9-54 An ideal dual cycle has a compression ratio of 14 and cutoff ratio of 1.2. The thermal efficiency, amount of heat added, and the maximum gas pressure and temperature are to be determined.

Assumptions 1 The air-standard assumptions are applicable. 2 Kinetic and potential energy changes are negligible. 3 Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are $c_p = 1.005 \text{ kJ/kg·K}$, $c_v = 0.718 \text{ kJ/kg·K}$, R = 0.287 kJ/kg·K, and k = 1.4 (Table A-2).

Analysis The specific volume of the air at the start of the compression is

$$\mathbf{v}_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(253 \text{ K})}{80 \text{ kPa}} = 0.9076 \text{ m}^3/\text{kg}$$

and the specific volume at the end of the compression is

$$\mathbf{v}_2 = \frac{\mathbf{v}_1}{r} = \frac{0.9076 \,\mathrm{m}^3/\mathrm{kg}}{14} = 0.06483 \,\mathrm{m}^3/\mathrm{kg}$$

The pressure at the end of the compression is

$$P_2 = P_1 \left(\frac{v_1}{v_2}\right)^k = P_1 r^k = (80 \text{ kPa})(14)^{1.4} = 3219 \text{ kPa}$$

and the maximum pressure is

$$P_x = P_3 = r_p P_2 = (1.5)(3219 \text{ kPa}) = 4829 \text{ kPa}$$

The temperature at the end of the compression is

$$T_2 = T_1 \left(\frac{\mathbf{v}_1}{\mathbf{v}_2}\right)^{k-1} = T_1 r^{k-1} = (253 \text{ K})(14)^{1.4-1} = 727.1 \text{ K}$$

and

$$T_x = T_2 \left(\frac{P_3}{P_2}\right) = (727.1 \text{ K}) \left(\frac{4829 \text{ kPa}}{3219 \text{ kPa}}\right) = 1091 \text{ K}$$

From the definition of cutoff ratio

$$\mathbf{v}_3 = r_c \mathbf{v}_x = r_c \mathbf{v}_2 = (1.2)(0.06483 \,\mathrm{m}^3/\mathrm{kg}) = 0.07780 \,\mathrm{m}^3/\mathrm{kg}$$

The remaining state temperatures are then

$$T_3 = T_x \left(\frac{\mathbf{v}_3}{\mathbf{v}_x} \right) = (1091 \text{ K}) \left(\frac{0.07780}{0.06483} \right) = \mathbf{1309 K}$$

$$T_4 = T_3 \left(\frac{\boldsymbol{v}_3}{\boldsymbol{v}_4}\right)^{k-1} = (1309 \text{ K}) \left(\frac{0.07780}{0.9076}\right)^{1.4-1} = 490.0 \text{ K}$$

Applying the first law and work expression to the heat addition processes gives

$$\begin{split} q_{\rm in} &= c_{\it v} (T_{\it x} - T_{\it 2}) + c_{\it p} (T_{\it 3} - T_{\it x}) \\ &= (0.718 \, {\rm kJ/kg \cdot K}) (1091 - 727.1) {\rm K} + (1.005 \, {\rm kJ/kg \cdot K}) (1309 - 1091) {\rm K} \\ &= \textbf{480.4 kJ/kg} \end{split}$$

The heat rejected is

$$q_{\text{out}} = c_v (T_4 - T_1) = (0.718 \text{ kJ/kg} \cdot \text{K})(490.0 - 253)\text{K} = 170.2 \text{ kJ/kg}$$

Then,
$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{170.2 \text{ kJ/kg}}{480.4 \text{ kJ/kg}} = \textbf{0.646}$$

