Homework 8

Problem 1. Find a 100-connected bipartite graph G for which $|V(G)|$ is minimum.

Problem 2. Prove or Disprove: If G is a 2-connected graph and $P \subseteq G$ is a path from u to v, then $G - E(P)$ contains a path from u to v.

Problem 3. Prove or Disprove: If G is a 2-connected graph and $x, y, z \in V(G)$, then there exists a path from x to z which contains y.

Problem 4. Let G be a connected graph with $|V(G)| \geq 2$, and assume that G has no cycle of even length. Prove that every block of G is either an edge or an odd cycle.

Problem 5. Let v be a vertex of a 2-connected graph G. Prove that v has a neighbour u so that $G - \{u, v\}$ is connected.

Problem 6. Let G be a connected graph with no cut-edge. Define a binary relation $\sim$ on $E(G)$ by the rule that $e, f \in E(G)$ satisfy $e \sim f$ if either $e = f$ or $G - \{e, f\}$ is disconnected.

1. Show that $e \sim f$ if and only if e and f belong to the same cycles.
2. Show that $\sim$ is an equivalence relation.
3. For each equivalence class F, show that there is a cycle containing all of F.

Problem 7. Let G be a 3-regular 3-connected graph and let $u, v \in V(G)$. Prove that G contains a path P from u to v with the property that $G - V(P)$ is connected. (Hint: choose a path P from u to v so that in the graph $G - V(P)$ the largest component is as large as possible, and subject to this the second largest component is as large as possible, and so on.)