Homework 3

Problem 1. Let G be a graph with the property that every subgraph of G has a vertex of degree ≤ 1. Show that G is a forest.

Problem 2. We proved in class (and in tutorial) that every connected graph G with $|V(G)| \geq 2$ has two vertices x_1, x_2 so that $G - x_i$ is connected for $i = 1, 2$. Find an alternative proof of this fact using spanning trees.

Problem 3. Let $k \geq 2$ and let T be a tree in which every vertex has degree 1 or degree k. Show that $|V(G)| - 2$ is a multiple of $k - 1$.

Problem 4. Let T be a tree in which every vertex has degree 1 or degree 3. Prove that there is a vertex adjacent to two leaves.

Problem 5 Let $n \geq 3$ and let G be an n vertex graph with the property that $G - v$ is a tree for every $v \in V(G)$. What is the graph G?

Problem 6 Let T be a tree with k leaves and set $t = \lceil \frac{k}{2} \rceil$. Prove that there exist paths $P_1, P_2, \ldots, P_t$ which satisfy both properties below.

(i) $\cup_{i=1}^{t} P_i = T$

(ii) $V(P_i) \cap V(P_j) \neq \emptyset$ for every $1 \leq i \leq j \leq t$.

(Hint: first prove that there exist paths satisfying (i))

Problem 7. Let $d_1, d_2, \ldots, d_n$ be a sequence of positive integers with $\sum_{i=1}^{n} d_i = 2n - 2$. Show that there exists a tree with vertex set $\{x_1, \ldots, x_n\}$ so that $deg(x_i) = d_i$ for every $1 \leq i \leq n$.