Homework 7

Problem 1. Find a graph on 100 vertices with ≥ 98 cut vertices.

Problem 2. Find a formula for the number of spanning trees of a graph in terms of the number of spanning trees of each block.

Problem 3. If C_1 and C_2 are cycles of maximum length in a 2-connected graph, show that $|V(C_1) \cap V(C_2)| \geq 2$.

Problem 4. Let G be a simple graph, and assume that every vertex has degree $\geq |V(G)| - 2$. Prove that $G - X$ is connected whenever $|X| < |V(G)| - 2$.

Problem 5. Prove that every vertex of G has even degree if and only if every block of G is Eulerian.

Definition: For a set of vertices A we let $\delta(A) = \{uv \in E(G) \mid u \in A \text{ and } v \notin A\}$ and we call any set of the form $\delta(A)$ an *edge cut*.

Problem 6. Prove that the symmetric difference of two edge cuts is an edge cut.

Problem 7. Let F be a nonempty set of edges in G. Prove that F is an edge cut if and only if F contains an even number of edges from every cycle in G. For example, when $G = C_n$ every even subset of edges is an edge cut, but no odd set of edges is an edge cut. Hint: For sufficiency, the task is to show that the components of $G - F$ can be grouped into two nonempty collections so that every edge in F has an endpoint in each collection.