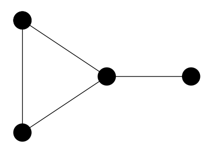
Homework 1

1. Prove or disprove: If every vertex of G has degree 2, then G is a cycle.

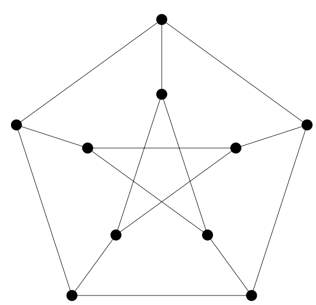
2. Prove that a bipartite graph has a unique bipartition (up to changing the two sets) if and only if it is connected.

3. We say that a set of vertices X in a graph is *independent* if no edge has both of its ends in X. In the graph below, find all maximal paths and maximal independent sets.



Definition: If $H_1, \ldots, H_m$ are subgraphs of G with $\cup_{i=1}^{k} E(H_i) = E(G)$ and $E(H_i) \cap E(H_j) = \emptyset$ for every $1 \leq i < j \leq m$ we say that $H_1, \ldots, H_m$ form a *decomposition* of G.

4. Find a decomposition of the Petersen graph (pictured below) into three pairwise isomorphic subgraphs. (Hint: it might help to find a drawing of Petersen with a 3-fold symmetry)



5. Prove that K_n has a decomposition into three pairwise isomorphic subgraphs if and only if $n+1$ is not divisible by 3. (Hint: for the case where n is divisible by 3, split the vertices into three sets of equal size)

6. Show that if K_n can be decomposed into triangles, then either $n-1$ or $n-3$ is a multiple of 6.

7. Prove that every simple connected graph with an even number of edges can be decomposed into paths of length 2. (Hint: induction).