Homework 6

1. Exhibit a Marriage System which has more than one stable marriage.

2. Use Theorem 11.2 (König Egerváry) to prove Theorem 9.2 (Hall's Marriage Theorem).

Recall that a *permutation matrix* is a square matrix with all entries 0 or 1 in which every row and every column has exactly one 1.

3. Let Q be an $n \times n$ nonnegative real matrix, let $t \in \mathbb{R}$, and assume that every row and every column sum to t. Prove that there exist permutation matrices $P_1, \ldots, P_k$ and real numbers $x_1, \ldots, x_k$ so that $Q = \sum_{i=1}^{k} x_i P_i$. (Hint: proceed by induction on the number of nonzero entries in Q, and show that there is a suitable permutation matrix P).

4. For every $k \geq 1$ construct a simple graph in which every vertex has degree $2k$ which has no perfect matching.

5. For every $k \geq 1$ construct a simple graph in which every vertex has degree $2k + 1$ which has no perfect matching. (Hint: start with $k = 1$ and then generalize).

6. Prove that a tree T has a perfect matching if and only if $odd(T - v) = 1$ for every $v \in V(T)$.

7. Let G be a simple graph in which every vertex has degree 3. Prove that G has a perfect matching if and only if there is a decomposition of G into 3-edge paths.