Thermal Joint Resistances of Conforming Rough Surfaces with Gas-Filled Gaps

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An approximate analytical model is developed for predicting the heat transfer of interstitial gases in the gap between conforming rough contacts. A simple relationship for the gap thermal resistance is derived by assuming that the contacting surfaces are of uniform temperature and that the gap heat transfer area and the apparent contact area are identical. The model covers the four regimes of gas heat conduction modes, that is, continuum, temperature jump or slip, transition, and free molecular. Effects of main input parameters on the gap and joint thermal resistances are investigated. The model is compared with other models and with more than 510 experimental data points in the open literature. Good agreement is shown over the entire range of the comparison.

Nomenclature

\[ A = \text{area, m}^2 \]
\[ a = \text{radius of contact, m} \]
\[ b_L = \text{specimens radius, m} \]
\[ c_1 = \text{Vickers microhardness coefficient, Pa} \]
\[ c_2 = \text{Vickers microhardness coefficient} \]
\[ d = \text{distance between two parallel plates, m} \]
\[ F = \text{external force, N} \]
\[ H_{mic} = \text{microhardness, Pa} \]
\[ H' = c_1(1.62\sigma^2/m)^2, \text{Pa} \]
\[ H'^* = c_1(\sigma'/m)^2, \text{Pa} \]
\[ K_n = \text{Knudsen number} \]
\[ k = \text{thermal conductivity, W/mK} \]
\[ l = \text{depth, m} \]
\[ M = \text{gas parameter, m} \]
\[ M_s = \text{molecular weight of solid, kg/kmol} \]
\[ m = \text{mean absolute surface slope} \]
\[ n_s = \text{number of microcontacts} \]
\[ P = \text{pressure, Pa} \]
\[ Pr = \text{Prandtl number} \]
\[ Q = \text{heat flow rate, W} \]
\[ q = \text{heat flux, W/m}^2 \]
\[ R = \text{thermal resistance, K/W} \]
\[ r, z = \text{cylindrical coordinates} \]
\[ T = \text{temperature, K} \]
\[ t = \text{dummy variable} \]
\[ Y = \text{mean surface plane separation, m} \]
\[ z = \text{surface height, m} \]
\[ \sigma_r = \text{thermal accommodation coefficient} \]
\[ \gamma = \text{ratio of gas specific heats} \]
\[ \Lambda = \text{mean free path, m} \]
\[ \lambda = \text{nondimensional separation} = \frac{Y}{\sqrt{2\sigma}} \]
\[ \mu = \text{ratio of molecular weights} = \frac{M_g}{M_s} \]
\[ \sigma = \text{rms surface roughness, m} \]
\[ \sigma' = \frac{\sigma}{\sigma_0}, \text{where} \sigma_0 = 1 \mu \text{m} \]
\[ \phi(z) = \text{Gaussian distribution of surface heights} \]
\[ \omega = \text{asperity deformation, m} \]

Subscripts

\[ a = \text{apparent} \]
\[ \text{cont} = \text{continuum} \]
\[ \text{fm} = \text{free molecular} \]
\[ g = \text{gas, gap} \]
\[ j = \text{joint} \]
\[ L = \text{large} \]
\[ \text{mic} = \text{micro} \]
\[ \text{mic, e} = \text{effective micro} \]
\[ r = \text{real} \]
\[ s = \text{solid, micro} \]
\[ \text{tr} = \text{transition} \]
\[ 0 = \text{reference value} \]
\[ 1, 2 = \text{solid 1, 2} \]

Introduction

HEAT transfer through interfaces formed by mechanical contacts has many important applications such as microelectronics cooling, nuclear engineering, and spacecraft structures design. Generally the heat transfer through the contact interfaces is associated with the presence of interstitial gases. The rate of heat transfer across the joint depends on a number of parameters: thermal properties of solids and gas, gas pressure, surface roughness characteristics, applied load, and contact microhardness.

When random rough surfaces are placed in mechanical contact, real contact occurs at the top of surface asperities called microcontacts. The microcontacts are distributed randomly in the apparent contact area \( A_a \) and located far from each other. In the real contact area \( A_r \), the summation of microcontacts forms a small portion of the nominal contact area typically a few percent of the nominal contact area.

The geometry of a typical conforming rough contact is shown in Fig. 1, where two cylindrical bodies with a radius of \( b_L \) are placed in mechanical contact. The gap between the microcontacts is filled with an interstitial gas, and heat is transferred from one body to the other. Conduction through microcontacts and the interstitial gas in the gap between the solids are the two main paths for transferring thermal energy between contacting bodies. Thermal radiation across the gap remains small as long as the surface temperatures are not too high, that is, less than 700 K, and in most applications can be neglected.\(^1\) As a result of the small real contact area and low thermal...
conducitivities of interstitial gases, heat flow experiences a relatively large thermal resistance passing through the joint; this phenomenon leads to a relatively high-temperature drop across the interface.

Natural convection does not occur within the fluid when the Grashof number is below 2500 (Ref. 2). (The Grashof number can be interpreted as the ratio of buoyancy to viscous forces.) In most practical situations concerning thermal contact resistance, the gap thickness between two contacting bodies is quite small (<0.01 mm); thus, the Grashof number based on the gap thickness is less than 2500. Consequently, in most instances the heat transfer through the interstitial gas in the gap occurs by conduction.

In applications where the contact pressure is relatively low, the real contact area is limited to an even smaller portion of the apparent area, on the order of 1% or less. Consequently, the heat transfer takes place mainly through the interstitial gas in the gap. The relative magnitude of the gap heat transfer varies greatly with the applied load, surface roughness, gas pressure, and ratio of the thermal conductivities between the gas and solids. As the contact pressure increases, the heat transfer through the microcontacts increases and becomes more significant. Many engineering applications of thermal contact resistance (TCR) are associated with low contact pressure where air (interstitial gas) is at atmospheric pressure; therefore, modeling of the gap resistance is an important issue.

The goal of this study is to develop an approximate, comprehensive, yet simple model for determining the heat transfer through the gap between conforming rough surfaces. This model will be used in the second part of this work to develop an analytical compact model for predicting the TCR of nonconforming rough contacts in the presence of an interstitial gas. The model covers the entire range of gas conduction heat transfer modes, that is, continuum, slip, transition, and free molecular.

**Theoretical Background**

TCR of conforming rough surfaces in the presence of interstitial gas includes two components, thermal constriction/spreading resistance of microcontacts, $R_s$, and gap thermal resistance $R_g$.

**Microcontacts Heat Transfer**

All solid surfaces are rough, where this roughness or surface texture can be thought of as the surface heights’ deviation from the nominal topography. If the asperities of a surface are isotropic and randomly distributed over the surface, the surface is called Gaussian. Williamson et al. have shown experimentally that many of the techniques used to produce engineering surfaces give a Gaussian distribution of surface heights. Many researchers including Greenwood and Williamson assumed that the contact between two Gaussian rough surfaces can be simplified to the contact between a single Gaussian surface, having effective surface characteristics, with a perfectly smooth surface, where the mean separation between the two contacting planes $Y$ remains the same (Fig. 2); for more details, see Bahrami et al.. The equivalent roughness $\sigma$ and surface slope $m$ can be found from

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}, \quad m = \sqrt{m_1^2 + m_2^2} \quad (1)$$

It is common to assume that the microcontacts are isothermal. Thermal constriction/spreading resistance of microcontacts can be modeled by using a flux tube geometry, or if microcontacts are considered to be located far (enough) from each other, the isothermal heat source on a half-space solution can be used. When comparison was made with the earlier mentioned solutions, that is, the flux tube and the half-space, Bahrami et al. showed that the microcontacts can be modeled as heat sources on a half-space for engineering TCR applications.

Bahrami et al., assuming plastically deformed asperities and using scaling analysis techniques, developed an analytical model to predict TCR of conforming rough contacts in a vacuum, $R_s$,

$$R_s = \frac{0.565HS(\sigma/m)}{k_F} \quad (2)$$

where

$$k_s = 2k_1k_2/(k_1 + k_2), \quad H^* = c_1(\sigma'/m)^2$$

They compared their model with more than 600 TCR experimental data points collected in a vacuum by many researchers and showed good agreement. The rms difference between Eq. (2) and the data was reported to be approximately 14%.

**Gap Heat Transfer**

According to Springer, conduction heat transfer in a gas layer between two parallel plates is commonly categorized into four heat-flow regimes: continuum, temperature jump or slip, transition, and free molecular. The parameter that characterizes the regimes is the
where \( T \) is the temperature of the gas, and \( A_g \) is the gas heat transfer area. The gas parameter \( M \) is defined as

\[
M = \left[ (2 - \alpha_{T1})/\alpha_{T1} + (2 - \alpha_{T2})/\alpha_{T2} \right] \left[ 2Y/((1 + Y))(1/Pr) \right] \Lambda
\]

where \( \alpha_{T1}, \alpha_{T2}, \) and \( \Lambda \) are thermal accommodation coefficients corresponding to the gas–solid combination of plates 1 and 2 and the molecular mean free path at \( P_1 \) and \( P_2 \), respectively. Thermal accommodation coefficient \( \alpha_T \) depends on the type of the gas–solid combination and is in general very sensitive to the condition of the solid surfaces. It represents the degree to which the kinetic energy of a gas molecule is exchanged while in collision with the solid wall. Song and Yovanovich\(^1\) proposed a correlation for predicting \( \alpha_T \) for engineering surfaces,

\[
\alpha_T = \exp \left[ -0.57 \left( \frac{T_r - T_0}{T_0} \right) \right] \left( M_g^* + 6.8 + M_g^* \right)
\]

where for monatomic gases\(^1\)

\[
M_g^* = M_g
\]

and for diatomic/polyatomic gases\(^1\)

\[
M_g^* = 1.4 M_g
\]

where \( T_0 = 273 \) K. Equation (8) is general and can be used for any combination of gases and solid surfaces for a wide temperature range. The agreement between the predicted values and the experimental data is within 25%.

Yovanovich et al.\(^1\) developed a statistical sophisticated model (here called the integral model) to predict thermal gap conductance between conforming rough surfaces. The integral model takes into consideration the variation in the local gap thickness due to the surface roughness. It assumes that the temperature of the two surfaces in contact are uniform and the interface gap consists of many elemental flux tubes of different thermal resistances. The resistances of these elemental flux tubes are then assumed to be in parallel, which results in an overall gap conductance in an integral form that may be represented in thermal resistance form as

\[
R_g = \sqrt{2\pi Y} \int_{0}^{\infty} \exp\left(\frac{-\left(Y/\sigma - t/\sigma\right)^2}{2}\right) \left(\frac{t}{\sigma}\right) d(\frac{t}{\sigma})
\]

where \( R_g, t, k_g, A_g, \) and \( Y \) are thermal gap resistance, length of the elemental flux tube or the local gap thickness, thermal conductivity of the gas, gap heat transfer area, and mean plane separation distance, respectively.

Equation (9) is in integral form, and its evaluation requires a numerical integration. Song\(^1\) correlated Eq. (9) and proposed an expression that can be written as follows:

\[
R_g = \frac{Y}{k_g A_g} \left[ 1 + \frac{M}{Y} + \frac{0.304(M Y)/2}{(1 + M/Y)^2} - 2.29(M Y)^2/(1 + M/Y)^2 \right]
\]

**Present Model**

Implementing Eq. (9) results in a complicated integral for the effective thermal resistance of the microgaps in the second part of this study\(^4\) that is, nonconforming rough joints in a gaseous environment. To avoid a numerical solution, an approximate analytical model is developed for predicting the heat transfer of interstitial gases in the gap between conforming rough joints.

The geometry of the contact is shown in Figs. 1 and 4, where the contact of two rough surfaces is simplified to the contact of an equivalent rough and a smooth plate. It is assumed that the contacting surfaces are Gaussian and the asperities deform plastically. Heat transferred through the joint includes the microcontacts \( Q_c \) and the gas \( Q_g \) heat flows.

**Fig. 3** Heat flux regimes as a function of Knudsen number.
As already mentioned, microcontacts can be modeled as isothermal heat sources on a half-space. Considering circular shape microcontacts with the radius \( a \), on the order of micrometers, isothermal planes with some temperatures \( T_{i,1} \) and \( T_{i,2} \) at depth \( l \) must exist in bodies one and two, respectively (Fig. 4). Under vacuum conditions, that is, \( Q_g = 0 \), the distance between the isothermal planes and the contact plane is \( l = 40a \sim 40 \mu m \) (Ref. 1). When the gas pressures is increased, heat flow through the joint increases and distance \( l \) decreases. Because microcontacts are assumed to be flat and located in the contact plane, isothermal planes \( T_{i,1} \) and \( T_{i,2} \) are parallel to the contact plane. Therefore, TCR can be represented by two sets of thermal resistances in parallel between isothermal planes \( T_{i,1} \) and \( T_{i,2} \),

\[
R_j = (1/R_1 + 1/R_2)^{-1}
\]

and \( n_i \) are the equivalent thermal resistance of the microcontacts and the number of microcontacts, respectively. The thermal resistance of the microcontacts \( R_i \) is determined using Eq. (2).

Figure 5 shows the thermal resistance network of the joint. Because the thermal resistances are considered to be in parallel between two isothermal plates \( T_{i,1} \) and \( T_{i,2} \), the gap resistance \( R_g \) has three components, the gap resistance and \( R_1 \) and \( R_2 \), which correspond to the bulk thermal resistance of the solid layers in bodies 1 and 2, respectively. The bulk resistances \( R_1 \) and \( R_2 \) can be considered negligible in relation to \( R_g \) because the gas thermal conductivity is much lower than the conductivity of the solids, that is, \( k_j/k_s \leq 0.01 \).

\[
R_g_{\text{total}} = R_1 + R_2 + (d + M)/(k_s A_g) = R_g
\]

where \( R_1 = l/k_{i,1} A_s, R_2 = l/k_{i,2} A_s \), and \( A_s = A_a - A_i \) is the gas heat transfer area.

The real contact area is a very small portion of the apparent contact area, that is, \( A_c \ll A_a \); thus, it can be assumed that \( A_s = A_a \). As a result, the heat transferred through the microgaps between rough surfaces can be replaced by the gas heat transfer between two isothermal parallel plates that are located at an effective distance \( d \) from each other. In addition, the gap heat transfer area becomes less than 1.7% over the entire range of \( \lambda \).

Therefore, TCR can be represented by two sets of components, the gap resistance \( R_g \) and the contact area, that is, \( R_{a} \), respectively. The bulk resistances \( R_1 \) and \( R_2 \) are negligible in relation to \( R_g \).

To determine the gap thermal resistance, the effective distance between contacting bodies, \( d \), is required. For contact of Gaussian rough surfaces with the mean separation \( Y \), the statistical effective plane separation over the contact area, \( d \), can be found from

\[
d = \int_{-\infty}^{Y} \phi(z) dz
\]

where \( \phi(z) \) is the Gaussian distribution defined as

\[
\phi(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp(-z^2/(2\sigma^2))
\]

and \( \sigma \) is surface heights and the equivalent rms surface roughness, respectively. When Eq. (14) is substituted into Eq. (13), after evaluating and simplifying, \( d \) becomes

\[
d = \sqrt{\frac{1}{\lambda}} \left[ 1 + \text{erf}\left(\frac{\lambda}{\sqrt{2}}\right) + \exp\left(\frac{-\lambda^2}{2}\right) \right]
\]

The maximum relative difference between Eqs. (15) and (16) is less than 1.7% over the entire range of \( \lambda \). Equation (16) indicates that \( d \) is identical to the mean separation between two planes.

For conforming rough contacts assuming plastic deformation of asperities, it can be shown that

\[
\frac{P}{H_{\text{mic}}} = \frac{1}{2} \text{erfc}\left(\frac{\lambda}{\sqrt{2}}\right)
\]

or

\[
\lambda = \text{erfc}^{-1}(2P/H_{\text{mic}})
\]

where \( H_{\text{mic}} \), \( P = F/A_s \), and \( \text{erfc}^{-1}(\cdot) \) are the effective microhardness of the softer material in contact, contact pressure, and inverse complementary error function, respectively.

Microhardness depends on several parameters: mean surface roughness \( \sigma \), mean absolute slope of asperities, \( m \), type of material, method of surface preparation, and applied pressure. Hegazy\textsuperscript{16} proposed correlations in the form of the Vickers microhardness for calculating surface microhardness. Song and Yovanovich\textsuperscript{17} developed an explicit expression relating microhardness to the applied pressure,

\[
\frac{P}{H_{\text{mic}}} = (P/H)^{(1+0.071\lambda)}
\]

The real contact area is a very small portion of the apparent contact area, that is, \( A_c \ll A_a \); thus, it can be assumed that \( A_s = A_a \). As a result, the heat transferred through the microgaps between rough surfaces can be replaced by the gas heat transfer between two isothermal parallel plates that are located at an effective distance \( d \) from each other. In addition, the gap heat transfer area becomes

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\]

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\[
\frac{P}{H_{\text{mic}}} = (P/H)^{(1+0.071\lambda)}
\]
where \( H' = c_1(1.62\sigma'/m)^{1/2} \), \( \sigma' = \sigma/\sigma_0 \), and \( \sigma_0 = 1 \mu \text{m} \). In situations where an effective value for microhardness \( H_{\text{mic,e}} \) is known, the microhardness coefficients can be replaced by \( c_2 \). When Eq. (19) is substituted in Eq. (18),

\[
\lambda = Y/\sqrt{2\sigma} = \text{erfc}^{-1}(2P/H')
\]

(20)

where for convenience parameter \( 1/(1 + 0.071c_2) \) is assumed to be 1. Note that \(-0.35 \leq c_2 \leq 0\).

Yovanovich\(^{12}\) proposed an accurate correlation for determining the inverse complementary error function, \( \text{erfc}^{-1}(x) = 0.837 - 1.566x + 1.65x^2 \) for \( x \leq 0.01 \), with the maximum relative error less than 0.25%. Because a broader range of \( \text{erfc}^{-1}(x) \) is needed in this study (especially the second part), with use of Maple,\(^{18}\) a set of expressions for determining \( \text{erfc}^{-1}(x) \) are developed that cover a range of \( 10^{-9} \leq x \leq 1.9 \). \( \text{erfc}^{-1}(x) = \)

\[
\begin{align*}
1 & \quad 10^{-9} \leq x \leq 0.02 \\
0.218 + 0.735x^{0.113} & \quad 0.02 < x \leq 0.5 \\
x^{0.12} & \quad 0.5 < x \leq 1.9 \\
1 - x & \quad 1.9 < x \leq \infty
\end{align*}
\]

The maximum relative difference between Eq. (21) and \( \text{erfc}^{-1}(x) \) is less than 2.8% for the range \( 10^{-9} \leq x \leq 1.9 \). Figure 7 shows the comparison between \( \text{erfc}^{-1}(x) \) and Eq. (21).

When Eqs. (16) and (20) are combined, the gas thermal resistance can be found from

\[
R_g = \left(1/k_yA_y\right) \left[ M + \sqrt{2\sigma} \text{erfc}^{-1}(2P/H') \right]
\]

(22)

The thermal joint resistance can be calculated combining Eqs. (11), (2), and (22).

Comparison Between Present and Integral Models

To compare the present model Eq. (22) with the integral model Eq. (10), both expressions are nondimensionalized and rewritten in the following form:

\[
\frac{k_yA_yR_g}{Y} = \begin{cases} 
1 + \frac{M}{Y} & \text{present model} \\
1 + \frac{M}{Y} + \frac{0.304(\sigma/Y)}{(1 + M/Y)} - \frac{2.29(\sigma/Y)^2}{(1 + M/Y)^2} & \text{integral model}
\end{cases}
\]

(23)

The ratio \( Y/\sigma \) appears in the integral model correlation, which can be interpreted as the level of loading. For a fixed contact geometry, as the applied load increases, \( Y \) decreases and this parameter becomes smaller. Three values of \( Y/\sigma \) in Eq. (23) are included in the comparison, 2.5, 3, and 3.5, which represent three levels of loading from high to low, respectively (Fig. 8). The other parameter, \( M/Y \), is varied over a wide range, \( 10^{-3} < M/Y \ll \infty \), from vacuum to atmospheric pressure conditions, respectively. Table 1 lists relative differences between the present and the integral gap models. As shown, the relative differences are negligible where \( M/Y \geq 1 \), that is, slip to free-molecular regimes. As the parameter \( M/Y \) becomes smaller, that is, continuum regime (atmospheric gas pressure condition), the relative difference becomes larger. It can also be seen that the relative difference is larger at smaller values of \( Y/\sigma \), that is, higher loads. As already mentioned, the total or joint resistance is the parallel combination of the microcontacts \( R_s \) and the microgaps \( R_g \) resistances. Note that the contribution of the gas heat transfer is relatively smaller in higher loads because the microcontact resistance is smaller and controls the joint resistance. As a result, the relative difference in the joint resistances determined from the present and the integral gap models becomes smaller.

<table>
<thead>
<tr>
<th>( M/Y )</th>
<th>( Y/\sigma )</th>
<th>( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.50</td>
<td>0.001</td>
<td>9.97</td>
</tr>
<tr>
<td>3.00</td>
<td>0.001</td>
<td>9.63</td>
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<td>2.50</td>
<td>0.001</td>
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<tr>
<td>1.00</td>
<td>0.84</td>
<td>0.17</td>
</tr>
<tr>
<td>0.50</td>
<td>1.00</td>
<td>1.68</td>
</tr>
<tr>
<td>0.10</td>
<td>1.54</td>
<td>0.17</td>
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<tr>
<td>0.05</td>
<td>4.58</td>
<td>0.17</td>
</tr>
<tr>
<td>0.01</td>
<td>23.64</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1 Relative percent difference between present and integral gap models

The ratio \( Y/\sigma \) appears in the integral model correlation, which can be interpreted as the level of loading. For a fixed contact geometry, as the applied load increases, \( Y \) decreases and this parameter becomes smaller. Three values of \( Y/\sigma \) in Eq. (23) are included in the comparison, 2.5, 3, and 3.5, which represent three levels of loading from high to low, respectively (Fig. 8). The other parameter, \( M/Y \), is varied over a wide range, \( 10^{-3} < M/Y \ll \infty \), from vacuum to atmospheric pressure conditions, respectively. Table 1 lists relative differences between the present and the integral gap models. As shown, the relative differences are negligible where \( M/Y \geq 1 \), that is, slip to free-molecular regimes. As the parameter \( M/Y \) becomes smaller, that is, continuum regime (atmospheric gas pressure condition), the relative difference becomes larger. It can also be seen that the relative difference is larger at smaller values of \( Y/\sigma \), that is, higher loads. As already mentioned, the total or joint resistance is the parallel combination of the microcontacts \( R_s \) and the microgaps \( R_g \) resistances. Note that the contribution of the gas heat transfer is relatively smaller in higher loads because the microcontact resistance is smaller and controls the joint resistance. As a result, the relative difference in the joint resistances determined from the present and the integral gap models becomes smaller.

Parametric Study

The ratio \( Y/\sigma \) appears in the integral model correlation, which can be interpreted as the level of loading. For a fixed contact geometry, as the applied load increases, \( Y \) decreases and this parameter becomes smaller. Three values of \( Y/\sigma \) in Eq. (23) are included in the comparison, 2.5, 3, and 3.5, which represent three levels of loading from high to low, respectively (Fig. 8). The other parameter, \( M/Y \), is varied over a wide range, \( 10^{-3} < M/Y \ll \infty \), from vacuum to atmospheric pressure conditions, respectively. Table 1 lists relative differences between the present and the integral gap models. As shown, the relative differences are negligible where \( M/Y \geq 1 \), that is, slip to free-molecular regimes. As the parameter \( M/Y \) becomes smaller, that is, continuum regime (atmospheric gas pressure condition), the relative difference becomes larger. It can also be seen that the relative difference is larger at smaller values of \( Y/\sigma \), that is, higher loads. As already mentioned, the total or joint resistance is the parallel combination of the microcontacts \( R_s \) and the microgaps \( R_g \) resistances. Note that the contribution of the gas heat transfer is relatively smaller in higher loads because the microcontact resistance is smaller and controls the joint resistance. As a result, the relative difference in the joint resistances determined from the present and the integral gap models becomes smaller.
Table 2 Input parameters for a typical SS-nitrogen joint

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_T$ (SS − $N_2$)</td>
<td>0.78</td>
</tr>
<tr>
<td>$b_L$</td>
<td>12.5 mm</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2 $\mu$m</td>
</tr>
<tr>
<td>$m$</td>
<td>0.12</td>
</tr>
<tr>
<td>$F_0$</td>
<td>35 N</td>
</tr>
<tr>
<td>$k_0$</td>
<td>62.8 nm</td>
</tr>
<tr>
<td>$c_1, c_2$</td>
<td>0.031, 20 W/mK</td>
</tr>
<tr>
<td>$c_3$</td>
<td>6.23 GPa, −0.23</td>
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Table 3 Range of parameters for experimental data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F$</td>
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</tr>
<tr>
<td>$P$</td>
<td>0.14−8.8 MPa</td>
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<tr>
<td>$k_j$</td>
<td>19.2−72.5 W/mK</td>
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<tr>
<td>$m$</td>
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<tr>
<td>$P_g$</td>
<td>10$^{-5}$−760 torr</td>
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<tr>
<td>$\alpha_T$</td>
<td>0.55−0.9</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.52−11.8 $\mu$m</td>
</tr>
</tbody>
</table>

Table 4 Properties of gases

<table>
<thead>
<tr>
<th>Gas</th>
<th>$k_g$, W/mK</th>
<th>$\alpha_T$</th>
<th>$\gamma$</th>
<th>$\Lambda_0$, nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar</td>
<td>0.018 + 4.05 × 10$^{-5}$T</td>
<td>0.67</td>
<td>0.90</td>
<td>1.67</td>
</tr>
<tr>
<td>He</td>
<td>0.147 + 3.24 × 10$^{-4}$T</td>
<td>0.67</td>
<td>0.55</td>
<td>1.67</td>
</tr>
<tr>
<td>N$_2$</td>
<td>0.028 + 5.84 × 10$^{-5}$T</td>
<td>0.69</td>
<td>0.78</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Figs. 10 and 11, respectively. Input parameters of a typical contact are shown in Table 2; contacting surfaces are stainless steel and the interstitial gas is nitrogen at 373.15 K and 50 torr.

The external load is varied over a wide range, 10 ≤ $F$ ≤ 180,000 N, to study the effect of load on thermal joint resistance. As shown in Fig. 10, at light loads the gap thermal resistance is the controlling component of thermal joint resistance. Thus, most of the heat transfer occurs through the gas. As the load increases, $R_s$, which is inversely proportional to the load [Eq. (2)], decreases. As a result, the mean separation between the two bodies, $Y$, decreases, which leads to a decrease in $R_g$. In higher loads, $R_s$ is smaller and controls the joint resistance.

To study the effect of gas pressure on the thermal joint resistance, the gas pressure is varied over the range of 10$^{-5}$ ≤ $P_g$ ≤ 760 torr, while all other parameters in Table 2 are held constant. As shown in Fig. 11, at very low gas pressures (vacuum) $R_g$ is large. Thus $R_s$ controls the joint resistance by increasing the gas pressure, thermal gas resistance decreases, and $R_j$ becomes the controlling component.

Comparison with Experimental Data

The present model is compared with more than 510 experimental data points collected by Hegazy$^{16}$ and Song.$^{15}$ The geometry of the experimental set up is shown in Fig. 1. Tests include two flat rough cylindrical specimens with the same radius $b_L = 12.5$ mm, which are placed in contact by applying an external load in a chamber filled with an interstitial gas. To minimize the radiation and convection heat transfer to the surroundings, the lateral surfaces of the specimens were insulated. Test specimens were made of stainless steel (SS) 304 and nickel 200, and interstitial gases were argon, helium, and nitrogen. The gas pressure was varied from atmospheric pressure 760 to vacuum 10$^{-5}$ torr. As summarized in Table 3, the experimental data cover a relatively wide range of mechanical, thermal, and surface characteristics.

Thermal properties of argon, helium, and nitrogen are listed in Table 4.$^{15,16}$ Note that the reference mean free paths, $\Lambda_0$ nm, are at 288 K and 760 torr and temperature in $k_g$ correlations must be in degrees Celsius.

Hegazy$^{16}$ Experimental Data

Hegazy$^{16}$ collected more than 160 data points during four sets of experiments performed on SS 304 joints tested in nitrogen and helium. Low thermal conductivity and high microhardness values of SS 304 provide a reasonable set of extremes for verification of the gap model. Table 5 lists the experiment numbers, solid–gas combinations, gas pressure, surface roughness, and slope of the Hegazy experimental data. The nominal contact pressure was varied from 0.459 to 8.769 MPa throughout the tests. The average gas temperature and thermal conductivity of SS 304 were reported in the range of 170−220°C and 20.2 W/mK, respectively.
Table 5 Summary of Hegazy experiments

<table>
<thead>
<tr>
<th>Test</th>
<th>Gas</th>
<th>$P_g$, torr</th>
<th>$\sigma$, $\mu$m</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>$N_2$</td>
<td>562–574</td>
<td>5.65</td>
<td>0.153</td>
</tr>
<tr>
<td>T2</td>
<td>$N_2$, He</td>
<td>Vacuum, 40</td>
<td>5.61</td>
<td>0.151</td>
</tr>
<tr>
<td>T3</td>
<td>$N_2$, He</td>
<td>Vacuum, 40</td>
<td>6.29</td>
<td>0.195</td>
</tr>
<tr>
<td>T4</td>
<td>$N_2$, He</td>
<td>Vacuum, 40</td>
<td>4.02</td>
<td>0.168</td>
</tr>
</tbody>
</table>

Table 6 Summary of Song experiments

<table>
<thead>
<tr>
<th>Test</th>
<th>Solid–gas</th>
<th>$P$, MPa</th>
<th>$\sigma$, $\mu$m</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>SS–$N_2$, Ar, He</td>
<td>0.595–0.615</td>
<td>1.53</td>
<td>0.090</td>
</tr>
<tr>
<td>T2</td>
<td>SS–$N_2$, Ar, He</td>
<td>0.467–0.491</td>
<td>4.83</td>
<td>0.128</td>
</tr>
<tr>
<td>T3</td>
<td>Ni–$N_2$, Ar, He</td>
<td>0.511–0.530</td>
<td>2.32</td>
<td>0.126</td>
</tr>
<tr>
<td>T4</td>
<td>Ni–$N_2$, Ar, He</td>
<td>0.371–0.389</td>
<td>11.8</td>
<td>0.206</td>
</tr>
<tr>
<td>T5</td>
<td>SS–$N_2$, He</td>
<td>0.403–7.739</td>
<td>6.45</td>
<td>0.132</td>
</tr>
<tr>
<td>T6</td>
<td>SS–$N_2$, He</td>
<td>0.526–8.713</td>
<td>2.09</td>
<td>0.904</td>
</tr>
<tr>
<td>T7</td>
<td>Ni–$N_2$, He</td>
<td>0.367–6.550</td>
<td>11.8</td>
<td>0.206</td>
</tr>
</tbody>
</table>

Fig. 12 Comparison of present model with Hegazy data.

The experimental data are nondimensionalized and compared with the present model in Fig. 12. The maximum uncertainty of the experimental data was reported to be 5.7%. As can be seen in Fig. 12, the present model shows good agreement, where the rms difference between the model and the data is approximately 6%.

Song Experimental Data

Song conducted seven sets of experiments performed on nickel 200 and SS 304 joints tested in argon, helium, and nitrogen. In addition to SS 304 specimens, nickel 200 was chosen, which has a thermal conductivity of about 3.5 times that of SS 304 (at 170°C). Thus, the contribution of the microcontacts to the joint heat transfer is significantly greater than that of a SS 304 joint of similar conditions.

Table 6 summarizes the experiment numbers, solid–gas combinations, range of the nominal contact pressure, and surface roughness and slope of the Song’s experimental data. The tests were conducted in the following order: 1) at least one vacuum test, 2) series of helium tests at various gas pressures, 3) vacuum test, 4) series of nitrogen tests at various gas pressures, 5) vacuum test, and 6) series of argon tests at various gas pressures. The gas pressure was varied from $10^{-5}$ to approximately 650 torr. The mean contact temperature, that is, the mean gas temperature was maintained at approximately 170°C, and the average thermal conductivities of SS 304 and Ni 200 were reported as 19.5 and 71.2 W/mK, respectively.

Experiments T5–T7 involved gas tests at several load levels, indicated by letters A, B, C, and D in Fig. 13. The purpose of these tests was to observe the load dependence of the thermal gap resistance. As can be seen in Table 6, only helium and nitrogen were used in these tests because it had been concluded from tests T1–T4 that argon behaves essentially the same as nitrogen.

Approximately 350 data points are nondimensionalized and compared with the present model in Fig. 13. The maximum uncertainty of the experimental data was reported to be less than 10%. As shown in Fig. 13, the present model shows good agreement with the data over the entire range of the comparison. The rms difference between the model and the data is 8.1 percent.

Figure 14 shows the comparison between the present model and both Hegazy and Song experimental data. The rms difference between the present model and experimental is approximately 7.3 percent.

Conclusions

Heat transfer of an interstitial gas between conforming random rough joints was studied. When the general expression for heat transfer between two isothermal parallel plates proposed by Yovanovich was used, an approximate analytical model was developed. The model covers the four regimes of heat conduction modes of gas, that is, continuum, temperature jump or slip, transition, and free molecular and accounts for gas and solid mechanical and thermal properties, gas pressure and temperature, surface roughness, and applied load.

It was shown that the gas and the microcontacts thermal resistances are in parallel. With use of a statistical relation for Gaussian rough surfaces, it was illustrated that for engineering applications the effective separation over the contact area, $d$, is identical to the mean separation between two contacting surfaces $Y$. With the knowledge that the real contact area is a very small portion of the apparent area, it was assumed that the gap heat transfer area is identical to the apparent area. Also uniform temperatures for the contacting surfaces were assumed. These assumptions simplified the analysis, and a simple relationship for the gap thermal resistance was derived. A
correlation for inverse complementary error function was developed that determines erfc\(^{-1}(\cdot)\) within 2.8% relative error.

The influence of the main input parameters on the gap and joint thermal resistances revealed the following:

1) With constant gas pressure, at light loads \(R_{s}\) was the controlling part of \(R_{j}\). Thus most of the heat transfer occurred through the gas. By the increase of the external load \(R_{g}\), \(R_{s}\) and \(R_{g}\) decreased and \(R_{j}\) became relatively small and controlled the joint resistance.

2) With constant load, at very low gas pressures (vacuum) \(R_{g}\) was large. Thus \(R_{j}\) controlled the joint resistance, by the increase of the gas pressure, \(R_{g}\) decreased and became the controlling component of \(R_{j}\).

The present model was compared with the integral model (Yovanovich et al.\(^{14}\)). It was shown that the relative differences between the present and the integral model were negligible for the slip to free-molecular regimes. The relative difference became larger for the continuum regime (atmospheric gas pressure condition) at relatively high loads. When it is considered that the contribution of the gas heat transfer is relatively small at higher loads, the relative difference in the total joint resistances determined from the present and the integral gap model became smaller.

The present model was compared with more than 510 experimental data points collected by Hegazy\(^{16}\) and Song.\(^{15}\) Tests were performed with SS 304 and nickel 200 with three gases, that is, argon, helium, and nitrogen. The data covered a wide range of surface characteristics, applied load, thermal and mechanical properties, and gas pressure, which was varied from vacuum to atmospheric pressure. The present model showed good agreement with the data over the entire range of the comparison. The rms relative difference between the model and data was determined to be approximately 7.3%.

\textbf{References}