Transverse Permeability of Fibrous Porous Media
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Abstract
In this study, the transverse permeability of fibrous porous media is studied both experimentally and theoretically. A scale analysis technique is employed for determining the transverse permeability of fibrous media with a variety of fibrous matrices including square, staggered, hexagonal unidirectional fiber arrangements, simple two directional mats, and simple cubic structures. In this approach, the permeability is related to the porosity, fiber diameter, and tortuosity of the medium. In addition, the pressure drop in several samples of tube banks of different arrangements and metal foams are measured in the creeping flow regime. The results are then used to calculate the permeability of the samples. The developed compact relationships are successfully verified through comparison with the present experimental results and the data reported by others. Our results suggest that the fiber orientation has an important effect on the permeability; however, these effects are more pronounced in low porosities, i.e., \( \varepsilon < 0.7 \).

Keywords: Transverse permeability; Scale analysis; Fibrous media; Modeling; Creeping flow; Experimental

1. Introduction
Study of flow in fibrous porous media is important in many natural and industrial processes such as: physiological transport phenomena [1], filtration [2-4], composite fabrication [5,6], compact heat exchangers [7,8], paper production [9], and fuel cell technology [10, 11]. As such, prediction of the flow properties of fibrous materials including permeability and inertial coefficient has drawn the attention of numerous researchers. Authors have employed various theoretical and experimental techniques to investigate the problem. Comprehensive reviews of the pertinent literature can be found in Refs. [4,6,12]. Permeability, which can be interpreted as the flow conductance of the solid matrix, is related to geometrical features of the solid matrix including particle size and shape, pore size, and pore distribution.

Fibrous materials can be divided into 1, 2, and 3 directional media. In one-directional (1D) structures the axes of fibers are parallel to each other. In two-directional (2D) fibrous matrices the fibers axes are located on planes parallel to each other with random positions and orientations on these planes. The axes of fibers in three-directional (3D) are randomly positioned and oriented in space. Unlike the previous configurations, 3D structures can be isotropic. Complex geometry of actual fibrous materials avoid from exact solutions for flow and permeability. However, in preliminary design and optimization processes, approximate solutions often suffice. To estimate the permeability of the fibrous structures, several researchers have modeled the complex micro-structure of the porous media with simplified 1D “unit cells” [13-21]. The models developed theoretically for transverse permeability of 1D structures are plotted against experimental data in Fig. 1. The comparison shows that most of these models have a limited range of accuracy. Only, the model of Tamayol and Bahrami [19] captures the trends of experimental data over the entire range of porosity. However, the model of Tamayol and Bahrami [19] is limited to square arrays of fibers.

Theoretical studies of permeability of 2D and 3D materials are not as frequent as 1D arrangements which is a result of geometrical complexity of these media. A selection of the existing models for 2D and 3D structures [2, 22-24] are plotted in Figs. 2 and 3, respectively. It can be seen that these models are not accurate over the entire range of porosity.

![Figure 1: Comparison of the existing models for square arrangements with experimental data.](image-url)
Therefore, the objectives of the present work are to:

- Develop a theoretical approach that is applicable to 1D, 2D, and 3D fibrous matrices and accurately captures the trends observed in experimental data.
- Investigate the effect of relevant geometrical parameters involved and identify the controlling parameters.
- Perform independent experimental studies on fibrous structures to verify the developed models.

A scale analysis technique is employed to predict the permeability of a variety of unit cells including square, staggered, and hexagonal arrangements of 1D fibers, simple 2D mats and simple cubic structures. This method that was originally applied by Clauge et al. [25] to fibrous media, is modified to improve its accuracy. Moreover, compact relationships are presented for determining the permeability of each category as a function of porosity and fiber diameter. In addition, pressure drop is measured for creeping flow through several samples of tube banks and aluminum foams with 1D and 3D structures, respectively. The developed solutions are successfully compared with experimental and numerical results for a wide range of geometries and materials.

2. Geometrical modeling

Depending on the orientation of fibers in the medium, the fibrous matrix could be divided into 1D, 2D, or 3D structures. The simplest representation of 1D structures or generally fibrous media is ordered arrangements of unidirectional cylinders. In the present study, several ordered structures including square, staggered, and hexagonal arrays of fibers are considered, see Fig. 4. The solid volume fractions, $\phi$, for the arrangements shown in Fig. 4 are related to the distance between the centers of adjacent fibers, $S$, and the fibers diameter, $d$:

$$
\phi = \begin{cases} 
\frac{\pi d^2}{4S^2} & \text{Square} \\
\frac{\pi d^2}{2\sqrt{3}S^2} & \text{Staggered} \\
\frac{\pi d^2}{3\sqrt{3}S^2} & \text{Hexagonal}
\end{cases}
$$

To obtain results for woven textile structures with non-overlapping fibers, the structure, shown in Fig. 5, is considered here. The relationship between $\phi$ and other geometrical parameters can be found from Fig. 5:

$$
\phi = \frac{\pi d}{4S}
$$

Figure 4: Considered unit cells for ordered 1D structures: a) square, b) staggered, and c) hexagonal arrays of cylinders.
In 3D structures such as metal foams, fibers can have any arbitrary orientation in space, see Fig. 6a. Many researchers have proposed simple cubic arrangements as the representing unit cell for these structures, see in Fig. 6b. The relationship between the solid volume fraction and other geometric parameters of SC arrangement considered in the present study is [26]:

$$\varphi = \frac{3\pi d^2}{4S^2} - \frac{\sqrt{2}}{S} d^3$$

3. Experimental approach

Experimental data for creeping flow through fibrous structures of our interests are not abundant in the open literature. As such, several samples of tube banks with square and staggered fiber arrangements and metal foams are tested using glycerol. The properties of the sample are summarized in Table 1.

The gravity driven test bed, illustrated in Fig. 7, is consisted of an elevated reservoir, an entry section, sample holder, and an exit section with a ball valve. The reservoir cross-section of 300X300 mm$^2$ is large enough to ensure that the variation of the driving pressure is negligible during the experiment. The pressure drop through the samples was measured using a differential pressure transducer (PX 154) provided by BEC Controls.

The bulk flow was calculated by weighting the collected test fluid over a specific period of time. The permeability of the samples is then calculated using Darcy equation described in the following section. Figure 8 shows that the measured pressure gradients for samples of tube bank with square fiber arrangement have linear relationships with the Reynolds number.

Figure 7: Schematic of the test setup.

Figure 8: measured pressure gradients for samples of tube bank with square fiber arrangement.
Table 1: Summary of the properties of the tested samples; glycerol used as test fluid.

<table>
<thead>
<tr>
<th>Sample type</th>
<th>ε</th>
<th>d (mm)</th>
<th>Orientation</th>
<th>Permeability (K,m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tube bank (square)</td>
<td>0.8</td>
<td>1.5</td>
<td>1D</td>
<td>1.38×10⁻⁷</td>
</tr>
<tr>
<td>Tube bank (square)</td>
<td>0.85</td>
<td>1.5</td>
<td>1D</td>
<td>3.74×10⁻⁷</td>
</tr>
<tr>
<td>Tube bank (square)</td>
<td>0.9</td>
<td>1.5</td>
<td>1D</td>
<td>5.44×10⁻⁷</td>
</tr>
<tr>
<td>Tube bank (staggered)</td>
<td>0.7</td>
<td>1.5</td>
<td>1D</td>
<td>1.00×10⁻⁷</td>
</tr>
<tr>
<td>Tube bank (staggered)</td>
<td>0.9</td>
<td>1.5</td>
<td>1D</td>
<td>7.75×10⁻⁷</td>
</tr>
<tr>
<td>Metalfoam (PPI=10)</td>
<td>0.93</td>
<td>0.4</td>
<td>3D</td>
<td>2.53×10⁻⁷</td>
</tr>
<tr>
<td>Metalfoam (PPI=20)</td>
<td>0.93</td>
<td>0.3</td>
<td>3D</td>
<td>1.45×10⁻⁷</td>
</tr>
<tr>
<td>Metalfoam (PPI=40)</td>
<td>0.94</td>
<td>0.2</td>
<td>3D</td>
<td>0.81×10⁻⁷</td>
</tr>
</tbody>
</table>

4. Model development

Experimental observations have shown that a linear relationship exists between the volume-averaged superficial fluid velocity, \( U_D \), and the pressure gradient; this is called Darcy’s law [4]:

\[
\frac{dP}{dx} = \frac{\mu U_D}{K} \tag{4}
\]

where, \( \mu \) is the fluid viscosity and \( K \) is the permeability of the medium. Darcy’s relationship is empirical, convenient, and widely accepted; this equation holds when flow is in creeping regime [5]. However, one should know the permeability beforehand to use the Darcy’s equation. Permeability can be calculated through the pore-scale analysis of flow in the solid matrix. In the creeping regime, the pore-scale velocity, \( \vec{V} \), is governed by Stokes equation:

\[
\nabla \vec{V} = 0 \tag{5}
\]

\[
\mu \nabla^2 \vec{V} = -\nabla P \tag{6}
\]

A scale analysis is followed for determining the resulting pressure drop. In this approach the scale or the range of variation of the parameters is substituted in governing equations, i.e., derivatives are approximated with differences [27]. Following Claque et al. [25] and Sobera and Klein [28], half of the minimum opening between two adjacent cylinders, \( \delta_{\text{min}} \), is selected as the characteristic length scale over which rapid changes of the velocity occurs, see Fig. 4. Sobera and Klein [28] proposed to use the average velocity in the section with minimum frontal area as the characteristic velocity scale. However, this assumption is only accurate for highly porous structures, \( \varepsilon > 0.8 \), and overpredicts the pressure drop in low porosities [28]. Carman [29] argued that a fluid particle should travel in a tortuous path of length \( L_e \) to path through a sample of size \( L \). Therefore, it is expected that the resulting velocity scale from applying a constant pressure difference to be inversely related to \( L_e / L \); this ratio is called tortuosity factor, \( \tau \). Thus, the pore-level velocity scale becomes:

\[
\vec{V} \approx \frac{U_D \beta}{\tau} \tag{7}
\]

where \( \beta \) is the ratio of the minimum to the total frontal areas in the unit cell. Substituting from Eq. (7) for velocity scale and using \( \delta_{\text{min}} \) as the length scale, permeability can be calculated as:

\[
K = C \beta \delta_{\text{min}}^2 \tau \tag{8}
\]

where \( C \) is a constant that should be determined through comparison with data. Therefore one needs to know the ratio between minimum to total frontal area, \( \beta \), and tortuosity factor, \( \tau \), to be able to calculate the permeability.

4.1. Tortuosity factor

The tortuosity factor is defined as the ratio of the average distance, \( e_L \), that a particle should travel to cover a direct distance of \( L \). Due to its importance in mass, thermal and electrical diffusion, several theoretical and empirical relationship have been proposed for tortuosity calculation in the literature; good reviews can be found elsewhere [30,31]. Any relationship proposed for tortuosity should satisfy three conditions [30]:

\[
\begin{align*}
\tau &> 1; \\
\lim_{\varepsilon \to 1} \tau &= 1; \\
\lim_{\varepsilon \to 0} \tau &\to \infty .
\end{align*}
\]

One of the most popular empirical models for determination of tortuosity, that satisfies all these conditions, is the Archie’s law [32]:

\[
\tau = \left( \frac{1}{\varepsilon} \right)^\alpha = \left( \frac{1}{1 - \phi} \right)^\alpha \tag{9}
\]

where \( \alpha \) is a constant and \( \varepsilon \) is the porosity. Boudreau [30] showed that \( \alpha = 0.5 \) provides a good estimate for tortuosity in packed beds. Due to similarity of flow in packed beds and flow in 1D and 2D fibers \( \alpha \) is assumed to be equal to 0.5. The study of Tomadakis and Robertson [12] showed that 3D fibrous structures are less tortuous in comparison with 1D and 2D matrices. Therefore for 3D structures \( \alpha \) is assumed to be equal to 0.3.

5. Results and discussions

Equation (8) relates the permeability of fibrous media to the minimum opening between adjacent fibers, \( \delta_{\text{min}} \), the
ratio between minimum to total frontal area, $\beta$, and tortuosity factor, $\tau$, that can be calculated from Eq. (9). In the following subsections, using geometrical properties of the considered microstructures, compact models will be developed that relate the permeability to the solid volume fraction.

### 5.1. Unidirectional arrangements

For the three different ordered 1D unit cells shown in Fig.1, it can be seen that $\beta = (S - d)/S$ and $\delta_{\min} = (S - d)$. Therefore, Eq. (8) can be rewritten as:

$$K = C \frac{(S - d)^3}{S \sqrt{1 - \varphi}}$$  \hspace{1cm} (10)

Substituting from Eq. (1) and comparing the model with experimental data, the dimensionless permeability of the ordered structures is:

$$K = \frac{0.16}{\sqrt{1 - \varphi}} \left[ \frac{\pi}{4\varphi} - 3 - \frac{\pi}{4\varphi} + 3 - \frac{4\varphi}{\pi} \right]$$ Square

$$K = \frac{0.16}{\sqrt{1 - \varphi}} \left[ \frac{\pi}{2\sqrt{3}\varphi} - 3 - \frac{\pi}{2\sqrt{3}\varphi} + 3 - \frac{2\sqrt{3}\varphi}{\pi} \right]$$ Staggered

$$K = \frac{0.16}{\sqrt{1 - \varphi}} \left[ \frac{\pi}{3\sqrt{3}\varphi} - 3 - \frac{\pi}{3\sqrt{3}\varphi} + 3 - \frac{3\sqrt{3}\varphi}{\pi} \right]$$ Hexagon

In Fig. 9, Eq. (11) is compared with the present experimental results and the data collected by others [33-39]. As one can see, the model is in agreement with experimental data over the entire range of porosity. These experiments were conducted using different fluids including: air, water, oil, and glycerol with a variety of porous materials such as metallic rods, acrylic cylinders, and carbon fibers.

In Fig. 10 the predicted results of Eq. (11) for staggered arrangement of fibers are compared with present experimental data and numerical results of Higdon and Ford [26]. It can be seen that the proposed model can accurately predict the numerical results in the entire range of porosity.

### 5.2. Two-directional structures

The ratio of the minimum frontal to the total unit cell areas for the 2D structure shown in Fig. 2b is not exactly known. Therefore, similar to 1D structures and using the Forchheimer law that relates average pore-scale velocity to $U_D/\varphi$ [5], the pore-level velocity scale is estimated as:

$$\bar{v} \approx U_D \frac{S - d}{S \varepsilon^{3/2}}$$ \hspace{1cm} (12)

Substituting for geometrical parameters from Eq. (2), the dimensionless permeability becomes:

$$K = \frac{0.008}{(1 - \varphi)^{3/2}} \left[ \frac{\pi}{4\varphi} - 3 - \frac{\pi}{4\varphi} + 3 - \frac{4\varphi}{\pi} \right]$$ \hspace{1cm} (13)

The constant value in Eq. (13), i.e., 0.008, is found through comparison with experimental data collected from different sources, see Fig. 11. It can be seen that Eq. (13) captures the trends of the experimental data collected from different sources over a wide range of porosity. The experiments were conducted on glass rods, glass wool, cotton wool, kapok with application in filtration [40], alloy fibers [41], fiber reinforcing mats with application in molding and composite fabrication [42,43], and gas diffusion layers [10]. Kostornov and shevchuk [41] performed experiments with several fluids and they observed that permeability was dependent on the working fluid, i.e., water resulted in higher permeability than alcohol.
5.3. Three directional arrangements

For simple cubic arrangement that is considered in this study as a simple representation of 3D fibrous materials, the ratio of the minimum frontal to the unit cell areas is $\beta = (S - d)^2 / S^2$. Therefore, the permeability of 3D structures becomes:

$$\frac{K}{d^2} = 0.08 \frac{(S - d)^4}{S^2 d^2 \varepsilon^{0.3}}$$

where the ratio of $S$ to $d$ is calculated from Eq. (3). The constant in Eq. (14) is found to be 0.08 through comparison of this equation with the numerical data reported by Higdon and Ford [26] for SC arrangements over a wide range of porosity. Figure 12 includes the present model, current experimental measurements, and experimental data collected from different sources. The plotted data are based on permeability results for polymer chain in solutions [44], glass wool randomly packed, stainless steel crimps [29], metallic fibers [45], and aluminum metal foams [46]. It can be seen that the present model is in agreement with the numerical results over the entire range of porosity.

6. Effect of fiber orientation of the permeability

To see the effect of fibers arrangement on the permeability of the fibrous structures, the proposed relationships for 1D, 2D, and 3D arrays are plotted in Fig. 13. It can be seen that the square arrangements and 2D structures have similar permeabilities and 3D structures is the most permeable microstructure; this is in agreement with the results reported by Tomadakis and Robertson [12]. It is noteworthy that the effect of microstructure is more significant in low porosities, where $\varepsilon < 0.7$ and the deviations are reduced in higher porosities; this is in line with our previous observations for parallel flow through 1D fibers [47].
8. Acknowledgements
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9. References


A. Tamayol, and M. Bahrami, 2009, “Parallel flow in ordered fibrous structures: an analytical approach,” ASME 2009 Fluids Engineering Division Summer Meeting, Vail, Colorado, USA.