Homework 2

1. Determine the maximum size of an independent set in *Petersen*.

2. Prove that a loopless graph G is bipartite if and only if for every subgraph $H \subseteq G$ the graph H has an independent set of size at least $\frac{1}{2}|V(H)|$.

3. For every $k \geq 1$ find a simple disconnected graph G_k on $2k$ vertices with highest possible minimum degree. This should include a proof that any graph with higher minimum degree is connected.

4. Let $u = v_1, e_1, \ldots, v_n, e_n, v_{n+1} = u$ be a closed walk in the graph G. Let S denote the set of all edges used in this walk and assume there is no cycle C of G for which $E(C) \subseteq S$. Prove that either $e_1 = e_n$ or there exists $1 \leq j \leq n$ so that $e_j = e_{j+1}$.

5. Let G be an Eulerian graph and let $\{X, Y\}$ be a partition of $V(G)$. Show that the number of edges with one end in X and one end in Y is even. (Hint: consider the degrees of vertices in X).

6. Let G be a loopless graph with every vertex of degree ≥ 3. Prove that G has a cycle of even length. (Hint: consider a path of maximum length)

7. Let G be a connected graph with $V(G) \geq 3$. Prove that G has two vertices x, y so that $G - \{x, y\}$ is connected, and further x and y are either adjacent or they have a common neighbour. (Hint: consider a path of maximum length)