10.45\ W/m^2.°C}$$

Repeating calculations for water:

$$\text{Re} = \frac{VD}{\nu} = \frac{(2 \text{ m/s})(0.08 \text{ m})}{8.937\times10^{-7} \text{ m}^2\text{/s}} = 179{,}035$$

$$Nu = \frac{hD}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(179{,}035)^{0.8}(6.14)^{0.4} = 757.4$$

$$h = \frac{k}{D}Nu = \frac{0.607 \text{ W/m.°C}}{0.08 \text{ m}}(757.4) = \mathbf{5747\ W/m^2.°C}$$

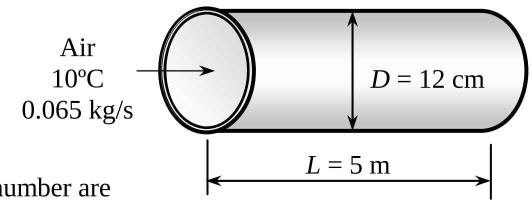
Discussion The heat transfer coefficient for water is 550 times that of air.

13-40 Air flows in a pipe whose inner surface is not smooth. The rate of heat transfer is to be determined using two different Nusselt number relations.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The pressure of air is 1 atm.

Properties Assuming a bulk mean fluid temperature of 20°C, the properties of air are (Table A-22)

$$\rho = 1.204\ \text{kg/m}^3$$
$$k = 0.02514\ \text{W/m.°C}$$
$$\nu = 1.516\times10^{-5}\ \text{m}^2/\text{s}$$
$$c_p = 1007\ \text{J/kg.°C}$$
$$\text{Pr} = 0.7309$$



Analysis The mean velocity of air and the Reynolds number are

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{0.065\ \text{kg/s}}{(1.204\ \text{kg/m}^3)\pi(0.12\ \text{m})^2/4} = 4.773\ \text{m/s}$$

$$\text{Re} = \frac{V_{\text{avg}}D}{\nu} = \frac{(4.773\ \text{m/s})(0.12\ \text{m})}{1.516\times10^{-5}\ \text{m}^2/\text{s}} = 37{,}785$$

which is greater than 10,0000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.12\ \text{m}) = 1.2\ \text{m}$$

which is much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire duct. The friction factor may be determined from Colebrook equation using EES to be

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \longrightarrow \frac{1}{\sqrt{f}} = -2\log\left(\frac{0.00022/0.12}{3.7} + \frac{2.51}{37{,}785\sqrt{f}}\right) \longrightarrow f = 0.02695$$

The Nusselt number from Eq. 13-66 is

$$Nu = 0.125 f\ \text{Re}\,\text{Pr}^{1/3} = 0.125(0.02695)(37{,}785)(0.7309)^{1/3} = 114.7$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.02514\ \text{W/m.°C}}{0.12\ \text{m}}(114.7) = 24.02\ \text{W/m}^2.\text{°C}$$

Next we determine the exit temperature of air

$$A = \pi DL = \pi(0.12\ \text{m})(5\ \text{m}) = 1.885\ \text{m}^2$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 50 - (50-10)e^{-\frac{(24.02)(1.885)}{(0.065)(1007)}} = 30.0\text{°C}$$

This result verifies our assumption of bulk mean fluid temperature that we used for property evaluation. Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.065\ \text{kg/s})(1007\ \text{J/kg.°C})(30.0-10)\text{°C} = \mathbf{1307\ W}$$

Repeating the calculations using the Nusselt number from Eq. 13-70:

$$Nu = \frac{(f/8)(\text{Re}-1000)\,\text{Pr}}{1+12.7(f/8)^{0.5}(\text{Pr}^{2/3}-1)} = \frac{(0.02695/8)(37{,}785-1000)(0.7309)}{1+12.7(0.02695/8)^{0.5}(0.7309^{2/3}-1)} = 105.2$$

$$h = \frac{k}{D}Nu = \frac{0.02514\ \text{W/m.°C}}{0.12\ \text{m}}(105.2) = 22.04\ \text{W/m}^2.\text{°C}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 50 - (50-10)e^{-\frac{(22.04)(1.885)}{(0.065)(1007)}} = 28.8\text{°C}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.065\ \text{kg/s})(1007\ \text{J/kg.°C})(28.8-10)\text{°C} = \mathbf{1230\ W}$$

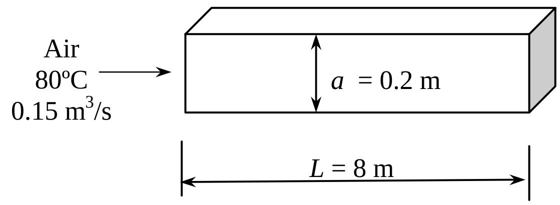
The result by Eq. 13-66 is about 6 percent greater than that by Eq. 13-70.

13-41 Air flows in a square cross section pipe. The rate of heat loss and the pressure difference between the inlet and outlet sections of the duct are to be determined.

Assumptions **1** Steady operating conditions exist. **2** Air is an ideal gas with constant properties. **3** The pressure of air is 1 atm.

Properties Taking a bulk mean fluid temperature of 80°C assuming that the air does not loose much heat to the attic, the properties of air are (Table A-22)

$$\rho = 0.9994 \text{ kg/m}^3$$
$$k = 0.02953 \text{ W/m.°C}$$
$$\nu = 2.097\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1008 \text{ J/kg.°C}$$
$$\Pr = 0.7154$$



Analysis The mean velocity of air, the hydraulic diameter, and the Reynolds number are

$$V = \frac{\dot{V}}{A} = \frac{0.15 \text{ m}^3\text{/s}}{(0.2 \text{ m})^2} = 3.75 \text{ m/s}$$

$$D_h = \frac{4A}{P} = \frac{4a^2}{4a} = a = 0.2 \text{ m}$$

$$\text{Re} = \frac{VD_h}{\nu} = \frac{(3.75 \text{ m/s})(0.2 \text{ m})}{2.097\times10^{-5}} = 35{,}765$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.2 \text{ m}) = 2 \text{ m}$$

which is much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\Pr^{0.3} = 0.023(35{,}765)^{0.8}(0.7154)^{0.3} = 91.4$$

Heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.02953 \text{ W/m.°C}}{0.2 \text{ m}}(91.4) = 13.5 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of air

$$A = 4aL = 4(0.2 \text{ m})(8 \text{ m}) = 6.4 \text{ m}^2$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 60 - (60-80)e^{-\frac{(13.5)(6.4)}{(0.9994)(0.15)(1008)}} = 71.3\text{°C}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.9994 \text{ kg/m}^3)(0.15 \text{ m}^3\text{/s})(1008 \text{ J/kg.°C})(80-71.3)\text{°C} = \mathbf{1315 \text{ W}}$$

From Moody chart:

Re = 35,765 and $\varepsilon/D = 0.001 \rightarrow f = 0.026$

Then the pressure drop is determined to be

$$\Delta P = f\frac{\rho V^2}{2D}L = (0.026)\frac{(0.9994 \text{ kg/m}^3)(3.75 \text{ m/s})^2}{2(0.2 \text{ m})}(8 \text{ m}) = \mathbf{7.3 \text{ Pa}}$$

13-42 A liquid is heated as it flows in a pipe that is wrapped by electric resistance heaters. The required surface heat flux, the surface temperature at the exit, and the pressure loss through the pipe and the minimum power required to overcome the resistance to flow are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth. **4** Heat transfer to the surroundings is negligible.

Properties The properties of the fluid are given to be ρ = 1000 kg/m^3, c_p = 4000 J/kg·K, μ = 2x10^{-3} kg/s·m, k = 0.48 W/m·K, and Pr = 10

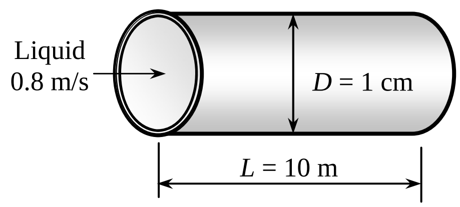
Analysis (a) The mass flow rate of the liquid is

$$\dot{m} = \rho AV = (1000\ \text{kg/m}^3)\left(\pi(0.01\ \text{m})^2/4\right)(0.8\ \text{m/s}) = 0.0628\ \text{kg/s}$$

The rate of heat transfer and the heat flux are

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.0628\ \text{kg/s})(4000\ \text{J/kg.°C})(75-25)\text{°C} = 12{,}560\ \text{W}$$

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{12{,}560\ \text{W}}{\pi(0.01\ \text{m})(10\ \text{m})} = \mathbf{40{,}000\ W/m^2}$$



(b) The Reynolds number is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(1000\ \text{kg/m}^3)(0.8\ \text{m/s})(0.01\ \text{m})}{0.002\ \text{kg/m}\cdot\text{s}} = 4000$$

which is greater than 2300 and smaller than 10,000. Therefore, we have transitional flow. However, we use turbulent flow relation. The entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.01\ \text{m}) = 0.1\ \text{m}$$

which is much shorter than the total length of the tube. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(4000)^{0.8}(10)^{0.4} = 44$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.48\ \text{W/m.°C}}{0.01\ \text{m}}(44) = 2112\ \text{W/m}^2\text{.°C}$$

The surface temperature at the exit is

$$\dot{q} = h(T_s - T_e) \longrightarrow 40{,}000\ \text{W} = (2112\ \text{W/m}^2\cdot\text{°C})(T_s - 75)\text{°C} \longrightarrow T_s = \mathbf{93.9°C}$$

(c) From Moody chart:

Re = 4000, ε = 0.046 mm, ε/D = 0.045/10 = 0.0045 → f = 0.044

Then the pressure drop and the minimum power required to overcome this pressure drop are determined to be

$$\Delta P = f\frac{\rho V^2}{2D}L = (0.044)\frac{(1000\ \text{kg/m}^3)(0.8\ \text{m/s})^2}{2(0.01\ \text{m})}(10\ \text{m}) = \mathbf{14{,}080\ Pa}$$

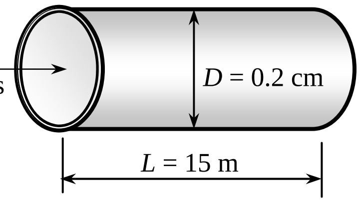
$$\dot{W} = \dot{V}\Delta P = \left(\pi(0.01\ \text{m})^2/4\right)(0.8\ \text{m/s})(14{,}080\ \text{Pa}) = \mathbf{0.88\ W}$$

13-43 The average flow velocity in a pipe is given. The pressure drop and the pumping power are to be determined.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The pipe involves no components such as bends, valves, and connectors. **4** The piping section involves no work devices such as pumps and turbines.

Properties The density and dynamic viscosity of water are given to be ρ = 999.7 kg/m^3 and μ = 1.307×10^{-3} kg/m·s, respectively.

Analysis (*a*) First we need to determine the flow regime. The Reynolds number of the flow is



$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{(999.7\ \text{kg/m}^3)(1.2\ \text{m/s})(2\times10^{-3}\ \text{m})}{1.307\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 1836$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the pressure drop become

$$f = \frac{64}{\text{Re}} = \frac{64}{1836} = 0.0349$$

$$\Delta P = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.0349\frac{15\ \text{m}}{0.002\ \text{m}}\frac{(999.7\ \text{kg/m}^3)(1.2\ \text{m/s})^2}{2}\left(\frac{1\,\text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\,\text{kPa}}{1\,\text{kN/m}^2}\right) = \mathbf{188\ kPa}$$

(*b*) The volume flow rate and the pumping power requirements are

$$\dot{V} = V_{\text{avg}} A_c = V_{\text{avg}}(\pi D^2/4) = (1.2\ \text{m/s})[\pi(0.002\ \text{m})^2/4] = 3.77\times10^{-6}\ \text{m}^3/s$$

$$\dot{W}_{\text{pump}} = \dot{V}\Delta P = (3.77\times10^{-6}\ \text{m}^3/s)(188\ \text{kPa})\left(\frac{1000\ \text{W}}{1\,\text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.71\ W}$$

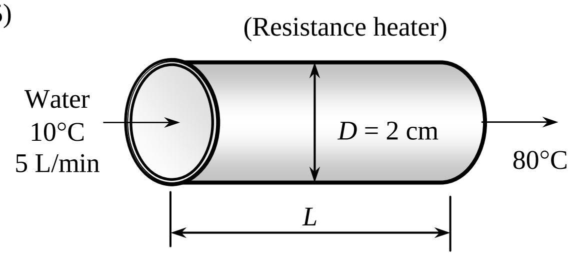
Therefore, power input in the amount of 0.71 W is needed to overcome the frictional losses in the flow due to viscosity.

13-44 Water is to be heated in a tube equipped with an electric resistance heater on its surface. The power rating of the heater and the inner surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface heat flux is uniform. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the average temperature of (80+10) / 2 = 45°C are (Table A-15)

$$\rho = 990.1\ \text{kg/m}^3$$
$$k = 0.637\ \text{W/m.°C}$$
$$\nu = \mu/\rho = 0.602\times10^{-6}\ \text{m}^2/\text{s}$$
$$c_p = 4180\ \text{J/kg.°C}$$
$$\text{Pr} = 3.91$$



Analysis The power rating of the resistance heater is

$$\dot{m} = \rho\dot{V} = (990.1\ \text{kg/m}^3)(0.005\ \text{m}^3/\text{min}) = 4.951\ \text{kg/min} = 0.0825\ \text{kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.0825\ \text{kg/s})(4180\ \text{J/kg.°C})(80-10)\text{°C} = \mathbf{24{,}140\ W}$$

The velocity of water and the Reynolds number are

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{(5\times10^{-3}/60)\ \text{m}^3/\text{s}}{\pi(0.02\ \text{m})^2/4} = 0.2653\ \text{m/s}$$

$$\text{Re} = \frac{V_{\text{avg}}D_h}{\nu} = \frac{(0.2653\ \text{m/s})(0.02\ \text{m})}{0.602\times10^{-6}\ \text{m}^2/\text{s}} = 8813$$

which is less than 10,000 but much greater than 2300. We assume the flow to be turbulent. The entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.02\ \text{m}) = 0.20\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(8813)^{0.8}(3.91)^{0.4} = 56.85$$

Heat transfer coefficient is

$$h = \frac{k}{D_h}Nu = \frac{0.637\ \text{W/m.°C}}{0.02\ \text{m}}(56.85) = 1811\ \text{W/m}^2.\text{°C}$$

Then the inner surface temperature of the pipe at the exit becomes

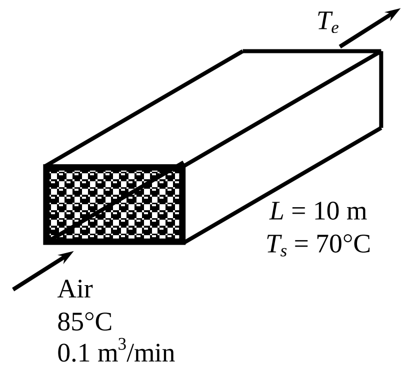
$$\dot{Q} = hA_s(T_{s,e} - T_e)$$
$$24{,}140\ \text{W} = (1811\ \text{W/m}^2.\text{°C})[\pi(0.02\ \text{m})(13\ \text{m})](T_s - 80)\text{°C}$$
$$T_{s,e} = \mathbf{96.3°C}$$

13-45 Flow of hot air through uninsulated square ducts of a heating system in the attic is considered. The exit temperature and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 80°C since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at 1 atm and this temperature are (Table A-22)

$$\rho = 0.9994 \text{ kg/m}^3$$
$$k = 0.02953 \text{ W/m.°C}$$
$$\nu = 2.097\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1008 \text{ J/kg.°C}$$
$$\Pr = 0.7154$$



Analysis The characteristic length that is the hydraulic diameter, the mean velocity of air, and the Reynolds number are

$$D_h = \frac{4A_c}{P} = \frac{4a^2}{4a} = a = 0.15 \text{ m}$$

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{0.10 \text{ m}^3\text{/s}}{(0.15 \text{ m})^2} = 4.444 \text{ m/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(4.444 \text{ m/s})(0.15 \text{ m})}{2.097\times10^{-5} \text{ m}^2\text{/s}} = 31{,}791$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.15 \text{ m}) = 1.5 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\Pr^{0.3} = 0.023(31{,}791)^{0.8}(0.7154)^{0.3} = 83.16$$

Heat transfer coefficient is

$$h = \frac{k}{D_h} Nu = \frac{0.02953 \text{ W/m.°C}}{0.15 \text{ m}}(83.16) = 16.37 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of air,

$$A_s = 4aL = 4(0.15 \text{ m})(10 \text{ m}) = 6 \text{ m}^2$$
$$\dot{m} = \rho\dot{V} = (0.9994 \text{ kg/m}^3)(0.10 \text{ m}^3\text{/s}) = 0.09994 \text{ kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 70 - (70-85)e^{-\frac{(16.37)(6)}{(0.09994)(1008)}} = \mathbf{75.7°C}$$

Then the logarithmic mean temperature difference and the rate of heat loss from the air becomes

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{75.7-85}{\ln\left(\frac{70-75.7}{70-85}\right)} = 9.58°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (16.37 \text{ W/m}^2\text{.°C})(6 \text{ m}^2)(9.58°\text{C}) = \mathbf{941\ W}$$

Note that the temperature of air drops by almost 10°C as it flows in the duct as a result of heat loss.

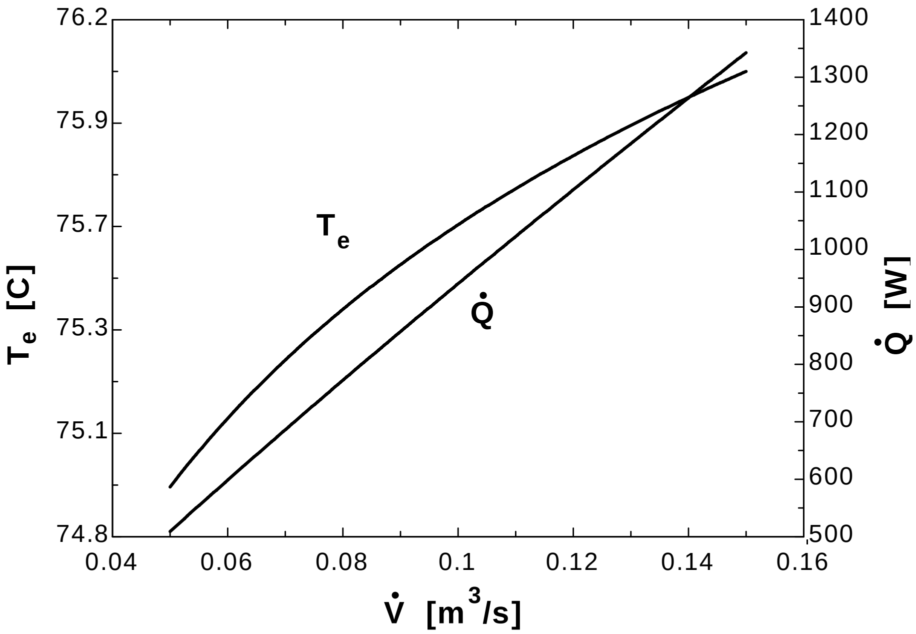
13-46 EES Prob. 13-45 is reconsidered. The effect of the volume flow rate of air on the exit temperature of air and the rate of heat loss is to be investigated.

Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=85 [C]
L=10 [m]
side=0.15 [m]
V_dot=0.10 [m^3/s]
T_s=70 [C]

"PROPERTIES"
Fluid$='air'
C_p=CP(Fluid$, T=T_ave)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
T_ave=1/2*(T_i+T_e)

"ANALYSIS"
D_h=(4*A_c)/p
A_c=side^2
p=4*side
Vel=V_dot/A_c
Re=(Vel*D_h)/nu "The flow is turbulent"
L_t=10*D_h "The entry length is much shorter than the total length of the duct."
Nusselt=0.023*Re^0.8*Pr^0.3
h=k/D_h*Nusselt
A=4*side*L
m_dot=rho*V_dot
T_e=T_s-(T_s-T_i)*exp((-h*A)/(m_dot*C_p))
DELTAT_ln=(T_e-T_i)/ln((T_s-T_e)/(T_s-T_i))
Q_dot=h*A*DELTAT_ln
```

V [m^3/s]	T_e [C]	Q [W]
0.05	74.89	509
0.055	75	554.1
0.06	75.09	598.6
0.065	75.18	642.7
0.07	75.26	686.3
0.075	75.34	729.5
0.08	75.41	772.4
0.085	75.48	814.8
0.09	75.54	857
0.095	75.6	898.9
0.1	75.66	940.4
0.105	75.71	981.7
0.11	75.76	1023
0.115	75.81	1063
0.12	75.86	1104

0.125	75.9	1144
0.13	75.94	1184
0.135	75.98	1224
0.14	76.02	1264
0.145	76.06	1303
0.15	76.1	1343



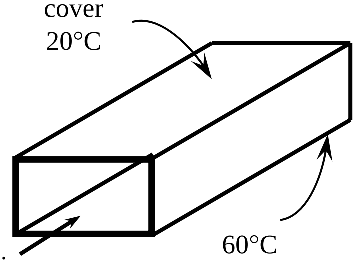
13-47 Air enters the constant spacing between the glass cover and the plate of a solar collector. The net rate of heat transfer and the temperature rise of air are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the spacing are smooth. **3** Air is an ideal gas with constant properties. **4** The local atmospheric pressure is 1 atm.

Properties The properties of air at 1 atm and estimated average temperature of 35°C are (Table A-22)

$$\rho = 1.145 \text{kg/m}^3, \quad k = 0.02625 \text{ W/m.°C}$$

$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}, \quad c_p = 1007 \text{ J/kg.°C}, \quad \Pr = 0.7268$$



Analysis Mass flow rate, cross sectional area, hydraulic diameter, mean velocity of air and the Reynolds number are

$$\dot{m} = \rho\dot{V} = (1.145 \text{ kg/m}^3)(0.15 \text{ m}^3\text{/s}) = 0.1718 \text{ kg/s}$$

$$A_c = (1\text{ m})(0.03\text{ m}) = 0.03\text{ m}^2$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.03\text{ m}^2)}{2(1\text{ m}+0.03\text{ m})} = 0.05825\text{ m}$$

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{0.15\text{ m}^3\text{/s}}{0.03\text{ m}^2} = 5\text{ m/s}$$

$$\text{Re} = \frac{V_{\text{avg}}D_h}{\nu} = \frac{(5\text{ m/s})(0.05825\text{ m})}{1.655\times10^{-5}\text{ m}^2\text{/s}} = 17{,}600$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.05825\text{ m}) = 0.5825\text{ m}$$

which are much shorter than the total length of the collector. Therefore, we can assume fully developed turbulent flow in the entire collector, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(17{,}600)^{0.8}(0.7268)^{0.4} = 50.43$$

and

$$h = \frac{k}{D_h}Nu = \frac{0.02625\text{ W/m.°C}}{0.05825\text{ m}}(50.43) = 22.73\text{ W/m}^2\text{.°C}$$

The exit temperature of air can be calculated using the "average" surface temperature as

$$A_s = 2(5\text{ m})(1\text{ m}) = 10\text{ m}^2, \qquad T_{s,avg} = \frac{60+20}{2} = 40°\text{C}$$

$$T_e = T_{s,avg} - (T_{s,avg} - T_i)\exp\left(-\frac{hA_s}{\dot{m}c_p}\right) = 40-(40-30)\exp\left(-\frac{22.73\times10}{0.1718\times1007}\right) = 37.31°\text{C}$$

The temperature rise of air is

$$\Delta T = 37.3°\text{C} - 30°\text{C} = \mathbf{7.3°C}$$

The logarithmic mean temperature difference and the heat loss from the glass are

$$\Delta T_{\text{ln},glass} = \frac{T_e - T_i}{\ln\frac{T_s - T_e}{T_s - T_i}} = \frac{37.31-30}{\ln\frac{20-37.31}{20-30}} = 13.32°\text{C}$$

$$\dot{Q}_{glass} = hA_s\Delta T_{\text{ln}} = (22.73\text{ W/m}^2\text{.°C})(5\text{ m}^2)(13.32°\text{C}) = 1514\text{ W}$$

The logarithmic mean temperature difference and the heat gain of the absorber are

$$\Delta T_{\text{ln},absorber} = \frac{T_e - T_i}{\ln\frac{T_s - T_e}{T_s - T_i}} = \frac{37.31-30}{\ln\frac{60-37.31}{60-30}} = 26.17°\text{C}$$

$$\dot{Q}_{absorber} = hA\Delta T_{\text{ln}} = (22.73\text{ W/m}^2\text{.°C})(5\text{ m}^2)(26.17°\text{C}) = 2975\text{ W}$$

Then the net rate of heat transfer becomes

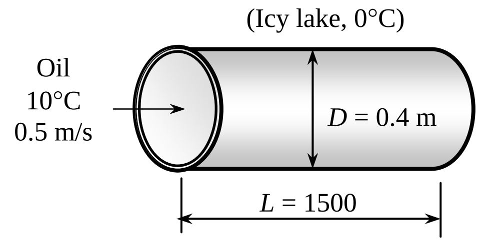
$$\dot{Q}_{net} = 2975 - 1514 = \mathbf{1461\,W}$$

13-48 Oil flows through a pipeline that passes through icy waters of a lake. The exit temperature of the oil and the rate of heat loss are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The surface temperature of the pipe is very nearly 0°C. **3** The thermal resistance of the pipe is negligible. **4** The inner surfaces of the pipeline are smooth. **5** The flow is hydrodynamically developed when the pipeline reaches the lake.

Properties The properties of oil at 10°C are (Table A-19)

$$\rho = 893.6\ \text{kg/m}^3, \qquad k = 0.1460\ \text{W/m.°C}$$
$$\mu = 2.326\ \text{kg/m.s}, \qquad \nu = 2.592\times10^{-3}\ \text{m}^2\text{/s}$$
$$c_p = 1839\ \text{J/kg.°C}, \qquad \text{Pr} = 28{,}750$$



Analysis (a) The Reynolds number in this case is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(0.5\ \text{m/s})(0.4\ \text{m})}{2.592\times10^{-3}\ \text{m}^2\text{/s}} = 77.16$$

which is less than 2300. Therefore, the flow is laminar, and the thermal entry length is roughly

$$L_t = 0.05\,\text{Re}\,\text{Pr}\,D = 0.05(77.16)(28{,}750)(0.4\ \text{m}) = 44{,}367\ \text{m}$$

which is much longer than the total length of the pipe. Therefore, we assume thermally developing flow, and determine the Nusselt number from

$$Nu = \frac{hD}{k} = 3.66 + \frac{0.065(D/L)\,\text{Re}\,\text{Pr}}{1+0.04\left[(D/L)\,\text{Re}\,\text{Pr}\right]^{2/3}} = 3.66 + \frac{0.065\left(\frac{0.4\ \text{m}}{1500\ \text{m}}\right)(77.16)(28{,}750)}{1+0.04\left[\left(\frac{0.4\ \text{m}}{1500\ \text{m}}\right)(77.16)(28{,}750)\right]^{2/3}} = 13.73$$

and $$h = \frac{k}{D}Nu = \frac{0.1460\ \text{W/m.°C}}{0.4\ \text{m}}(13.73) = 5.011\ \text{W/m}^2\text{.°C}$$

Next we determine the exit temperature of oil

$$A_s = \pi DL = \pi(0.4\ \text{m})(1500\ \text{m}) = 1885\ \text{m}^2$$

$$\dot{m} = \rho\dot{V} = \rho A_c V_{\text{avg}} = \rho\left(\frac{\pi D^2}{4}\right)V_{\text{avg}} = (893.6\ \text{kg/m}^3)\frac{\pi(0.4\ \text{m})^2}{4}(0.5\ \text{m/s}) = 56.15\ \text{kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}c_p)} = 0-(0-10)e^{-\frac{(5.011)(1885)}{(56.15)(1839)}} = \mathbf{9.13\ °C}$$

(*b*) The logarithmic mean temperature difference and the rate of heat loss from the oil are

$$\Delta T_{\text{ln}} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{9.13-10}{\ln\left(\frac{0-9.13}{0-10}\right)} = 9.56°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\text{ln}} = (5.011\ \text{W/m}^2\text{.°C})(1885\ \text{m}^2)(9.56°\text{C}) = 90{,}300\ \text{W} = \mathbf{90.3\ kW}$$

The friction factor is

$$f = \frac{64}{\text{Re}} = \frac{64}{77.16} = 0.8294$$

Then the pressure drop in the pipe and the required pumping power become

$$\Delta P = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.8294\frac{1500\ \text{m}}{0.4\ \text{m}}\frac{(893.6\ \text{kg/m}^3)(0.5\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = 347.4\ \text{kPa}$$

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = A_c V_{avg}\Delta P = \frac{\pi(0.4\ \text{m})^2}{4}(0.5\ \text{m/s})(347.4\ \text{kPa})\left(\frac{1\ \text{kW}}{1\ \text{kPa}\cdot\text{m}^3\text{/s}}\right) = \mathbf{21.8\ kW}$$

Discussion The power input determined is the mechanical power that needs to be imparted to the fluid. The shaft power will be much more than this due to pump inefficiency; the electrical power input will be even more due to motor inefficiency.

13-49 Laminar flow of a fluid through an isothermal square channel is considered. The change in the pressure drop and the rate of heat transfer are to be determined when the mean velocity is doubled.

Assumptions **1** The flow is fully developed. **2** The effect of the change in ΔT_{ln} on the rate of heat transfer is not considered.

Analysis The pressure drop of the fluid for laminar flow is expressed as

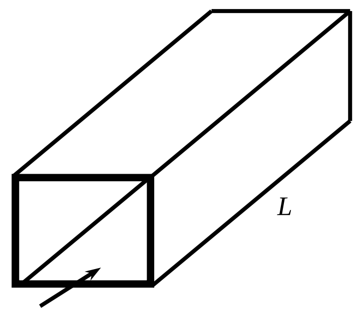
$$\Delta P_1 = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} = \frac{64}{\text{Re}}\frac{L}{D}\frac{\rho V_{avg}^2}{2} = \frac{64\nu}{V_{avg}D}\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 32V_{avg}\frac{\nu L\rho}{D^2}$$

When the free-stream velocity of the fluid is doubled, the pressure drop becomes

$$\Delta P_2 = f\frac{L}{D}\frac{\rho(2V_{avg})^2}{2} = \frac{64}{\text{Re}}\frac{L}{D}\frac{\rho 4V_{avg}^2}{2} = \frac{64\nu}{2V_{avg}D}\frac{L}{D}\frac{\rho 4V_{avg}^2}{2} = 64V_{avg}\frac{\nu L\rho}{D^2}$$

Their ratio is

$$\frac{\Delta P_2}{\Delta P_1} = \frac{64}{32} = \mathbf{2}$$



The rate of heat transfer between the fluid and the walls of the channel is expressed as

$$\dot{Q}_1 = hA_s\Delta T_{ln} = \frac{k}{D}NuA_s\Delta T_{ln} = \frac{k}{D}2.98A_s\Delta T_{ln}$$

When the effect of the change in ΔT_{ln} on the rate of heat transfer is disregarded, the rate of heat transfer remains the same. Therefore,

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \mathbf{1}$$

Therefore, doubling the velocity will double the pressure drop but it will not affect the heat transfer rate.

13-50 Turbulent flow of a fluid through an isothermal square channel is considered. The change in the pressure drop and the rate of heat transfer are to be determined when the free-stream velocity is doubled.

Assumptions **1** The flow is fully developed. **2** The effect of the change in ΔT_{ln} on the rate of heat transfer is not considered.

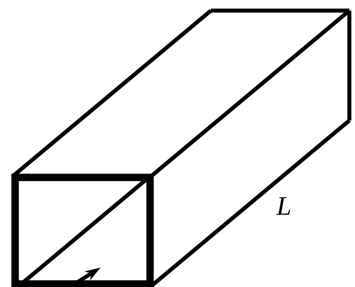
Analysis The pressure drop of the fluid for turbulent flow is expressed as

$$\Delta P_1 = f\,\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.184\,\text{Re}^{-0.2}\,\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.184\,\frac{V_{avg}^{-0.2}D^{-0.2}}{\nu^{-0.2}}\,\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.092V_{avg}^{1.8}\left(\frac{D}{\nu}\right)^{-0.2}\frac{L\rho}{D}$$

When the free-stream velocity of the fluid is doubled, the pressure drop becomes

$$\Delta P_2 = f\,\frac{L}{D}\frac{\rho(2V_{avg})^2}{2} = 0.184\,\text{Re}^{-0.2}\,\frac{L}{D}\frac{\rho 4V_{avg}^2}{2} = 0.184\,\frac{(2V_{avg})^{-0.2}D^{-0.2}}{\nu^{-0.2}}\,\frac{L}{D}\frac{\rho 4V_{avg}^2}{2}$$

$$= 0.368(2)^{-0.2}V_{avg}^{1.8}\left(\frac{D}{\nu}\right)^{-0.2}\frac{L\rho}{D}$$



Their ratio is

$$\frac{\Delta P_2}{\Delta P_1} = \frac{0.368(2)^{-0.2}V_{avg}^{1.8}}{0.092V_{avg}^{1.8}} = 4(2)^{-0.2} = \mathbf{3.48}$$

The rate of heat transfer between the fluid and the walls of the channel is expressed as

$$\dot{Q}_1 = hA\Delta T_{ln} = \frac{k}{D}NuA\Delta T_{ln} = \frac{k}{D}0.023\,\text{Re}^{0.8}\,\text{Pr}^{1/3}\,A\Delta T_{ln}$$

$$= 0.023V_{avg}^{0.8}\left(\frac{D}{\nu}\right)^{0.8}\frac{k}{D}\text{Pr}^{1/3}\,A\Delta T_{ln}$$

When the free-stream velocity of the fluid is doubled, the heat transfer rate becomes

$$\dot{Q}_2 = 0.023(2V_{avg})^{0.8}\left(\frac{D}{\nu}\right)^{0.8}\frac{k}{D}\text{Pr}^{1/3}\,A\Delta T_{ln}$$

Their ratio is

$$\frac{\dot{Q}_2}{\dot{Q}_1} = \frac{(2V_{avg})^{0.8}}{V_{avg}^{0.8}} = 2^{0.8} = \mathbf{1.74}$$

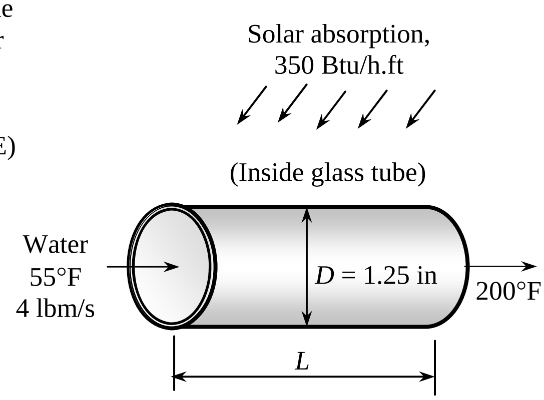
Therefore, doubling the velocity will increase the pressure drop 3.8 times but it will increase the heat transfer rate by only 74%.

13-51E Water is heated in a parabolic solar collector. The required length of parabolic collector and the surface temperature of the collector tube are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The thermal resistance of the tube is negligible. **3** The inner surfaces of the tube are smooth.

Properties The properties of water at the average temperature of (55+200)/2 = 127.5°F are (Table A-15E)

$$\rho = 61.59 \text{ lbm/ft}^3$$
$$k = 0.374 \text{ Btu/ft.°F}$$
$$\nu = \mu/\rho = 0.5681\times10^{-5} \text{ ft}^2\text{/s}$$
$$c_p = 0.999 \text{Btu/lbm.°F}$$
$$\Pr = 3.368$$



Analysis The total rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (4 \text{ lbm/s})(0.999 \text{ Btu/lbm.°F})(200-55)\text{°F} = 579.4 \text{ Btu/s} = 2.086\times10^6 \text{ Btu/h}$$

The length of the tube required is

$$L = \frac{\dot{Q}_{total}}{\dot{Q}} = \frac{2.086\times10^6 \text{ Btu/h}}{350 \text{ Btu/h.ft}} = \mathbf{5960 \text{ ft}}$$

The velocity of water and the Reynolds number are

$$V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{4 \text{ lbm/s}}{(61.59 \text{ lbm/m}^3)\pi \dfrac{(1.25/12 \text{ ft})^2}{4}} = 7.621 \text{ ft/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(7.621 \text{ m/s})(1.25/12 \text{ ft})}{0.5681\times10^{-5} \text{ ft}^2\text{/s}} = 1.397\times10^5$$

which is greater than 10,000. Therefore, we can assume fully developed turbulent flow in the entire tube, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023 \text{Re}^{0.8} \Pr^{0.4} = 0.023(1.397\times10^5)^{0.8}(3.368)^{0.4} = 488.5$$

The heat transfer coefficient is

$$h = \frac{k}{D_h} Nu = \frac{0.374 \text{ Btu/h.ft.°F}}{1.25/12 \text{ ft}}(488.5) = 1754 \text{ Btu/h.ft}^2\text{.°F}$$

The heat flux on the tube is

$$\dot{q} = \frac{\dot{Q}}{A_s} = \frac{2.086\times10^6 \text{ Btu/h}}{\pi(1.25/12 \text{ ft})(5960 \text{ ft})} = 1070 \text{ Btu/h.ft}^2$$

Then the surface temperature of the tube at the exit becomes

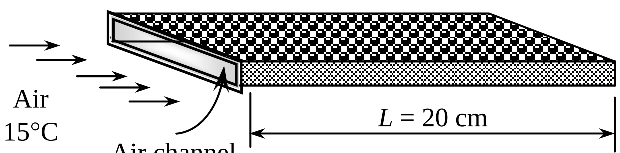
$$\dot{q} = h(T_s - T_e) \longrightarrow T_s = T_e + \frac{\dot{q}}{h} = 200\text{°F} + \frac{1070 \text{ Btu/h.ft}^2}{1754 \text{ Btu/h.ft}^2\text{.°F}} = \mathbf{200.6°F}$$

13-52 A circuit board is cooled by passing cool air through a channel drilled into the board. The maximum total power of the electronic components is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat flux at the top surface of the channel is uniform, and heat transfer through other surfaces is negligible. **3** The inner surfaces of the channel are smooth. **4** Air is an ideal gas with constant properties. **5** The pressure of air in the channel is 1 atm.

Properties The properties of air at 1 atm and estimated average temperature of 25°C are (Table A-22)

$$\rho = 1.184 \text{ kg/m}^3$$
$$k = 0.02551 \text{ W/m.°C}$$
$$\nu = 1.562\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\Pr = 0.7296$$



Analysis The cross-sectional and heat transfer surface areas are

$$A_c = (0.002 \text{ m})(0.14 \text{ m}) = 0.00028 \text{ m}^2$$
$$A_s = (0.14 \text{ m})(0.2 \text{ m}) = 0.028 \text{ m}^2$$

To determine heat transfer coefficient, we first need to find the Reynolds number,

$$D_h = \frac{4A_c}{P} = \frac{4(0.00028 \text{ m}^2)}{2(0.002 \text{ m} + 0.14 \text{ m})} = 0.003944 \text{ m}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(4 \text{ m/s})(0.003944 \text{ m})}{1.562\times10^{-5} \text{ m}^2\text{/s}} = 1010$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length is

$$L_t = 0.05 \,\text{Re}\,\text{Pr}\, D_h = 0.05(1010)(0.7296)(0.003944 \text{ m}) = 0.1453 \text{ m} < 0.20 \text{ m}$$

Therefore, we have developing flow through most of the channel. However, we take the conservative approach and assume fully developed flow, and from Table 13-1 we read Nu = 8.24. Then the heat transfer coefficient becomes

$$h = \frac{k}{D_h} Nu = \frac{0.02551 \text{ W/m.°C}}{0.003944 \text{ m}}(8.24) = 53.30 \text{ W/m}^2\text{.°C}$$

Also,

$$\dot{m} = \rho V A_c = (1.184 \text{ kg/m}^3)(4 \text{ m/s})(0.00028 \text{ m}^2) = 0.001326 \text{ kg/s}$$

Heat flux at the exit can be written as $\dot{q} = h(T_s - T_e)$ where $T_s = 50°\text{C}$ at the exit. Then the heat transfer rate can be expressed as $\dot{Q} = \dot{q}A_s = hA_s(T_s - T_e)$, and the exit temperature of the air can be determined from

$$hA_s(T_s - T_e) = \dot{m}c_p(T_e - T_i)$$
$$(53.30 \text{ W/m}^2\text{.°C})(0.028 \text{ m}^2)(50°\text{C} - T_e) = (0.001326 \text{ kg/s})(1007 \text{ J/kg.°C})(T_e - 15°\text{C})$$
$$T_e = 33.5°\text{C}$$

Then the maximum total power of the electronic components that can safely be mounted on this circuit board becomes

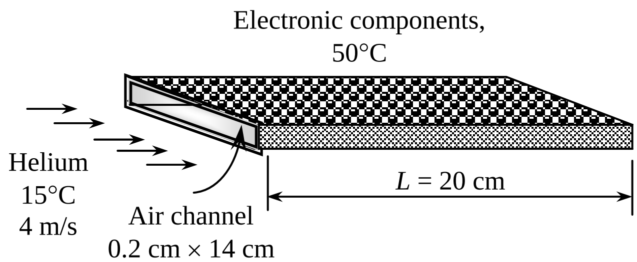
$$\dot{Q}_{\max} = \dot{m}c_p(T_e - T_i) = (0.001326 \text{ kg/s})(1007 \text{ J/kg.°C})(33.5 - 15°\text{C}) = \mathbf{24.7 \text{ W}}$$

13-53 A circuit board is cooled by passing cool helium gas through a channel drilled into the board. The maximum total power of the electronic components is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The heat flux at the top surface of the channel is uniform, and heat transfer through other surfaces is negligible. **3** The inner surfaces of the channel are smooth. **4** Helium is an ideal gas. **5** The pressure of helium in the channel is 1 atm.

Properties The properties of helium at the estimated average temperature of 25°C are (from EES)

$$\rho = 0.1635\ \text{kg/m}^3$$
$$k = 0.1553\ \text{W/m.°C}$$
$$\nu = 1.214\times10^{-4}\ \text{m}^2\text{/s}$$
$$c_p = 5193\ \text{J/kg.°C}$$
$$\text{Pr} = 0.6636$$



Analysis The cross-sectional and heat transfer surface areas are

$$A_c = (0.002\ \text{m})(0.14\ \text{m}) = 0.00028\ \text{m}^2$$
$$A_s = (0.14\ \text{m})(0.2\ \text{m}) = 0.028\ \text{m}^2$$

To determine heat transfer coefficient, we need to first find the Reynolds number

$$D_h = \frac{4A_c}{p} = \frac{4(0.00028\ \text{m}^2)}{2(0.002\ \text{m} + 0.14\ \text{m})} = 0.003944\ \text{m}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(4\ \text{m/s})(0.003944\ \text{m})}{1.214\times10^{-4}\ \text{m}^2\text{/s}} = 130.0$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length is

$$L_t = 0.05\,\text{Re}\,\text{Pr}\,D_h = 0.05(130.0)(0.6636)(0.003944\ \text{m}) = 0.0170\ \text{m} << 0.20\ \text{m}$$

Therefore, the flow is fully developed flow, and from Table 13-3 we read Nu = 8.24. Then the heat transfer coefficient becomes

$$h = \frac{k}{D_h} Nu = \frac{0.1553\ \text{W/m.°C}}{0.003944\ \text{m}}(8.24) = 324.5\ \text{W/m}^2\text{.°C}$$

Also,

$$\dot{m} = \rho V A_c = (0.1635\ \text{kg/m}^3)(4\ \text{m/s})(0.00028\ \text{m}^2) = 0.0001831\ \text{kg/s}$$

Heat flux at the exit can be written as $\dot{q} = h(T_s - T_e)$ where $T_s = 50°\text{C}$ at the exit. Then the heat transfer rate can be expressed as $\dot{Q} = \dot{q}A_s = hA_s(T_s - T_e)$, and the exit temperature of the air can be determined from

$$\dot{m}c_p(T_e - T_i) = hA_s(T_s - T_e)$$
$$(0.0001831\ \text{kg/s})(5193\ \text{J/kg.°C})(T_e - 15°\text{C}) = (324.5\ \text{W/m}^2\text{.°C})(0.028\ \text{m}^2)(50°\text{C} - T_e)$$
$$T_e = 46.68°\text{C}$$

Then the maximum total power of the electronic components that can safely be mounted on this circuit board becomes

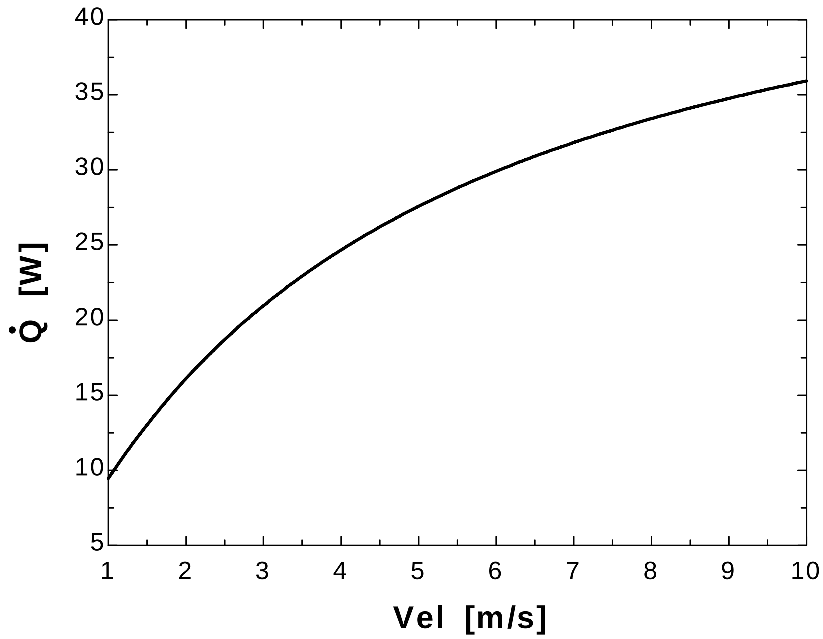
$$\dot{Q}_{\max} = \dot{m}c_p(T_e - T_i) = (0.0001831\ \text{kg/s})(5193\ \text{J/kg.°C})(46.68 - 15°\text{C}) = \mathbf{30.1\ W}$$

13-54 EES Prob. 13-52 is reconsidered. The effects of air velocity at the inlet of the channel and the maximum surface temperature on the maximum total power dissipation of electronic components are to be investigated.

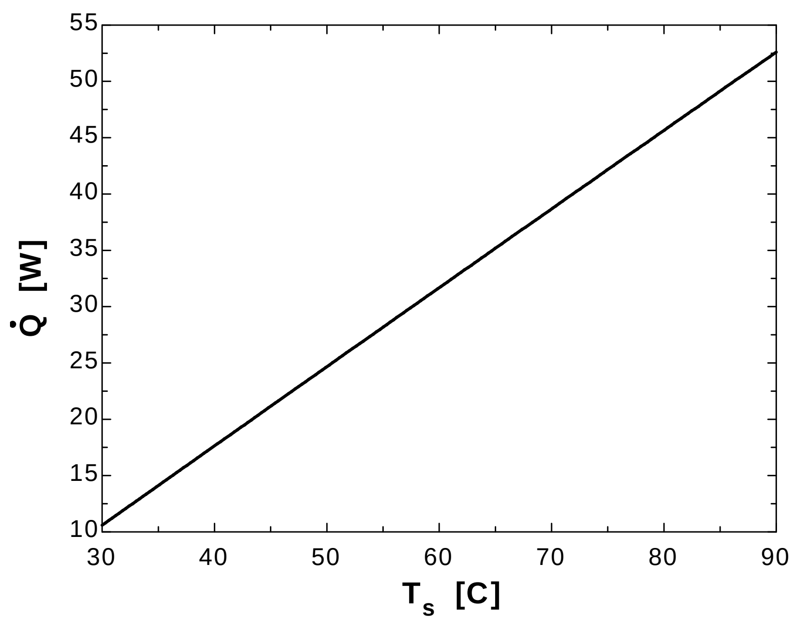
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=0.20 [m]
width=0.14 [m]
height=0.002 [m]
T_i=15 [C]
Vel=4 [m/s]
T_s=50 [C]

"PROPERTIES"
Fluid$='air'
c_p=CP(Fluid$, T=T_ave)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
T_ave=1/2*(T_i+T_e)

"ANALYSIS"
A_c=width*height
A=width*L
p=2*(width+height)
D_h=(4*A_c)/p
Re=(Vel*D_h)/nu "The flow is laminar"
L_t=0.05*Re*Pr*D_h
"Taking conservative approach and assuming fully developed laminar flow, from Table 13-1
we read"
Nusselt=8.24
h=k/D_h*Nusselt
m_dot=rho*Vel*A_c
Q_dot=h*A*(T_s-T_e)
Q_dot=m_dot*c_p*(T_e-T_i)
```

Vel [m/s]	Q [W]
1	9.453
2	16.09
3	20.96
4	24.67
5	27.57
6	29.91
7	31.82
8	33.41
9	34.76
10	35.92



T_s [C]	Q [W]
30	10.59
35	14.12
40	17.64
45	21.15
50	24.67
55	28.18
60	31.68
65	35.18
70	38.68
75	42.17
80	45.65
85	49.13
90	52.6

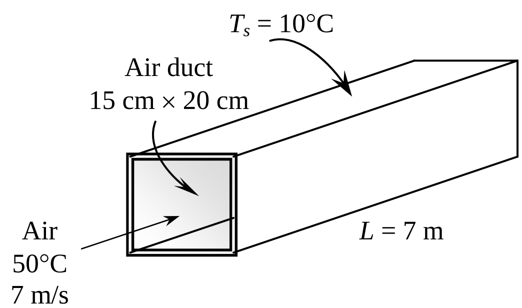


13-55 Air enters a rectangular duct. The exit temperature of the air, the rate of heat transfer, and the fan power are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The inner surfaces of the duct are smooth. **3** Air is an ideal gas with constant properties. **4** The pressure of air in the duct is 1 atm.

Properties We assume the bulk mean temperature for air to be 40°C since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at this temperature and 1 atm are (Table A-22)

$$\rho = 1.127 \text{ kg/m}^3 \qquad c_p = 1007 \text{ J/kg.°C}$$
$$k = 0.02662 \text{ W/m.°C} \qquad \Pr = 0.7255$$
$$\nu = 1.702\times10^{-5} \text{ m}^2/\text{s}$$



Analysis (*a*) The hydraulic diameter, the mean velocity of air, and the Reynolds number are

$$D_h = \frac{4A_c}{p} = \frac{4(0.15 \text{ m})(0.20 \text{ m})}{2[(0.15 \text{ m})+(0.20 \text{ m})]} = 0.1714 \text{ m}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(7 \text{ m/s})(0.1714 \text{ m})}{1.702\times10^{-5} \text{ m}^2/\text{s}} = 70{,}500$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.1714 \text{ m}) = 1.714 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\Pr^{0.3} = 0.023(70{,}500)^{0.8}(0.7255)^{0.3} = 157.9$$

Heat transfer coefficient is

$$h = \frac{k}{D_h} Nu = \frac{0.02662 \text{ W/m.°C}}{0.1714 \text{ m}}(157.9) = 24.52 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of air

$$A_s = 2\times 7[(0.15 \text{ m})+(0.20 \text{ m})] = 4.9 \text{ m}^2$$

$$A_c = (0.15 \text{ m})(0.20 \text{ m}) = 0.03 \text{ m}^2$$

$$\dot{m} = \rho V A_c = (1.127 \text{ kg/m}^3)(7 \text{ m/s})(0.03 \text{ m}^2) = 0.2367 \text{ kg/s}$$

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}c_p)} = 10-(10-50)e^{-\frac{(24.52)(4.9)}{(0.2367)(1007)}} = \mathbf{34.16°C}$$

(*b*) The logarithmic mean temperature difference and the rate of heat loss from the air are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{34.16-50}{\ln\left(\frac{10-34.16}{10-50}\right)} = 31.42°\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (24.52 \text{ W/m}^2\text{.°C})(4.9 \text{ m}^2)(31.42°\text{C}) = \mathbf{3775\ W}$$

(*c*) The friction factor, the pressure drop, and then the fan power can be determined for the case of fully developed turbulent flow to be

$$f = 0.184\,\text{Re}^{-0.2} = 0.184(70{,}500)^{-0.2} = 0.01973$$

$$\Delta P = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.01973\frac{(7 \text{ m})}{(0.1714 \text{ m})}\frac{(1.127 \text{ kg/m}^3)(7 \text{ m/s})^2}{2} = 22.25 \text{ N/m}^2$$

$$\dot{W}_{pump} = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.2367 \text{ kg/s})(22.25 \text{ N/m}^2)}{1.127 \text{ kg/m}^3} = \mathbf{4.67\ W}$$

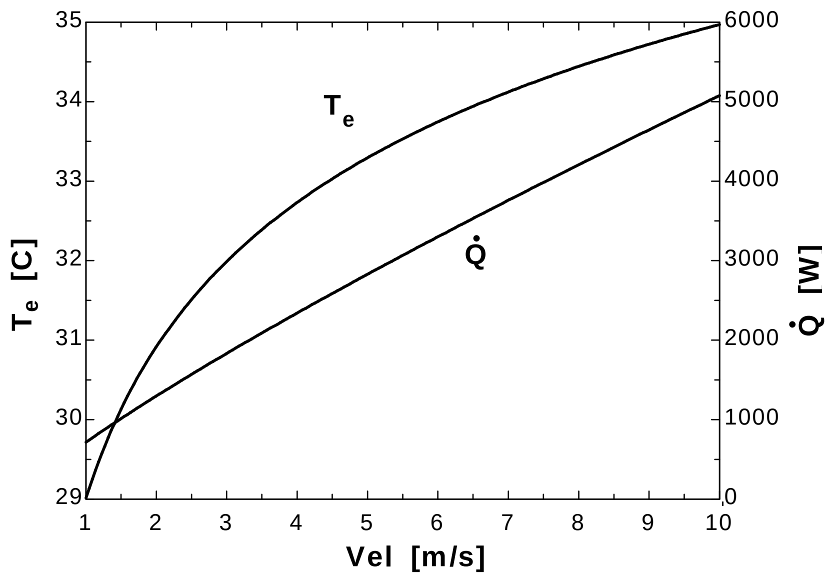
13-56 EES Prob. 13-55 is reconsidered. The effect of air velocity on the exit temperature of air, the rate of heat transfer, and the fan power is to be investigated.

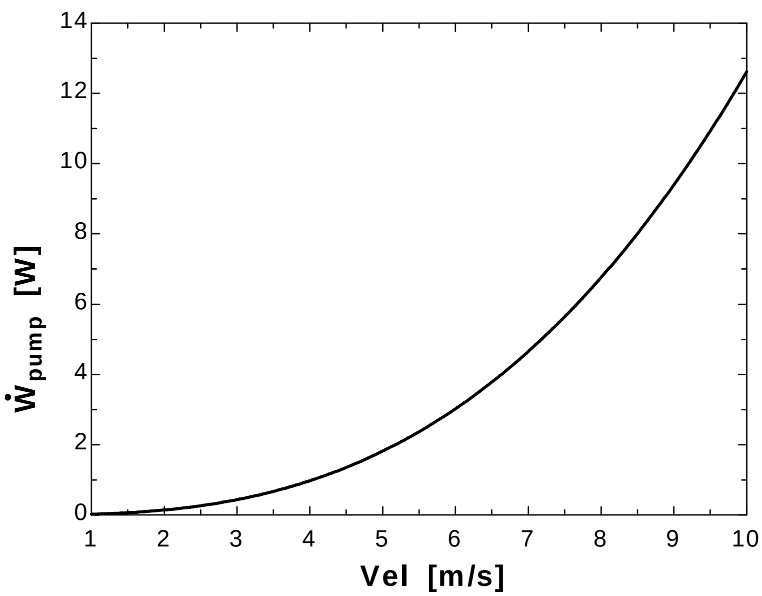
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
L=7 [m]
height=0.15 [m]
width=0.20 [m]
T_i=50 [C]
Vel=7 [m/s]
T_s=10 [C]

"PROPERTIES"
Fluid$='air'
c_p=CP(Fluid$, T=T_ave)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
T_ave=1/2*(T_i+T_e)

"ANALYSIS"
"(a)"
A_c=width*height
p=2*(width+height)
D_h=(4*A_c)/p
Re=(Vel*D_h)/nu "The flow is turbulent"
L_t=10*D_h "The entry length is much shorter than the total length of the duct."
Nusselt=0.023*Re^0.8*Pr^0.3
h=k/D_h*Nusselt
A=2*L*(width+height)
m_dot=rho*Vel*A_c
T_e=T_s-(T_s-T_i)*exp((-h*A)/(m_dot*c_p))
"(b)"
DELTAT_ln=(T_e-T_i)/ln((T_s-T_e)/(T_s-T_i))
Q_dot=h*A*DELTAT_ln
"(c)"
f=0.184*Re^(-0.2)
DELTAP=f*L/D_h*(rho*Vel^2)/2
W_dot_pump=(m_dot*DELTAP)/rho
```

Vel [m/s]	T_e [C]	Q [W]	W_{pump} [W]
1	29.01	715.6	0.02012
1.5	30.14	1014	0.06255
2	30.92	1297	0.1399
2.5	31.51	1570	0.2611
3	31.99	1833	0.4348
3.5	32.39	2090	0.6692
4	32.73	2341	0.9722
4.5	33.03	2587	1.352
5	33.29	2829	1.815
5.5	33.53	3066	2.369

6	33.75	3300	3.022
6.5	33.94	3531	3.781
7	34.12	3759	4.652
7.5	34.29	3984	5.642
8	34.44	4207	6.759
8.5	34.59	4427	8.008
9	34.72	4646	9.397
9.5	34.85	4862	10.93
10	34.97	5076	12.62



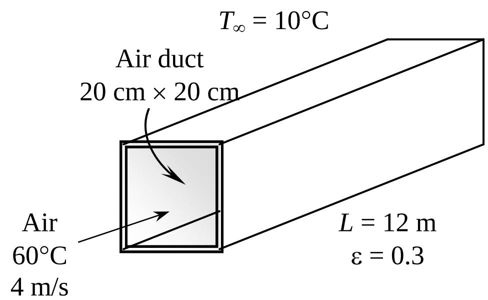


13-57 Hot air enters a sheet metal duct located in a basement. The exit temperature of hot air and the rate of heat loss are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We expect the air temperature to drop somewhat, and evaluate the air properties at 1 atm and the estimated bulk mean temperature of 50°C (Table A-22),

$$\rho = 1.092 \text{ kg/m}^3; \qquad k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2\text{/s}; \quad c_p = 1007 \text{ J/kg.°C}$$
$$\text{Pr} = 0.7228$$



Analysis The surface area and the Reynolds number are

$$A_s = 4aL = 4\times(0.2 \text{ m})(12 \text{ m}) = 9.6 \text{ m}$$

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 0.2 \text{ m}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(4 \text{ m/s})(0.20 \text{ m})}{1.798\times10^{-5} \text{ m}^2\text{/s}} = 44{,}494$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D_h = 10(0.2 \text{ m}) = 2.0 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow for the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(44{,}494)^{0.8}(0.7228)^{0.3} = 109.2$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.02735 \text{ W/m.°C}}{0.2 \text{ m}}(109.2) = 14.93 \text{ W/m}^2\text{.°C}$$

The mass flow rate of air is

$$\dot{m} = \rho A_c V = (1.092 \text{ kg/m}^3)(0.2\times0.2 \text{ m}^2)(4 \text{ m/s}) = 0.1747 \text{ kg/s}$$

In steady operation, heat transfer from hot air to the duct must be equal to the heat transfer from the duct to the surrounding (by convection and radiation), which must be equal to the energy loss of the hot air in the duct. That is,

$$\dot{Q} = \dot{Q}_{\text{conv,in}} = \dot{Q}_{\text{conv+rad,out}} = \Delta\dot{E}_{\text{hot air}}$$

Assuming the duct to be at an average temperature of T_s , the quantities above can be expressed as

$\dot{Q}_{\text{conv,in}}$:
$$\dot{Q} = h_i A_s \Delta T_{\ln} = h_i A_s \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} \rightarrow \dot{Q} = (14.93 \text{ W/m}^2\text{.°C})(9.6 \text{ m}^2)\frac{T_e - 60}{\ln\left(\frac{T_s - T_e}{T_s - 60}\right)}$$

$\dot{Q}_{\text{conv+rad,out}}$:
$$\dot{Q} = h_o A_s (T_s - T_o) + \varepsilon A_s \sigma\left(T_s^4 - T_o^4\right) \rightarrow \dot{Q} = (10 \text{ W/m}^2\text{.°C})(9.6 \text{ m}^2)(T_s - 10)\text{°C}$$
$$+ 0.3(9.6 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(T_s + 273)^4 - (10+273)^4\right]\text{K}^4$$

$\Delta\dot{E}_{\text{hot air}}$:
$$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow \dot{Q} = (0.1747 \text{ kg/s})(1007 \text{ J/kg.°C})(60 - T_e)\text{°C}$$

This is a system of three equations with three unknowns whose solution is

$$\dot{Q} = \mathbf{2622\ W},\ T_e = \mathbf{45.1°C}, \text{ and } T_s = 33.3\text{°C}$$

Therefore, the hot air will lose heat at a rate of 2622 W and exit the duct at 45.1°C.

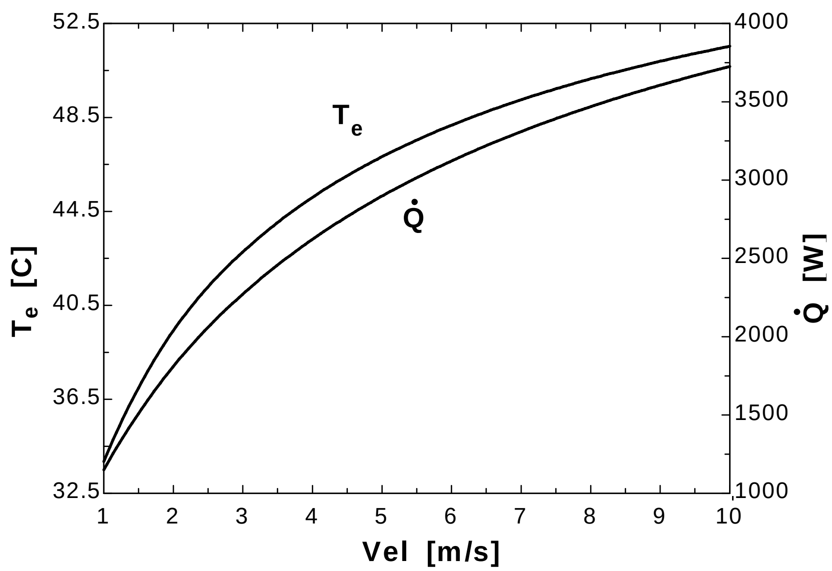
13-58 EES Prob. 13-57 is reconsidered. The effects of air velocity and the surface emissivity on the exit temperature of air and the rate of heat loss are to be investigated.

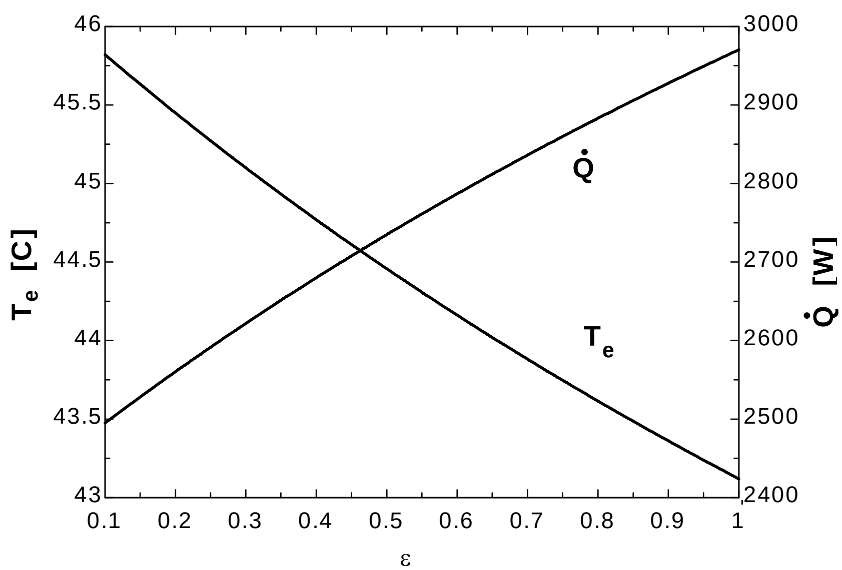
Analysis The problem is solved using EES, and the solution is given below.

```
"GIVEN"
T_i=60 [C]
L=12 [m]
side=0.20 [m]
Vel=4 [m/s]
epsilon=0.3
T_o=10 [C]
h_o=10 [W/m^2-C]
T_surr=10 [C]

"PROPERTIES"
Fluid$='air'
c_p=CP(Fluid$, T=T_ave)*Convert(kJ/kg-C, J/kg-C)
k=Conductivity(Fluid$, T=T_ave)
Pr=Prandtl(Fluid$, T=T_ave)
rho=Density(Fluid$, T=T_ave, P=101.3)
mu=Viscosity(Fluid$, T=T_ave)
nu=mu/rho
T_ave=T_i-10 "assumed average bulk mean temperature"

"ANALYSIS"
A=4*side*L
A_c=side^2
p=4*side
D_h=(4*A_c)/p
Re=(Vel*D_h)/nu "The flow is turbulent"
L_t=10*D_h "The entry length is much shorter than the total length of the duct."
Nusselt=0.023*Re^0.8*Pr^0.3
h_i=k/D_h*Nusselt
m_dot=rho*Vel*A_c
Q_dot=Q_dot_conv_in
Q_dot_conv_in=Q_dot_conv_out+Q_dot_rad_out
Q_dot_conv_in=h_i*A*DELTAT_ln
DELTAT_ln=(T_e-T_i)/ln((T_s-T_e)/(T_s-T_i))
Q_dot_conv_out=h_o*A*(T_s-T_o)
Q_dot_rad_out=epsilon*A*sigma*((T_s+273)^4-(T_surr+273)^4)
sigma=5.67E-8 "[W/m^2-K^4], Stefan-Boltzmann constant"
Q_dot=m_dot*c_p*(T_i-T_e)
```

Vel [m/s]	T_e [C]	Q [W]
1	33.85	1150
2	39.43	1810
3	42.78	2273
4	45.1	2622
5	46.83	2898
6	48.17	3122
7	49.25	3310
8	50.14	3469
9	50.89	3606
10	51.53	3726

ε	T_e [C]	Q [W]
0.1	45.82	2495
0.2	45.45	2560
0.3	45.1	2622
0.4	44.77	2680
0.5	44.46	2735
0.6	44.16	2787
0.7	43.88	2836
0.8	43.61	2883
0.9	43.36	2928
1	43.12	2970



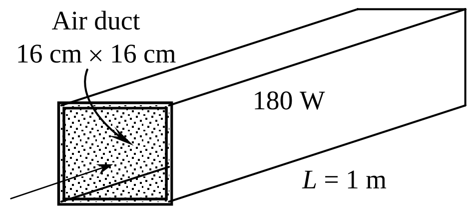


13-59 The components of an electronic system located in a rectangular horizontal duct are cooled by forced air. The exit temperature of the air and the highest component surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 35°C since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the duct whose surface is exposed to a constant heat flux. The properties of air at 1 atm and this temperature are (Table A-22)

$$\rho = 1.145 \text{ kg/m}^3$$
$$k = 0.02625 \text{ W/m.°C}$$
$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\text{Pr} = 0.7268$$



Analysis (*a*) The mass flow rate of air and the exit temperature are determined from

$$\dot{m} = \rho\dot{V} = (1.145 \text{ kg/m}^3)(0.65 \text{ m}^3\text{/min}) = 0.7443 \text{ kg/min} = 0.0124 \text{ kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m}c_p} = 27°\text{C} + \frac{(0.85)(180 \text{ W})}{(0.0124 \text{ kg/s})(1007 \text{ J/kg.°C})} = \mathbf{39.3°C}$$

(*b*) The mean fluid velocity and hydraulic diameter are

$$V_m = \frac{\dot{V}}{A_c} = \frac{0.65 \text{ m/min}}{(0.16 \text{ m})(0.16 \text{ m})} = 25.4 \text{ m/min} = 0.4232 \text{ m/s}$$

$$D_h = \frac{4A_c}{p} = \frac{4(0.16 \text{ m})(0.16 \text{ m})}{4(0.16 \text{ m})} = 0.16 \text{ m}$$

Then

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(0.4232 \text{ m/s})(0.16 \text{ m})}{1.655\times10^{-5} \text{ m}^2\text{/s}} = 4091$$

which is not greater than 10,000 but the components will cause turbulence and thus we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(4091)^{0.8}(0.7268)^{0.4} = 15.69$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.02625 \text{ W/m.°C}}{0.16 \text{ m}}(15.69) = 2.574 \text{ W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Assuming uniform surface heat flux, its value is determined from

$$\dot{Q}/A_s = h(T_{s,highest} - T_e)$$

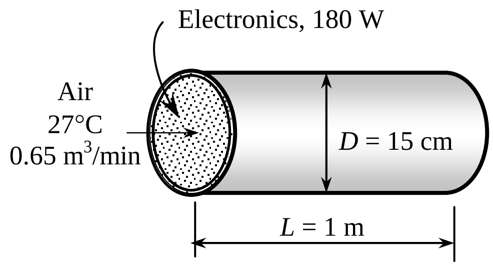
$$T_{s,highest} = T_e + \frac{\dot{Q}/A_s}{h} = 39.2°\text{C} + \frac{(0.85)(180 \text{ W})/[4(0.16 \text{ m})(1 \text{ m})]}{2.574 \text{ W/m}^2\text{.°C}} = \mathbf{132°C}$$

13-60 The components of an electronic system located in a circular horizontal duct are cooled by forced air. The exit temperature of the air and the highest component surface temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 35°C since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the duct whose surface is exposed to a constant heat flux. The properties of air at 1 atm and this temperature are (Table A-22)

$$\rho = 1.145 \text{ kg/m}^3$$
$$k = 0.02625 \text{ W/m.°C}$$
$$\nu = 1.655\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\Pr = 0.7268$$



Analysis (a) The mass flow rate of air and the exit temperature are determined from

$$\dot{m} = \rho\dot{V} = (1.145 \text{ kg/m}^3)(0.65 \text{ m}^3\text{/min}) = 0.7443 \text{ kg/min} = 0.0124 \text{ kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m}c_p} = 27\text{ °C} + \frac{(0.85)(180 \text{ W})}{(0.0124 \text{ kg/s})(1007 \text{ J/kg.°C})} = \mathbf{39.3\text{ °C}}$$

(b) The mean fluid velocity is

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{0.65 \text{ m/min}}{\pi(0.15 \text{ m})^2/4} = 36.8 \text{ m/min} = 0.613 \text{ m/s}$$

Then,

$$\text{Re} = \frac{V_{\text{avg}}D_h}{\nu} = \frac{(0.613 \text{ m/s})(0.15 \text{ m})}{1.655\times10^{-5} \text{ m}^2\text{/s}} = 5556$$

which is not greater than 10,000 but the components will cause turbulence and thus we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(5556)^{0.8}(0.7268)^{0.4} = 20.05$$

and

$$h = \frac{k}{D_h}Nu = \frac{0.02625 \text{ W/m.°C}}{0.15 \text{ m}}(20.05) = 3.51 \text{ W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Assuming uniform heat flux, its value is determined from

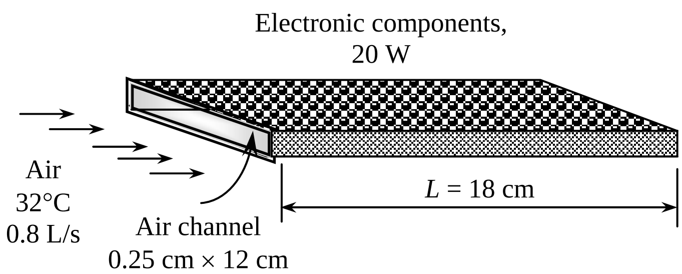
$$\dot{q} = h(T_{s,highest} - T_e) \rightarrow T_{s,highest} = T_e + \frac{\dot{q}}{h} = 39.2\text{°C} + \frac{(0.85)(180 \text{ W})/[\pi(0.15 \text{ m})(1 \text{ m})]}{3.51 \text{ W/m}^2\text{.°C}} = \mathbf{131.7\text{°C}}$$

13-61 Air enters a hollow-core printed circuit board. The exit temperature of the air and the highest temperature on the inner surface are to be determined.

Assumptions **1** Steady flow conditions exist. **2** Heat generated is uniformly distributed over the two surfaces of the PCB. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 40°C since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the hollow core whose surface is exposed to a constant heat flux. The properties of air at 1 atm and this temperature are (Table A-22)

$$\rho = 1.127 \text{ kg/m}^3$$
$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\Pr = 0.7255$$
$$\mu_b = 1.918\times10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@\,60°C} = 2.008\times10^{-5} \text{ kg/m.s}$$



Analysis (a) The mass flow rate of air and the exit temperature are determined from

$$\dot{m} = \rho\dot{V} = (1.127 \text{ kg/m}^3)(0.8\times10^{-3} \text{ m}^3\text{/s}) = 9.02\times10^{-4} \text{ kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m}c_p} = 32\text{ °C} + \frac{20\text{ W}}{(9.02\times10^{-4}\text{ kg/s})(1007\text{ J/kg.°C})} = \mathbf{54.0°C}$$

(b) The mean fluid velocity and hydraulic diameter are

$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{0.8\times10^{-3}\text{ m}^3\text{/s}}{(0.12\text{ m})(0.0025\text{ m})} = 2.67\text{ m/s}$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.12\text{ m})(0.0025\text{ m})}{2[(0.12\text{ m}) + (0.0025\text{ m})]} = 0.0049\text{ m}$$

Then,

$$\text{Re} = \frac{V_{avg}D_h}{\nu} = \frac{(2.67\text{ m/s})(0.0049\text{ m})}{1.702\times10^{-5}\text{ m}^2\text{/s}} = 769$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length in this case is

$$L_t = 0.05\,\text{Re}\,\text{Pr}\,D_h = 0.05(769)(0.7255)(0.0049\text{ m}) = 0.14\text{ m}$$

which is shorter than the total length of the duct. Therefore, we assume thermally developing flow , and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 1.86\left(\frac{\text{Re}\,\text{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left[\frac{(769)(0.7255)(0.0049)}{0.18}\right]^{1/3}\left(\frac{1.918\times10^{-5}}{2.008\times10^{-5}}\right)^{0.14} = 4.58$$

and,

$$h = \frac{k}{D_h}Nu = \frac{0.02662\text{ W/m.°C}}{0.0049\text{ m}}(4.58) = 24.9\text{ W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Its value is determined from

$$\dot{Q} = hA_s(T_{s,highest} - T_e) \rightarrow T_{s,highest} = T_e + \frac{\dot{Q}}{hA_s}$$

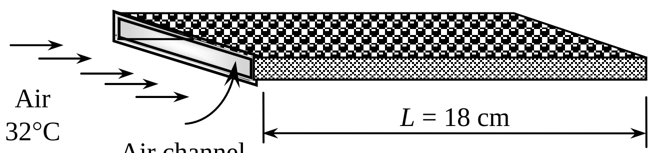
$$= 54.0°\text{C} + \frac{20\text{ W}}{(24.9\text{ W/m}^2\text{.°C})\left[2(0.12\times0.18 + 0.0025\times0.18)\text{m}^2\right]} = \mathbf{72.2°C}$$

13-62 Air enters a hollow-core printed circuit board. The exit temperature of the air and the highest temperature on the inner surface are to be determined.

Assumptions **1** Steady flow conditions exist. **2** Heat generated is uniformly distributed over the two surfaces of the PCB. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 40°C since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the hollow core whose surface is exposed to a constant heat flux. The properties of air at 1 atm and this temperature are (Table A-22)

$$\rho = 1.127 \text{ kg/m}^3$$
$$k = 0.02662 \text{ W/m.°C}$$
$$\nu = 1.702\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\Pr = 0.7255$$
$$\mu_b = 1.918\times10^{-5} \text{ kg/m.s}$$
$$\mu_{s,@80°C} = 2.096\times10^{-5} \text{ kg/m.s}$$



Analysis (a) The mass flow rate of air and the exit temperature are determined from

$$\dot{m} = \rho\dot{V} = (1.127 \text{ kg/m}^3)(0.8\times10^{-3} \text{ m}^3\text{/s}) = 9.02\times10^{-4} \text{ kg/s}$$

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow T_e = T_i + \frac{\dot{Q}}{\dot{m}C_p} = 32°\text{C} + \frac{35 \text{ W}}{(9.02\times10^{-4} \text{ kg/s})(1007 \text{ J/kg.°C})} = \mathbf{70.5°C}$$

(b) The mean fluid velocity and hydraulic diameter are

$$V_{\text{avg}} = \frac{\dot{V}}{A_c} = \frac{0.8\times10^{-3} \text{ m}^3\text{/s}}{(0.12 \text{ m})(0.0025 \text{ m})} = 2.67 \text{ m/s}$$

$$D_h = \frac{4A_c}{P} = \frac{4(0.12 \text{ m})(0.0025 \text{ m})}{2[(0.12 \text{ m}) + (0.0025 \text{ m})]} = 0.0049 \text{ m}$$

Then,

$$\text{Re} = \frac{V_{\text{avg}}D_h}{\nu} = \frac{(2.67 \text{ m/s})(0.0049 \text{ m})}{1.702\times10^{-5} \text{ m}^2\text{/s}} = 769$$

which is less than 2300. Therefore, the flow is laminar and the thermal entry length in this case is

$$L_t = 0.05 \text{ Re Pr } D_h = 0.05(783)(0.71)(0.0049 \text{ m}) = 0.14 \text{ m}$$

which is shorter than the total length of the duct. Therefore, we assume thermally developing flow , and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 1.86\left(\frac{\text{Re Pr } D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14} = 1.86\left[\frac{(769)(0.7255)(0.0049)}{0.18}\right]^{1/3}\left(\frac{1.918\times10^{-5}}{2.096\times10^{-5}}\right)^{0.14} = 4.55$$

and,

$$h = \frac{k}{D_h}Nu = \frac{0.02662 \text{ W/m.°C}}{0.0049 \text{ m}}(4.55) = 24.7 \text{ W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Its value is determined from

$$\dot{Q} = hA_s(T_{s,highest} - T_e) \longrightarrow T_{s,highest} = T_e + \frac{\dot{Q}}{hA_s}$$

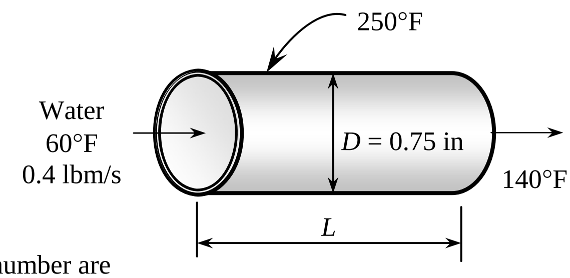
$$= 70.5°\text{C} + \frac{35 \text{ W}}{(24.7 \text{ W/m}^2\text{.°C})[2(0.12\times0.18 + 0.0025\times0.18)\text{m}^2]} = \mathbf{102.6°C}$$

13-63E Water is heated by passing it through thin-walled copper tubes. The length of the copper tube that needs to be used is to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the tube are smooth. **3** The thermal resistance of the tube is negligible. **4** The temperature at the tube surface is constant.

Properties The properties of water at the bulk mean fluid temperature of $T_{b,avg} = (60+140)/2 = 100°\text{F}$ are (Table A-15E)

$$\rho = 62.0\ \text{lbm/ft}^3$$
$$k = 0.363\ \text{Btu/h.ft.°F}$$
$$\nu = \mu/\rho = 0.738\times10^{-5}\ \text{ft}^2/\text{s}$$
$$c_p = 0.999\ \text{Btu/lbm.°F}$$
$$\text{Pr} = 4.54$$



Analysis (*a*) The mass flow rate and the Reynolds number are

$$\dot{m} = \rho A_c V_{\text{avg}} \rightarrow V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{0.4\ \text{lbm/s}}{(62\ \text{lbm/ft}^3)[\pi(0.75/12\ \text{ft})^2/4]} = 2.10\ \text{ft/s}$$

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(2.10\ \text{ft/s})(0.75/12\ \text{ft})}{0.738\times10^{-5}\ \text{ft}^2/\text{s}} = 17{,}810$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.75\ \text{in}) = 7.5\ \text{in}$$

which is probably shorter than the total length of the pipe we will determine. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(17{,}810)^{0.8}(4.54)^{0.4} = 105.9$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.363\ \text{Btu/h.ft.°F}}{(0.75/12)\ \text{ft}}(105.9) = 615\ \text{Btu/h.ft}^2.\text{°F}$$

The logarithmic mean temperature difference and then the rate of heat transfer per ft length of the tube are

$$\Delta T_{\ln} = \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{140-60}{\ln\left(\frac{250-140}{250-60}\right)} = 146.4°\text{F}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (615\ \text{Btu/h.ft}^2.\text{°F})[\pi(0.75/12\ \text{ft})(1\ \text{ft})](146.4°\text{F}) = 17{,}675\ \text{Btu/h}$$

The rate of heat transfer needed to raise the temperature of water from 60°F to 140°F is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.4\times3600\ \text{lbm/h})(0.999\ \text{Btu/lbm.°F})(140-60)°\text{F} = 115{,}085\ \text{Btu/h}$$

Then the length of the copper tube that needs to be used becomes

$$Length = \frac{115{,}085\ \text{Btu/h}}{17{,}675\ \text{Btu/h}} = \mathbf{6.51\ ft}$$

(*b*) The friction factor, the pressure drop, and then the pumping power required to overcome this pressure drop can be determined for the case of fully developed turbulent flow to be

$$f = 0.184\,\text{Re}^{-0.2} = 0.184(17{,}810)^{-0.2} = 0.02598$$

$$\Delta P = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.02598\frac{(7.69\ \text{ft})}{(0.75/12\ \text{ft})}\frac{(62\ \text{lbm/ft}^3)(2.10\ \text{ft/s})^2}{2}\left(\frac{1\ \text{lbf}}{32.174\ \text{lbm}\cdot\text{ft/s}^2}\right) = 13.58\ \text{lbf/ft}^2$$

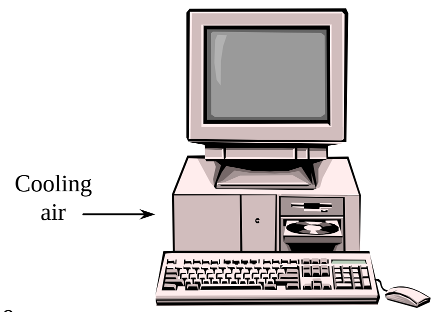
$$\dot{W}_{pump} = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.4\ \text{lbm/s})(13.58\ \text{lbf/ft}^2)}{62\ \text{lbm/ft}^3}\left(\frac{1\ \text{hp}}{550\ \text{lbf}\cdot\text{ft/s}}\right) = \mathbf{0.00016\ hp}$$

13-64 A computer is cooled by a fan blowing air through its case. The flow rate of the air, the fraction of the temperature rise of air that is due to heat generated by the fan, and the highest allowable inlet air temperature are to be determined.

Assumptions **1** Steady flow conditions exist. **2** Heat flux is uniformly distributed. **3** Air is an ideal gas with constant properties. **4** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 25°C. The properties of air at 1 atm and this temperature are (Table A-22)

$$\rho = 1.184\ \text{kg/m}^3$$
$$k = 0.02551\ \text{W/m.°C}$$
$$\nu = 1.562\times10^{-5}\ \text{m}^2\text{/s}$$
$$c_p = 1007\ \text{J/kg.°C}$$
$$\text{Pr} = 0.7296$$



Analysis (*a*) Noting that the electric energy consumed by the fan is converted to thermal energy, the mass flow rate of air is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow \dot{m} = \frac{\dot{Q} + \dot{W}_{\text{elect, fan}}}{c_p(T_e - T_i)} = \frac{(8\times10 + 25)\ \text{W}}{(1007\ \text{J/kg.°C})(10°\text{C})} = \mathbf{0.01043\ kg/s}$$

(*b*) The fraction of temperature rise of air that is due to the heat generated by the fan and its motor is

$$\dot{Q} = \dot{m}c_p\Delta T \rightarrow \Delta T = \frac{\dot{Q}}{\dot{m}c_p} = \frac{25\ \text{W}}{(0.01043\ \text{kg/s})(1007\ \text{J/kg.°C})} = \mathbf{2.38°C}$$

$$f = \frac{2.38°\text{C}}{10°\text{C}} = 0.238 = \mathbf{23.8\%}$$

(*c*) The mean velocity of air is

$$\dot{m} = \rho A_c V_{\text{avg}} \rightarrow V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{(0.01043/8)\ \text{kg/s}}{(1.184\ \text{kg/m}^3)[(0.003\ \text{m})(0.12\ \text{m})]} = 3.06\ \text{m/s}$$

and $$D_h = \frac{4A_c}{P} = \frac{4(0.003\ \text{m})(0.12\ \text{m})}{2(0.003\ \text{m} + 0.12\ \text{m})} = 0.00585\ \text{m}$$

Therefore,

$$\text{Re} = \frac{V_{\text{avg}}D_h}{\nu} = \frac{(3.06\ \text{m/s})(0.00585\ \text{m})}{1.562\times10^{-5}\ \text{m}^2\text{/s}} = 1146$$

which is less than 2300. Therefore, the flow is laminar. Assuming fully developed flow, the Nusselt number is determined from Table 13-4 corresponding to a/b = 12/0.3 = 40 to be Nu = 8.24. Then,

$$h = \frac{k}{D_h}Nu = \frac{0.02551\ \text{W/m.°C}}{0.00585\ \text{m}}(8.24) = 35.9\ \text{W/m}^2\text{.°C}$$

The highest component surface temperature will occur at the exit of the duct. Assuming uniform heat flux, the air temperature at the exit is determined from

$$\dot{q} = h(T_{s,\max} - T_e) \rightarrow T_e = T_{s,\max} - \frac{\dot{q}}{h} = 70°\text{C} - \frac{[(80+25)\ \text{W}]/[8\times2(0.12\times0.18 + 0.003\times0.18)\ \text{m}^2]}{35.9\ \text{W/m}^2\text{.°C}} = 61.7°\text{C}$$

The highest allowable inlet temperature then becomes

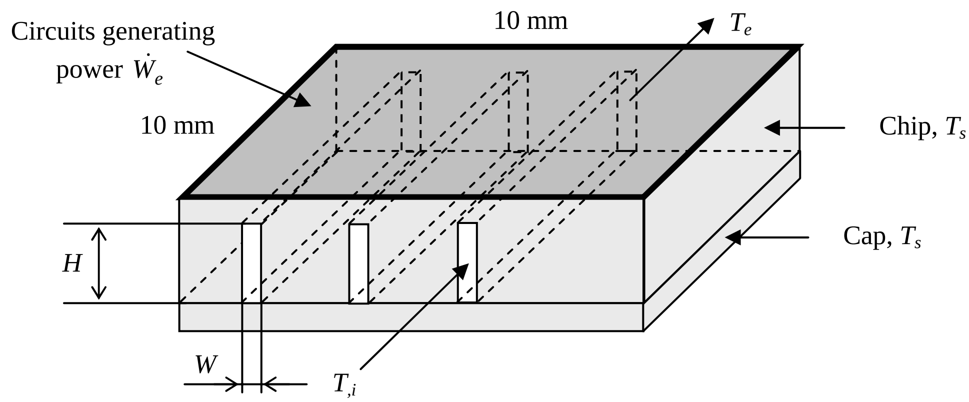
$$T_e - T_i = 10°\text{C} \rightarrow T_i = T_e - 10°\text{C} = 61.7°\text{C} - 10°\text{C} = \mathbf{51.7°C}$$

Discussion Although the Reynolds number is less than 2300, the flow in this case will most likely be turbulent because of the electronic components that that protrude into flow. Therefore, the heat transfer coefficient determined above is probably conservative.

Review Problems

13-65 A silicon chip is cooled by passing water through microchannels etched in the back of the chip. The outlet temperature of water and the chip power dissipation are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The flow of water is fully developed. **3** All the heat generated by the circuits on the top surface of the chip is transferred to the water.



Properties Assuming a bulk mean fluid temperature of 25°C, the properties of water are (Table A-15)

$$\rho = 997 \text{ kg/m}^3 \qquad c_p = 4180 \text{ J/kg.°C}$$
$$k = 0.607 \text{ W/m.°C} \qquad \text{Pr} = 6.14$$
$$\mu = 0.891\times10^{-3} \text{ m}^2\text{/s}$$

Analysis (a) The mass flow rate for one channel, the hydraulic diameter, and the Reynolds number are

$$\dot{m} = \frac{\dot{m}_{\text{total}}}{n_{\text{channel}}} = \frac{0.005 \text{ kg/s}}{50} = 0.0001 \text{ kg/s}$$

$$D_h = \frac{4A}{p} = \frac{4(H\times W)}{2(H+W)} = \frac{4(50\times200)}{2(50+200)} = 80\ \mu\text{m} = 8\times10^{-5} \text{ m}$$

$$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{\rho \dot{m} V D_h}{\rho A_c \mu} = \frac{\dot{m} D_h}{A_c \mu} = \frac{(0.0001 \text{ kg/s})(8\times10^{-5} \text{ m})}{(50\times200\times10^{-12} \text{ m}^2)(0.891\times10^{-3} \text{ kg/m}\cdot\text{s})} = 898$$

which is smaller than 2300. Therefore, the flow is laminar. We take fully developed laminar flow in the entire duct. The Nusselt number in this case is

$$Nu = 3.66$$

Heat transfer coefficient is

$$h = \frac{k}{D} Nu = \frac{0.607 \text{ W/m.°C}}{8\times10^{-5} \text{ m}}(3.66) = 27{,}770 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of water

$$A = 2WL + 2HL = 2(0.05\times0.01) + 2(0.05\times0.2) = 0.021 \text{ mm}^2 = 2.1\times10^{-6} \text{ m}^2$$

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 350 - (350-290)\exp\left[-\frac{(27{,}770)(2.1\times10^{-6})}{(0.0001)(4180)}\right] = \mathbf{297.8\ K}$$

Then the rate of heat transfer becomes

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (0.0001 \text{ kg/s})(4180 \text{ J/kg.°C})(350-297.8)\text{°C} = 21.82 \text{ W}$$

(b) Noting that there are 50 such channels, the chip power dissipation becomes

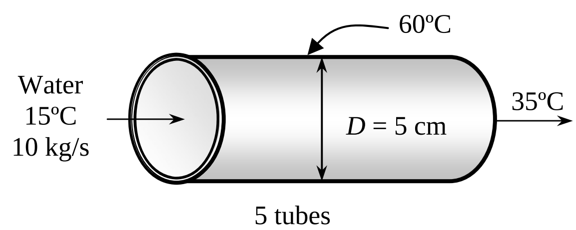
$$\dot{W}_e = n_{\text{channel}}\dot{Q}_{\text{one channel}} = 50(21.82 \text{ W}) = \mathbf{1091\ W}$$

13-66 Water is heated by passing it through five identical tubes that are maintained at a specified temperature. The rate of heat transfer and the length of the tubes necessary are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface temperature is constant and uniform. **3** The inner surfaces of the tubes are smooth. **4** Heat transfer to the surroundings is negligible.

Properties The properties of water at the bulk mean fluid temperature of (15+35)/2=25ºC are (Table A-15)

$$\rho = 997 \text{ kg/m}^3$$
$$k = 0.607 \text{ W/m.°C}$$
$$\mu = 0.891\times10^{-3} \text{ m}^2\text{/s}$$
$$c_p = 4180 \text{ J/kg.°C}$$
$$\Pr = 6.14$$



Analysis (*a*) The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (10 \text{ kg/s})(4180 \text{ J/kg.°C})(35-15)\text{°C} = \mathbf{836{,}000 \text{ W}}$$

(*b*) The water velocity is

$$V = \frac{\dot{m}}{\rho A_c} = \frac{(10/5) \text{ kg/s}}{(997 \text{ kg/m}^3)\pi(0.05 \text{ m})^2/4} = 1.02 \text{ m/s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(997 \text{ kg/m}^3)(1.02 \text{ m/s})(0.05 \text{ m})}{0.891\times10^{-3} \text{ kg/m}\cdot\text{s}} = 57{,}067$$

which is greater than 10,000. Therefore, we have turbulent flow. Assuming fully developed flow in the entire tube, the Nusselt number is determined from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\Pr^{0.4} = 0.023(57{,}067)^{0.8}(6.14)^{0.4} = 303.5$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.607 \text{ W/m.°C}}{0.05 \text{ m}}(303.5) = 3684 \text{ W/m}^2\text{.°C}$$

Using the average fluid temperature and considering that there are 5 tubes, the length of the tubes is determined as follows:

$$\dot{Q} = hA(T_s - T_{b,\text{avg}}) \longrightarrow 836{,}000 \text{ W} = (3684 \text{ W/m}^2\cdot\text{°C})A(60-25)\text{°C} \longrightarrow A_s = 6.484 \text{ m}^2$$

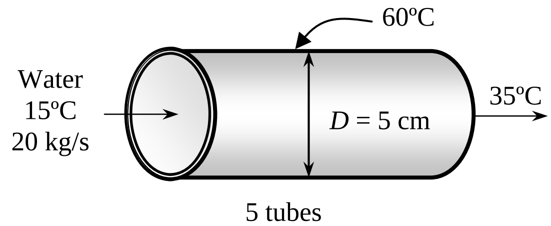
$$A = 5\pi DL \longrightarrow L = \frac{A}{5\pi D} = \frac{6.484 \text{ m}^2}{5\pi(0.05 \text{ m})} = \mathbf{8.26 \text{ m}}$$

13-67 Water is heated by passing it through five identical tubes that are maintained at a specified temperature. The rate of heat transfer and the length of the tubes necessary are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface temperature is constant and uniform. **3** The inner surfaces of the tubes are smooth. **4** Heat transfer to the surroundings is negligible.

Properties The properties of water at the bulk mean fluid temperature of (15+35)/2=25ºC are (Table A-15)

$$\rho = 997 \text{ kg/m}^3$$
$$k = 0.607 \text{ W/m.°C}$$
$$\mu = 0.891\times10^{-3} \text{ m}^2\text{/s}$$
$$c_p = 4180 \text{ J/kg.°C}$$
$$\text{Pr} = 6.14$$



Analysis (*a*) The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (20 \text{ kg/s})(4180 \text{ J/kg.°C})(35-15)\text{°C} = \mathbf{1{,}672{,}000\ W}$$

(*b*) The water velocity is

$$V = \frac{\dot{m}}{\rho A_c} = \frac{(20/5) \text{ kg/s}}{(997 \text{ kg/m}^3)\left[\pi(0.05 \text{ m})^2/4\right]} = 2.04 \text{ m/s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(997 \text{ kg/m}^3)(2.04 \text{ m/s})(0.05 \text{ m})}{0.891\times10^{-3} \text{ kg/m}\cdot\text{s}} = 114{,}320$$

which is greater than 10,000. Therefore, we have turbulent flow. Assuming fully developed flow in the entire tube, the Nusselt number is determined from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(114{,}320)^{0.8}(6.14)^{0.4} = 529.0$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.607 \text{ W/m.°C}}{0.05 \text{ m}}(529.0) = 6423 \text{ W/m}^2\text{.°C}$$

Using the average fluid temperature and considering that there are 5 tubes, the length of the tubes is determined as follows:

$$\dot{Q} = hA(T_s - T_{b,\text{avg}}) \longrightarrow 1{,}672{,}000 \text{ W} = (6423 \text{ W/m}^2\cdot\text{°C})A(60-25)\text{°C} \longrightarrow A_s = 7.438 \text{ m}^2$$

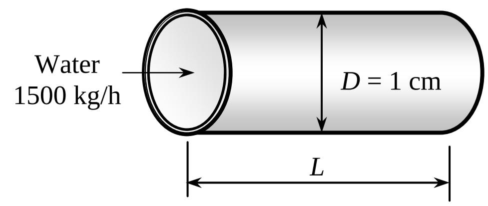
$$A = 5\pi DL \longrightarrow L = \frac{A}{5\pi D} = \frac{7.438 \text{ m}^2}{5\pi(0.05 \text{ m})} = \mathbf{9.47\ m}$$

13-68 Water is heated as it flows in a smooth tube that is maintained at a specified temperature. The necessary tube length and the water outlet temperature if the tube length is doubled are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The surface temperature is constant and uniform. **3** The inner surfaces of the tube are smooth. **4** Heat transfer to the surroundings is negligible.

Properties The properties of water at the bulk mean fluid temperature of (10+40)/2=25ºC are (Table A-15)

$$\rho = 997 \text{ kg/m}^3$$
$$k = 0.607 \text{ W/m.°C}$$
$$\mu = 0.891\times10^{-3} \text{ m}^2\text{/s}$$
$$c_p = 4180 \text{ J/kg.°C}$$
$$\text{Pr} = 6.14$$



Analysis (*a*) The rate of heat transfer is

$$\dot{Q} = \dot{m}c_p(T_e - T_i) = (1500/3600 \text{ kg/s})(4180 \text{ J/kg.°C})(40-10)\text{°C} = 52{,}250 \text{ W}$$

The water velocity is

$$V = \frac{\dot{m}}{\rho A_c} = \frac{(1500/3600) \text{ kg/s}}{(997 \text{ kg/m}^3)\left[\pi(0.01 \text{ m})^2/4\right]} = 5.32 \text{ m/s}$$

The Reynolds number is

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(997 \text{ kg/m}^3)(5.32 \text{ m/s})(0.01 \text{ m})}{0.891\times10^{-3} \text{ kg/m}\cdot\text{s}} = 59{,}542$$

which is greater than 10,000. Therefore, we have turbulent flow. Assuming fully developed flow in the entire tube, the Nusselt number is determined from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.4} = 0.023(59{,}542)^{0.8}(6.14)^{0.4} = 313.9$$

Heat transfer coefficient is

$$h = \frac{k}{D}Nu = \frac{0.607 \text{ W/m.°C}}{0.01 \text{ m}}(313.9) = 19{,}056 \text{ W/m}^2\text{.°C}$$

Using the average fluid temperature, the length of the tubes is determined as follows:

$$\dot{Q} = hA(T_s - T_{b,\text{avg}}) \longrightarrow 52{,}250 \text{ W} = (19{,}056 \text{ W/m}^2\cdot\text{°C})A(49-25)\text{°C} \longrightarrow A_s = 0.1142 \text{ m}^2$$
$$A = \pi DL \longrightarrow 0.1142 \text{ m}^2 = \pi(0.01 \text{ m})L \longrightarrow L = \mathbf{3.6\ m}$$

(b) If the tube length is doubled, the surface area doubles, and the outlet water temperature may be obtained from an energy balance to be

$$\dot{m}c_p(T_e - T_i) = hA_s(T_s - T_{b,\text{avg}})$$
$$(1500/3600)(4180)(T_e - 10) = (19{,}056)(2\times0.1142)\left(49 - \frac{10+T_e}{2}\right)$$
$$T_e = 53.3\text{°C}$$

which is greater than the surface temperature of the wall. This is impossible. It shows that the water reaches the surface temperature before the entire length of tube is covered and in reality the water will leave the tube at the surface temperature of 49ºC. This example demonstrates that the use of unnecessarily long tubes should be avoided.

13-69 Geothermal water is supplied to a city through stainless steel pipes at a specified rate. The electric power consumption and its daily cost are to be determined, and it is to be assessed if the frictional heating during flow can make up for the temperature drop caused by heat loss.

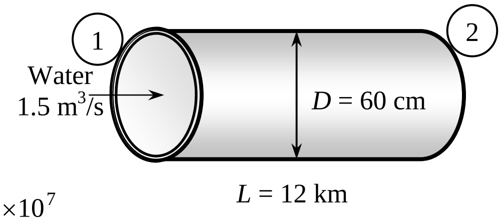
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are ρ = 950.6 kg/m^3, μ = 0.255×10^{-3} kg/m·s, and c_p = 4.229 kJ/kg·°C (Table A-15). The roughness of stainless steel pipes is 2×10^{-6} m (Table 13-3).

Analysis (*a*) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($V_1 = V_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{pump,u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The mean velocity and the Reynolds number are



$$V_{\text{avg}}=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{1.5\text{ m}^3/\text{s}}{\pi(0.60\text{ m})^2/4}=5.305\text{ m/s}$$

$$\text{Re}=\frac{\rho V_{\text{avg}}D}{\mu}=\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})(0.60\text{ m})}{0.255\times10^{-3}\text{ kg/m}\cdot\text{s}}=1.186\times10^7$$

which is greater than 10,000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{2\times10^{-6}\text{ m}}{0.60\text{ m}}=3.33\times10^{-6}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\ \rightarrow\ \frac{1}{\sqrt{f}}=-2.0\log\left(\frac{3.33\times10^{-6}}{3.7}+\frac{2.51}{1.187\times10^7\sqrt{f}}\right)$$

It gives f = 0.00829. Then the pressure drop, the head loss, and the required power input become

$$\Delta P=f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2}=0.00829\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(950.6\text{ kg/m}^3)(5.305\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=2218\text{ kPa}$$

$$\dot{W}_{\text{elect}}=\frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{(1.5\text{ m}^3/\text{s})(2218\text{ kPa})}{0.65}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{5118\text{ kW}}$$

Therefore, the pumps will consume 5118 kW of electric power to overcome friction and maintain flow.

(*b*) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount}=\dot{W}_{\text{elect,in}}\Delta t=(5118\text{ kW})(24\text{ h/day})=122{,}832\text{ kWh/day}$$

$$\text{Cost}=\text{Amount}\times\text{Unit cost}=(122{,}832\text{ kWh/day})(\$0.06/\text{kWh})=\mathbf{\$7370/day}$$

(*c*) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 5118 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{elect}} = \rho\dot{V}c_p\Delta T \ \rightarrow\ \Delta T = \frac{\eta_{\text{pump-motor}}\dot{W}_{\text{elect,in}}}{\rho\dot{V}c_p} = \frac{0.65\times(5118\ \text{kJ/s})}{(950.6\ \text{kg/m}^3)(1.5\ \text{m}^3\text{/s})(4.229\ \text{kJ/kg}\cdot{}^\circ\text{C})} = \mathbf{0.55^\circ C}$$

Therefore, the temperature of water will rise at least 0.55°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

13-70 Geothermal water is supplied to a city through cast iron pipes at a specified rate. The electric power consumption and its daily cost are to be determined, and it is to be assessed if the frictional heating during flow can make up for the temperature drop caused by heat loss.

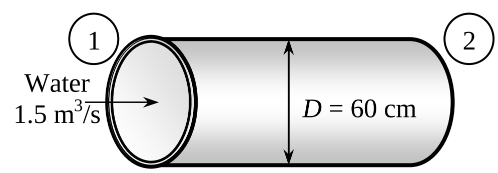
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are ρ = 950.6 kg/m^3, μ = 0.255×10^{-3} kg/m·s, and c_p = 4.229 kJ/kg·°C (Table A-15). The roughness of cast iron pipes is 0.00026 m (Table 13-3).

Analysis (*a*) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($V_1 = V_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_{\text{turbine}}+h_L \quad\rightarrow\quad h_{\text{pump,u}}=h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The mean velocity and the Reynolds number are



$$V_{\text{avg}}=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{1.5\ \text{m}^3/\text{s}}{\pi(0.60\ \text{m})^2/4}=5.305\ \text{m/s}$$

$$\text{Re}=\frac{\rho V_{\text{avg}}D}{\mu}=\frac{(950.6\ \text{kg/m}^3)(5.305\ \text{m/s})(0.60\ \text{m})}{0.255\times10^{-3}\ \text{kg/m}\cdot\text{s}}=1.187\times10^7$$

which is greater than 10,000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D=\frac{0.00026\ \text{m}}{0.60\ \text{m}}=4.33\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}}=-2.0\log\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\text{Re}\sqrt{f}}\right)\ \rightarrow\ \frac{1}{\sqrt{f}}=-2.0\log\left(\frac{4.33\times10^{-4}}{3.7}+\frac{2.51}{1.187\times10^7\sqrt{f}}\right)$$

It gives f = 0.01623. Then the pressure drop, the head loss, and the required power input become

$$\Delta P=f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2}=0.01623\frac{12{,}000\ \text{m}}{0.60\ \text{m}}\frac{(950.6\ \text{kg/m}^3)(5.305\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right)=4342\ \text{kPa}$$

$$\dot{W}_{\text{elect}}=\frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}}=\frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}}=\frac{(1.5\ \text{m}^3/\text{s})(4342\ \text{kPa})}{0.65}\left(\frac{1\ \text{kW}}{1\ \text{kPa}\cdot\text{m}^3/\text{s}}\right)=\mathbf{10{,}020\ kW}$$

Therefore, the pumps will consume 10,017 W of electric power to overcome friction and maintain flow.

(*b*) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount} = \dot{W}_{\text{elect,in}}\Delta t = (10{,}020\text{ kW})(24\text{ h/day}) = 240{,}480\text{ kWh/day}$$

$$\text{Cost} = \text{Amount} \times \text{Unit cost} = (240{,}480\text{ kWh/day})(\$0.06/\text{kWh}) = \mathbf{\$14{,}430/day}$$

(*c*) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 10,020 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{elect}} = \rho\dot{V}c_p\Delta T \;\rightarrow\; \Delta T = \frac{\eta_{\text{pump-motor}}\dot{W}_{\text{elect,in}}}{\rho\dot{V}c_p} = \frac{0.65\times(10{,}020\text{ kJ/s})}{(950.6\text{ kg/m}^3)(1.5\text{ m}^3/\text{s})(4.229\text{ kJ/kg}\cdot{}^\circ\text{C})} = \mathbf{1.08^\circ C}$$

Therefore, the temperature of water will rise at least 1.08°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

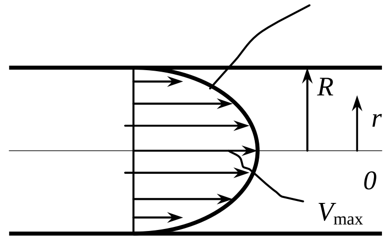
Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

13-71 The velocity profile in fully developed laminar flow in a circular pipe is given. The radius of the pipe, the mean velocity, and the maximum velocity are to be determined.

Assumptions The flow is steady, laminar, and fully developed.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$u(r) = V_{\max}\left(1 - \frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$u(r) = 6(1 - 100r^2)$$

Comparing the two relations above gives the pipe radius, the maximum velocity, and the mean velocity to be

$$R^2 = \frac{1}{100} \quad \rightarrow \quad R = \mathbf{0.10\ m}$$

$$V_{\max} = \mathbf{6\ m/s}$$

$$V_{\text{avg}} = \frac{V_{\max}}{2} = \frac{6\text{ m/s}}{2} = \mathbf{3\ m/s}$$

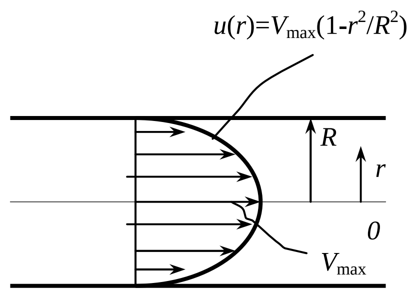
13-72E The velocity profile in fully developed laminar flow in a circular pipe is given. The volume flow rate, the pressure drop, and the useful pumping power required to overcome this pressure drop are to be determined.

Assumptions **1** The flow is steady, laminar, and fully developed. **2** The pipe is horizontal.

Properties The density and dynamic viscosity of water at 40°F are ρ = 62.42 lbm/ft^3 and $\mu = 1.308\times10^{-3}$ lbm/ft·s, respectively (Table A-15E).

Analysis The velocity profile in fully developed laminar flow in a circular pipe is

$$u(r) = V_{\max}\left(1 - \frac{r^2}{R^2}\right)$$



The velocity profile in this case is given by

$$u(r) = 0.8(1 - 625r^2)$$

Comparing the two relations above gives the pipe radius, the maximum velocity, and the mean velocity to be

$$R^2 = \frac{1}{625} \quad \rightarrow \quad R = 0.04 \text{ ft}$$

$V_{\max}$ = 0.8 ft/s

$$V_{\text{avg}} = \frac{V_{\max}}{2} = \frac{0.8 \text{ ft/s}}{2} = 0.4 \text{ ft/s}$$

Then the volume flow rate and the pressure drop become

$$\dot{V} = V_{avg}A_c = V_{avg}(\pi R^2) = (0.4 \text{ ft/s})[\pi(0.04 \text{ ft})^2] = \mathbf{0.00201\ ft^3/s}$$

$$\dot{V}_{\text{horiz}} = \frac{\Delta P \pi D^4}{128\mu L} \quad \rightarrow \quad 0.00201 \text{ ft}^3\text{/s} = \frac{(\Delta P)\pi(0.08 \text{ ft})^4}{128(1.308\times10^{-3} \text{ lbm/ft}\cdot\text{s})(140 \text{ ft})}\left(\frac{32.2 \text{ lbm}\cdot\text{ft/s}^2}{1 \text{ lbf}}\right)$$

It gives

$$\Delta P = 11.37 \text{ lbf/ft}^2 = \mathbf{0.0790\ psi}$$

Then the useful pumping power requirement becomes

$$\dot{W}_{\text{pump,u}} = \dot{V}\Delta P = (0.00201 \text{ ft}^3\text{/s})(11.37 \text{ lbf/ft}^2)\left(\frac{1 \text{ W}}{0.737 \text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.031\ W}$$

Checking The flow was assumed to be laminar. To verify this assumption, we determine the Reynolds number:

$$\text{Re} = \frac{\rho V_{avg} D}{\mu} = \frac{(62.42 \text{ lbm/ft}^3)(0.4 \text{ ft/s})(0.08 \text{ ft})}{1.308\times10^{-3} \text{ lbm/ft}\cdot\text{s}} = 1527$$

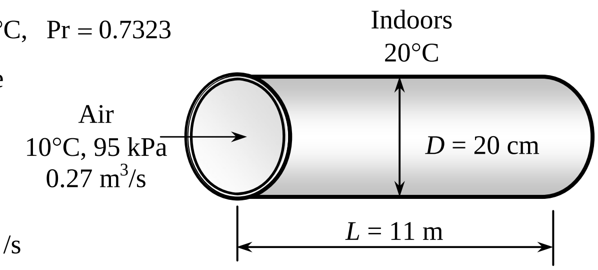
which is less than 2300. Therefore, the flow is laminar.

Discussion Note that the pressure drop across the water pipe and the required power input to maintain flow is negligible. This is due to the very low flow velocity. Such water flows are the exception in practice rather than the rule.

13-73 A compressor is connected to the outside through a circular duct. The power used by compressor to overcome the pressure drop, the rate of heat transfer, and the temperature rise of air are to be determined.
Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties.
Properties We take the bulk mean temperature for air to be 15°C since the mean temperature of air at the inlet will rise somewhat as a result of heat gain through the duct whose surface is exposed to a higher temperature. The properties of air at this temperature and 1 atm pressure are (Table A-22)

$$\rho = 1.225 \text{ kg/m}^3, \quad k = 0.02476 \text{ W/m.°C}$$

$$\nu = 1.470\times10^{-5} \text{ m}^2\text{/s}, \quad c_p = 1007 \text{ J/kg.°C}, \quad \text{Pr} = 0.7323$$



The density and kinematic viscosity at 95 kPa are

$$P = \frac{95 \text{ kPa}}{101.325 \text{ kPa}} = 0.938 \text{ atm}$$

$$\rho = (1.225 \text{ kg/m}^3)(0.938) = 1.149 \text{ kg/m}^3$$

$$\nu = (1.470\times10^{-5} \text{ m}^2\text{/s})/(0.938) = 1.378\times10^{-5} \text{ m}^2\text{/s}$$

Analysis The mean velocity of air is

$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{0.27 \text{ m}^3\text{/s}}{\pi(0.2 \text{ m})^2/4} = 8.594 \text{ m/s}$$

Then $\quad \text{Re} = \dfrac{V_{avg}D_h}{\nu} = \dfrac{(8.594 \text{ m/s})(0.2 \text{ m})}{1.378\times10^{-5} \text{ m}^2\text{/s}} = 1.247\times10^5$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.2 \text{ m}) = 2 \text{ m}$$

which is shorter than the total length of the duct. Therefore, we assume fully developed flow in a smooth pipe, and determine friction factor from

$$f = (0.790 \ln \text{Re} - 1.64)^{-2} = \left[0.790 \ln(1.247\times10^5) - 1.64\right]^{-2} = 0.01718$$

The pressure drop and the compressor power required to overcome this pressure drop are

$$\dot{m} = \rho\dot{V} = (1.149 \text{ kg/m}^3)(0.27 \text{ m}^3\text{/s}) = 0.3102 \text{ kg/s}$$

$$\Delta P = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} = (0.01718)\frac{(11 \text{ m})}{(0.2 \text{ m})}\frac{(1.149 \text{ kg/m}^3)(8.594 \text{ m/s})^2}{2} = 40.09 \text{ N/m}^2$$

$$\dot{W}_{pump} = \frac{\dot{m}\Delta P}{\rho} = \frac{(0.3102 \text{ kg/s})(40.09 \text{ N/m}^2)}{1.149 \text{ kg/m}^3} = \mathbf{10.8 \text{ W}}$$

(*b*) For the fully developed turbulent flow, the Nusselt number is

$$Nu = \frac{hD}{k} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} = 0.023(1.247\times10^5)^{0.8}(0.7323)^{0.4} = 242.3$$

and $\quad h = \dfrac{k}{D_h}Nu = \dfrac{0.02476 \text{ W/m.°C}}{0.2 \text{ m}}(242.3) = 30.00 \text{ W/m}^2\text{.°C}$

Disregarding the thermal resistance of the duct, the rate of heat transfer to the air in the duct becomes

$$A_s = \pi DL = \pi(0.2 \text{ m})(11 \text{ m}) = 6.912 \text{ m}^2$$

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{\dfrac{1}{h_1 A_s} + \dfrac{1}{h_2 A_s}} = \frac{20 - 10}{\dfrac{1}{(30.00)(6.912)} + \dfrac{1}{(10)(6.912)}} = \mathbf{518.4 \text{ W}}$$

(*c*) The temperature rise of air in the duct is

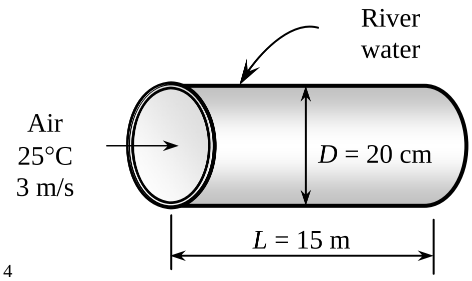
$$\dot{Q} = \dot{m}c_p\Delta T \rightarrow 518.4 \text{ W} = (0.3102 \text{ kg/s})(1007 \text{ J/kg.°C})\Delta T \rightarrow \Delta T = \mathbf{1.7°C}$$

13-74 Air enters the underwater section of a duct. The outlet temperature of the air and the fan power needed to overcome the flow resistance are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** The surface of the duct is at the temperature of the water. **5** Air is an ideal gas with constant properties. **6** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 20°C since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at 1 atm and this temperature are (Table A-22)

$$\rho = 1.204 \text{ kg/m}^3$$
$$k = 0.02514 \text{ W/m.°C}$$
$$\nu = 1.516\times10^{-5} \text{ m}^2\text{/s}$$
$$c_p = 1007 \text{ J/kg.°C}$$
$$\text{Pr} = 0.7309$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(3 \text{ m/s})(0.2 \text{ m})}{1.516\times10^{-5} \text{ m}^2\text{/s}} = 3.958\times10^4$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.2 \text{ m}) = 2 \text{ m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(3.958\times10^4)^{0.8}(0.7309)^{0.3} = 99.76$$

and

$$h = \frac{k}{D_h} Nu = \frac{0.02514 \text{ W/m.°C}}{0.2 \text{ m}}(99.76) = 12.54 \text{ W/m}^2\text{.°C}$$

Next we determine the exit temperature of air,

$$A_s = \pi DL = \pi(0.2 \text{ m})(15 \text{ m}) = 9.425 \text{ m}^2$$

$$\dot{m} = \rho V_{\text{avg}} A_c = (1.204 \text{ kg/m}^3)(3 \text{ m/s})\left(\frac{\pi(0.2 \text{ m})^2}{4}\right) = (1.204 \text{ kg/m}^3)(0.09425 \text{ m}^3\text{/s}) = 0.1135 \text{ kg/s}$$

and

$$T_e = T_s - (T_s - T_i)e^{-hA_s/(\dot{m}c_p)} = 15 - (15-25)e^{-\frac{(12.54)(9.425)}{(0.1135)(1007)}} = \mathbf{18.6°C}$$

The friction factor, pressure drop, and the fan power required to overcome this pressure drop can be determined for the case of fully developed turbulent flow in smooth pipes to be

$$f = (0.790 \ln \text{Re} - 1.64)^{-2} = \left[0.790 \ln(3.958\times10^4) - 1.64\right]^{-2} = 0.02212$$

$$\Delta P = f\,\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} = 0.02212\,\frac{15 \text{ m}}{0.2 \text{ m}}\,\frac{(1.204 \text{ kg/m}^3)(3 \text{ m/s})^2}{2}\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ Pa}}{1 \text{ N/m}^2}\right) = 8.988 \text{ Pa}$$

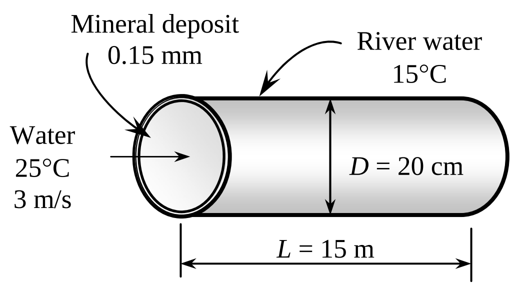
$$\dot{W}_{\text{fan}} = \frac{\dot{W}_{\text{pump,u}}}{\eta_{\text{pump-motor}}} = \frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}} = \frac{(0.09425 \text{ m}^3\text{/s})(8.988 \text{ Pa})}{0.55} = \left(\frac{1 \text{ W}}{1 \text{ Pa}\cdot\text{m}^3\text{/s}}\right) = \mathbf{1.54\ W}$$

13-75 Air enters the underwater section of a duct. The outlet temperature of the air and the fan power needed to overcome the flow resistance are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the duct are smooth. **3** The thermal resistance of the duct is negligible. **4** Air is an ideal gas with constant properties. **5** The pressure of air is 1 atm.

Properties We assume the bulk mean temperature for air to be 20°C since the mean temperature of air at the inlet will drop somewhat as a result of heat loss through the duct whose surface is at a lower temperature. The properties of air at 1 atm and this temperature are (Table A-22)

$$\rho = 1.204\ \text{kg/m}^3$$
$$k = 0.02514\ \text{W/m.°C}$$
$$\nu = 1.516\times10^{-5}\ \text{m}^2/\text{s}$$
$$c_p = 1007\ \text{J/kg.°C}$$
$$\text{Pr} = 0.7309$$



Analysis The Reynolds number is

$$\text{Re} = \frac{V_{avg}D_h}{\nu} = \frac{(3\ \text{m/s})(0.2\ \text{m})}{1.516\times10^{-5}\ \text{m}^2/\text{s}} = 3.958\times10^4$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.2\ \text{m}) = 2\ \text{m}$$

which is much shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number and h from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{0.3} = 0.023(3.959\times10^4)^{0.8}(0.7309)^{0.3} = 99.76$$

and $$h = \frac{k}{D_h}Nu = \frac{0.02514\ \text{W/m.°C}}{0.2\ \text{m}}(99.76) = 12.54\ \text{W/m}^2.\text{°C}$$

Next we determine the exit temperature of air,

$$A_s = \pi DL = \pi(0.2\ \text{m})(15\ \text{m}) = 9.425\ \text{m}^2$$

$$\dot{m} = \rho V_{avg}A_c = (1.204\ \text{kg/m}^3)(3\ \text{m/s})\left(\frac{\pi(0.2\ \text{m})^2}{4}\right) = (1.204\ \text{kg/m}^3)(0.09425\ \text{m}^3/\text{s}) = 0.1135\ \text{kg/s}$$

The unit thermal resistance of the mineral deposit is

$$R_{mineral} = \frac{L}{k} = \frac{0.0025\ \text{m}}{3\ \text{W/m.°C}} = 0.00083\ \text{m}^2.\text{°C/W}$$

which is much less than (about 1%) the unit convection resistance,

$$R_{conv} = \frac{1}{h} = \frac{1}{12.54\ \text{W/m}^2.\text{°C}} = 0.0797\ \text{m}^2.\text{°C/W}$$

Therefore, the effect of 0.25 mm thick mineral deposit on heat transfer is negligible.

Next we determine the exit temperature of air

$$T_e = T_s - (T_s - T_i)e^{-hA/(\dot{m}c_p)} = 15 - (15-25)e^{-\frac{(12.54)(9.425)}{(0.1135)(1007)}} = \mathbf{18.6°C}$$

The friction factor, pressure drop, and the fan power required to overcome this pressure drop can be determined for the case of fully developed turbulent flow in smooth pipes to be

$$f = (0.790\ln\text{Re} - 1.64)^{-2} = \left[0.790\ln(3.958\times10^4) - 1.64\right]^{-2} = 0.02212$$

$$\Delta P = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.02212\frac{15\ \text{m}}{0.2\ \text{m}}\frac{(1.204\ \text{kg/m}^3)(3\ \text{m/s})^2}{2}\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{Pa}}{1\ \text{N/m}^2}\right) = 8.988\ \text{Pa}$$

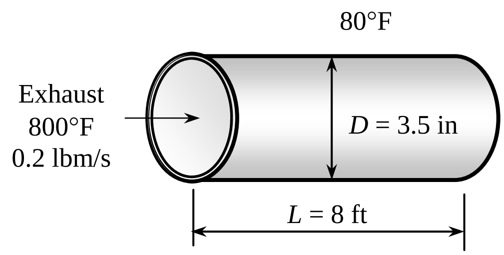
$$\dot{W}_{fan} = \frac{\dot{W}_{pump,u}}{\eta_{pump\text{-}motor}} = \frac{\dot{V}\Delta P}{\eta_{pump\text{-}motor}} = \frac{(0.09425\ \text{m}^3/\text{s})(8.988\ \text{Pa})}{0.55} = \left(\frac{1\ \text{W}}{1\ \text{Pa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{1.54\ W}$$

13-76E The exhaust gases of an automotive engine enter a steel exhaust pipe. The velocity of exhaust gases at the inlet and the temperature of exhaust gases at the exit are to be determined.

Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the pipe are smooth. **3** The thermal resistance of the pipe is negligible. **4** Exhaust gases have the properties of air, which is an ideal gas with constant properties.

Properties We take the bulk mean temperature for exhaust gases to be 700°F since the mean temperature of gases at the inlet will drop somewhat as a result of heat loss through the exhaust pipe whose surface is at a lower temperature. The properties of air at this temperature and 1 atm pressure are (Table A-22E)

$$\rho = 0.03421 \text{ lbm/ft}^3 \qquad c_p = 0.2535 \text{ Btu/lbm.°F}$$
$$k = 0.0280 \text{ Btu/h.ft.°F} \qquad \Pr = 0.694$$
$$\nu = 6.225\times10^{-4} \text{ ft}^2\text{/s}$$



Noting that 1 atm = 14.7 psia, the pressure in atm is
P = (15.5 psia)/(14.7 psia) = 1.054 atm. Then,

$$\rho = (0.03421 \text{ lbm/ft}^3)(1.054) = 0.03606 \text{ lbm/ft}^3$$
$$\nu = (6.225\times10^{-4} \text{ ft}^2\text{/s})/(1.054) = 5.906\times10^{-4} \text{ ft}^2\text{/s}$$

Analysis (a) The velocity of exhaust gases at the inlet of the exhaust pipe is

$$\dot{m} = \rho V_{\text{avg}} A_c \longrightarrow V_{\text{avg}} = \frac{\dot{m}}{\rho A_c} = \frac{0.2 \text{ lbm/s}}{(0.03606 \text{ lbm/ft}^3)\left(\pi(3.5/12 \text{ ft})^2/4\right)} = \mathbf{83.01 \text{ ft/s}}$$

(b) The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(83.01 \text{ ft/s})(3.5/12 \text{ ft})}{5.906\times10^{-4} \text{ ft}^2\text{/s}} = 40{,}990$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(3.5/12 \text{ ft}) = 2.917 \text{ ft}$$

which are shorter than the total length of the duct. Therefore, we can assume fully developed turbulent flow in the entire duct, and determine the Nusselt number from

$$Nu = \frac{hD_h}{k} = 0.023\,\text{Re}^{0.8}\,\Pr^{0.3} = 0.023(40{,}990)^{0.8}(0.6940)^{0.3} = 101.0$$

and $$h_i = h = \frac{k}{D_h} Nu = \frac{0.0280 \text{ Btu/h.ft.°F}}{(3.5/12) \text{ ft}}(101.0) = 9.70 \text{ Btu/h.ft}^2\text{.°F}$$

$$A_s = \pi DL = \pi(3.5/12 \text{ ft})(8 \text{ ft}) = 7.33 \text{ ft}^2$$

In steady operation, heat transfer from exhaust gases to the duct must be equal to the heat transfer from the duct to the surroundings, which must be equal to the energy loss of the exhaust gases in the pipe. That is,

$$\dot{Q} = \dot{Q}_{\text{internal}} = \dot{Q}_{\text{external}} = \Delta\dot{E}_{\text{exhaust gases}}$$

Assuming the duct to be at an average temperature of T_s, the quantities above can be expressed as

$\dot{Q}_{\text{internal}}$: $$\dot{Q} = h_i A_s \Delta T_{\ln} = h_i A_s \frac{T_e - T_i}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} \rightarrow \dot{Q} = (9.70 \text{ Btu/h.ft}^2\text{.°F})(7.33 \text{ ft}^2)\frac{T_e - 800\text{°F}}{\ln\left(\frac{T_s - T_e}{T_s - 800}\right)}$$

$\dot{Q}_{\text{external}}$: $$\dot{Q} = h_o A_s (T_s - T_o) \rightarrow \dot{Q} = (3 \text{ Btu/h.ft}^2\text{.°F})(7.33 \text{ ft}^2)(T_s - 80)\text{°F}$$

$\Delta\dot{E}_{\text{exhaust gases}}$: $$\dot{Q} = \dot{m}c_p(T_e - T_i) \rightarrow \dot{Q} = (0.2\times3600 \text{ lbm/h})(0.2535 \text{ Btu/lbm.°F})(800 - T_e)\text{°F}$$

This is a system of three equations with three unknowns whose solution is

$$\dot{Q} = 11{,}528 \text{ Btu/h},\; T_e = \mathbf{736.8°F},\; \text{and}\; T_s = 604.2\text{°F}$$

13-77 Hot water enters a cast iron pipe whose outer surface is exposed to cold air with a specified heat transfer coefficient. The rate of heat loss from the water and the exit temperature of the water are to be determined.

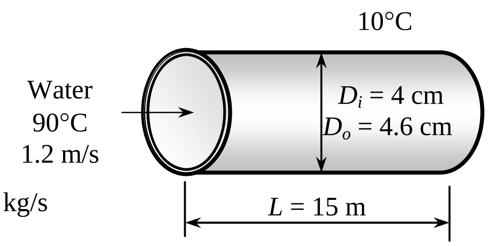
Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the pipe are smooth.

Properties We assume the water temperature not to drop significantly since the pipe is not very long. We will check this assumption later. The properties of water at 90°C are (Table A-15)

$\rho = 965.3\ \text{kg/m}^3$; $\quad k = 0.675\ \text{W/m.°C}$

$\nu = \mu/\rho = 0.326\times10^{-6}\ \text{m}^2/\text{s}$; $\quad c_p = 4206\ \text{J/kg.°C}$

$\text{Pr} = 1.96$



Analysis (a) The mass flow rate of water is

$$\dot{m} = \rho A_c V_{\text{avg}} = (965.3\ \text{kg/m}^3)\frac{\pi(0.04\ \text{m})^2}{4}(1.2\ \text{m/s}) = 1.456\ \text{kg/s}$$

The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(1.2\ \text{m/s})(0.04\ \text{m})}{0.326\times10^{-6}\ \text{m}^2/\text{s}} = 147{,}240$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.04\ \text{m}) = 0.4\ \text{m}$$

which are much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe. The friction factor corresponding to Re = 147,240 and ε/D = (0.026 cm)/(4 cm) = 0.0065 is determined from the Moody chart to be f = 0.032. Then the Nusselt number becomes

$$Nu = \frac{hD_h}{k} = 0.125 f\ \text{Re}\,\text{Pr}^{1/3} = 0.125\times0.032\times147{,}240\times1.96^{1/3} = 737.1$$

and $$h_i = h = \frac{k}{D_h}Nu = \frac{0.675\ \text{W/m.°C}}{0.04\ \text{m}}(737.1) = 12{,}440\ \text{W/m}^2.\text{°C}$$

which is much greater than the convection heat transfer coefficient of 12 W/m^2.°C. Therefore, the convection thermal resistance inside the pipe is negligible, and thus the inner surface temperature of the pipe can be taken to be equal to the water temperature. Also, we expect the pipe to be nearly isothermal since it is made of thin metal (we check this later). Then the rate of heat loss from the pipe will be the sum of the convection and radiation from the outer surface at a temperature of 90°C, and is determined to be

$$A_o = \pi D_0 L = \pi(0.046\ \text{m})(15\ \text{m}) = 2.168\ \text{m}^2$$

$$\dot{Q}_{conv} = h_o A_o (T_s - T_{surr}) = (12\ \text{W/m}^2.\text{°C})(2.168\ \text{m}^2)(90-10)\text{°C} = 2081\ \text{W}$$

$$\dot{Q}_{rad} = \varepsilon A_0 \sigma (T_s^{\,4} - T_{surr}^{\,4})$$
$$= (0.7)(2.168\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)\left[(90+273\ \text{K})^4 - (10+273\ \text{K})^4\right] = 942\ \text{W}$$

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 2081 + 942 = \mathbf{3023\ W}$$

(b) The temperature at which water leaves the basement is

$$\dot{Q} = \dot{m}c_p(T_i - T_e) \longrightarrow T_e = T_i - \frac{\dot{Q}}{\dot{m}c_p} = 90\text{°C} - \frac{3023\ \text{W}}{(1.456\ \text{kg/s})(4206\ \text{J/kg.°C})} = \mathbf{89.5°C}$$

The result justifies our assumption that the temperature drop of water is negligible. Also, the thermal resistance of the pipe and temperature drop across it are

$$R_{pipe} = \frac{\ln(D_2/D_1)}{2\pi kL} = \frac{\ln(4.6/4)}{4\pi(52\ \text{W/m.°C})(15\ \text{m})} = 2.85\times10^{-5}\ \text{°C/W}$$

$$\Delta T_{pipe} = \dot{Q}_{total} R_{pipe} = (3023\ \text{W})(2.85\times10^{-5}\ \text{°C/W}) = 0.09\text{°C}$$

which justifies our assumption that the temperature drop across the pipe is negligible.

13-78 Hot water enters a copper pipe whose outer surface is exposed to cold air with a specified heat transfer coefficient. The rate of heat loss from the water and the exit temperature of the water are to be determined.

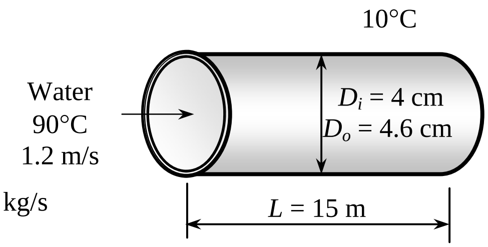
Assumptions **1** Steady flow conditions exist. **2** The inner surfaces of the pipe are smooth.

Properties We assume the water temperature not to drop significantly since the pipe is not very long. We will check this assumption later. The properties of water at 90°C are (Table A-22)

$\rho = 965.3 \text{ kg/m}^3$; $k = 0.675 \text{ W/m.°C}$

$\nu = \mu/\rho = 0.326\times10^{-6} \text{ m}^2\text{/s}$; $c_p = 4206 \text{ J/kg.°C}$

$\text{Pr} = 1.96$



Analysis (a) The mass flow rate of water is

$$\dot{m} = \rho A_c V_{\text{avg}} = (965.3 \text{ kg/m}^3)\frac{\pi(0.04 \text{ m})^2}{4}(1.2 \text{ m/s}) = 1.456 \text{ kg/s}$$

The Reynolds number is

$$\text{Re} = \frac{V_{\text{avg}} D_h}{\nu} = \frac{(1.2 \text{ m/s})(0.04 \text{ m})}{0.326\times10^{-6} \text{ m}^2\text{/s}} = 147{,}240$$

which is greater than 10,000. Therefore, the flow is turbulent and the entry lengths in this case are roughly

$$L_h \approx L_t \approx 10D = 10(0.04 \text{ m}) = 0.4 \text{ m}$$

which are much shorter than the total length of the pipe. Therefore, we can assume fully developed turbulent flow in the entire pipe. The friction factor corresponding to Re = 147,240 and ε/D = (0.026 cm)/(4 cm) = 0.0065 is determined from the Moody chart to be f = 0.032. Then the Nusselt number becomes

$$Nu = \frac{hD_h}{k} = 0.125 f \text{ Re Pr}^{1/3} = 0.125\times0.032\times147{,}240\times1.96^{1/3} = 737.1$$

and $$h_i = h = \frac{k}{D_h} Nu = \frac{0.675 \text{ W/m.°C}}{0.04 \text{ m}}(737.1) = 12{,}440 \text{ W/m}^2\text{.°C}$$

which is much greater than the convection heat transfer coefficient of 12 W/m^2.°C. Therefore, the convection thermal resistance inside the pipe is negligible, and thus the inner surface temperature of the pipe can be taken to be equal to the water temperature. Also, we expect the pipe to be nearly isothermal since it is made of thin metal (we check this later). Then the rate of heat loss from the pipe will be the sum of the convection and radiation from the outer surface at a temperature of 90°C, and is determined to be

$$A_o = \pi D_0 L = \pi(0.046 \text{ m})(15 \text{ m}) = 2.168 \text{ m}^2$$

$$\dot{Q}_{conv} = h_o A_o (T_s - T_{surr}) = (12 \text{ W/m}^2\text{.°C})(2.168 \text{ m}^2)(90-10)\text{°C} = 2081 \text{ W}$$

$$\dot{Q}_{rad} = \varepsilon A_0 \sigma (T_s^{\,4} - T_{surr}^{\,4})$$
$$= (0.7)(2.168 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(90+273 \text{ K})^4 - (10+273 \text{ K})^4\right] = 942 \text{ W}$$

$$\dot{Q}_{total} = \dot{Q}_{conv} + \dot{Q}_{rad} = 2081 + 942 = \mathbf{3023 \text{ W}}$$

(b) The temperature at which water leaves the basement is

$$\dot{Q} = \dot{m} c_p (T_i - T_e) \longrightarrow T_e = T_i - \frac{\dot{Q}}{\dot{m} c_p} = 90\text{°C} - \frac{3023 \text{ W}}{(1.456 \text{ kg/s})(4206 \text{ J/kg.°C})} = \mathbf{89.5°C}$$

The result justifies our assumption that the temperature drop of water is negligible. Also, the thermal resistance of the pipe and temperature drop across it are

$$R_{pipe} = \frac{\ln(D_2/D_1)}{4\pi kL} = \frac{\ln(4.6/4)}{2\pi(386 \text{ W/m.°C})(15 \text{ m})} = 3.84\times10^{-6} \text{ °C/W}$$

$$\Delta T_{pipe} = \dot{Q}_{total} R_{pipe} = (3023 \text{ W})(3.84\times10^{-6} \text{ °C/W}) = 0.012\text{°C}$$

which justifies our assumption that the temperature drop across the pipe is negligible.

13-79 …. 13-81 Design and Essay Problems

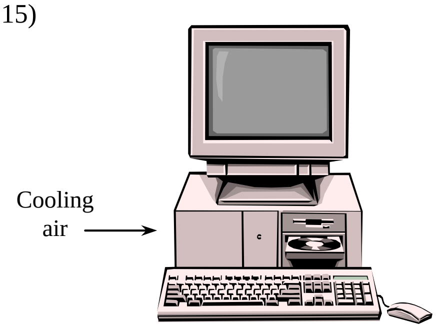
13-81 A computer is cooled by a fan blowing air through the case of the computer. The flow rate of the fan and the diameter of the casing of the fan are to be specified.

Assumptions **1** Steady flow conditions exist. **2** Heat flux is uniformly distributed. **3** Air is an ideal gas with constant properties.

Properties The relevant properties of air are (Tables A-1 and A-15)

$$c_p = 1007 \text{ J/kg.°C}$$

$$R = 0.287 \text{ kPa.m}^3\text{/kg.K}$$



Analysis We need to determine the flow rate of air for the worst case scenario. Therefore, we assume the inlet temperature of air to be 50°C, the atmospheric pressure to be 70.12 kPa, and disregard any heat transfer from the outer surfaces of the computer case. The mass flow rate of air required to absorb heat at a rate of 80 W can be determined from

$$\dot{Q} = \dot{m}c_p(T_{out} - T_{in}) \longrightarrow \dot{m} = \frac{\dot{Q}}{c_p(T_{out} - T_{in})} = \frac{80 \text{ J/s}}{(1007 \text{ J/kg.°C})(60-50)\text{°C}} = 0.007944 \text{ kg/s}$$

In the worst case the exhaust fan will handle air at 60°C. Then the density of air entering the fan and the volume flow rate becomes

$$\rho = \frac{P}{RT} = \frac{70.12 \text{ kPa}}{(0.287 \text{ kPa.m}^3\text{/kg.K})(60+273)\text{K}} = 0.7337 \text{ kg/m}^3$$

$$\dot{V} = \frac{\dot{m}}{\rho} = \frac{0.007944 \text{ kg/s}}{0.7337 \text{ kg/m}^3} = 0.01083 \text{ m}^3\text{/s} = \mathbf{0.6497 \text{ m}^3\text{/min}}$$

For an average velocity of 120 m/min, the diameter of the duct in which the fan is installed can be determined from

$$\dot{V} = A_c V = \frac{\pi D^2}{4} V \longrightarrow D = \sqrt{\frac{4\dot{V}}{\pi V}} = \sqrt{\frac{4(0.6497 \text{ m}^3\text{/min})}{\pi(120 \text{ m/min})}} = 0.083 \text{ m} = \mathbf{8.3 \text{ cm}}$$