10-117 Circular aluminum fins are to be attached to the tubes of a heating system. The increase in heat transfer from the tubes per unit length as a result of adding fins is to be determined.

Assumptions 1 Steady operating conditions exist. 2 The heat transfer coefficient is constant and uniform over the entire fin surfaces. 3 Thermal conductivity is constant. 4 Heat transfer by radiation is negligible.

Properties The thermal conductivity of the fins is given to be $k = 186 \text{ W/m} \cdot ^{\circ}\text{C}$.

Analysis In case of no fins, heat transfer from the tube per meter of its length is

$$A_{\text{no fin}} = \pi D_1 L = \pi (0.05 \text{ m})(1 \text{ m}) = 0.1571 \text{ m}^2$$

$$\dot{Q}_{\text{no fin}} = h A_{\text{no fin}} (T_b - T_\infty) = (40 \text{ W/m}^2.^{\circ}\text{C})(0.1571 \text{ m}^2)(180 - 25)^{\circ}\text{C} = 974 \text{ W}$$
The efficiency of these circular fins is, from the efficiency curve, Fig. 10-43
$$L = (D_2 - D_1)/2 = (0.06 - 0.05)/2 = 0.005 \text{ m}$$

$$\frac{r_2 + (t/2)}{r_1} = \frac{0.03 + (0.001/2)}{0.025} = 1.22$$

$$L_c^{3/2} \left(\frac{h}{kA_p}\right)^{1/2} = \left(L + \frac{t}{2}\right) \sqrt{\frac{h}{kt}}$$

$$= \left(0.005 + \frac{0.001}{2}\right) \sqrt{\frac{40 \text{ W/m}^{2 \circ}\text{C}}{(186 \text{ W/m}^{\circ}\text{C})(0.001 \text{ m})}} = 0.08$$

Heat transfer from a single fin is

$$A_{\text{fin}} = 2\pi (r_2^2 - r_1^2) + 2\pi r_2 t = 2\pi (0.03^2 - 0.025^2) + 2\pi (0.03)(0.001) = 0.001916 \text{ m}^2$$

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}} \dot{Q}_{\text{fin,max}} = \eta_{\text{fin}} h A_{\text{fin}} (T_b - T_\infty)$$

$$= 0.97 (40 \text{ W/m}^2.^{\circ}\text{C}) (0.001916 \text{ m}^2) (180 - 25)^{\circ}\text{C}$$

$$= 11.53 \text{ W}$$

Heat transfer from a single unfinned portion of the tube is

$$A_{\text{unfin}} = \pi D_1 s = \pi (0.05 \text{ m})(0.003 \text{ m}) = 0.0004712 \text{ m}^2$$

$$\dot{Q}_{\text{unfin}} = h A_{\text{unfin}} (T_b - T_{\infty}) = (40 \text{ W/m}^2.^{\circ}\text{C})(0.0004712 \text{ m}^2)(180 - 25)^{\circ}\text{C} = 2.92 \text{ W}$$

There are 250 fins and thus 250 interfin spacings per meter length of the tube. The total heat transfer from the finned tube is then determined from

$$\dot{Q}_{\text{total,fin}} = n(\dot{Q}_{\text{fin}} + \dot{Q}_{\text{unfin}}) = 250(11.53 + 2.92) = 3613 \text{ W}$$

Therefore the increase in heat transfer from the tube per meter of its length as a result of the addition of the fins is

$$\dot{Q}_{\text{increase}} = \dot{Q}_{\text{total,fin}} - \dot{Q}_{\text{no fin}} = 3613 - 974 = 2639 \text{ W}$$