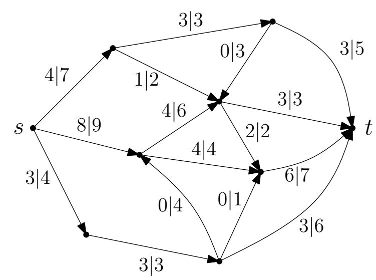
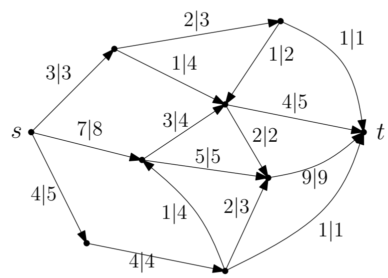
Homework 10

Problem 1. In the following digraph, each edge is labelled as $\phi(e) \mid c(e)$ where ϕ is a (s,t)-flow and $c(e)$ is the capacity of the edge e. Either prove that there is an augmenting path from s to t of find a cut with capacity equal to the existing flow value.



Problem 2. Repeat the previous exercise for the following graph.



Problem 3. Prove or disprove: every loopless graph G has an orientation which is acyclic.

Problem 4. Let D be a digraph, let $s,t \in V(D)$, and assume that $deg^+(v) = deg^-(v)$ for every vertex $v \in V(D) \setminus \{s,t\}$ and $deg^+(s) - deg^-(s) = k = deg^-(t) - deg^+(t)$. Show that there exist k edge disjoint directed paths in D from s to t.

Problem 5. Prove or disprove: every graph G has an orientation D so that $|\delta^+(X)|$ and $|\delta^-(X)|$ differ by at most one for every $X \subseteq V(D)$.

Problem 6. Prove that every connected graph G has an orientation in which the number of vertices with odd outdegree is at most 1. (Hint: choose a spanning tree T and orient the edges in $E(G) \setminus E(T)$ arbitrarily)

Problem 7. Let D be an orientation of a tree with the property that $deg^-(v) \leq 1$ for every $v \in V(D)$. Prove that there exists a vertex r so that every other vertex in the tree can be reached from r by a directed path.