Topological Graph Theory

Math 820 - Spring 2006

Homework assignment #2

1. Prove (by using the Jordan Curve Theorem) that a planar graph is bipartite if and only if all its faces are of even length. (Make appropriate definition of the "face length" yourself.)
2. Let G be a (2-)connected planar graph with vertices $v_1,\ldots,v_n$ and faces $F_1,\ldots,F_n$. Prove that:

$$\sum (6 - \deg(v_i)) \geq 12$$

$$\sum (6 - \deg(v_i)) + \sum (6 - 2\deg(F_j)) = 12$$

$$\sum (4 - \deg(v_i)) + \sum (4 - \deg(F_j)) = 8.$$

   When do we have equality in the first formula? How do these relations change if we allow multigraphs?
3. Let G be a planar graph of minimum degree at least 4. Show that G has at least eight triangular faces. Find an infinite set of 3-connected planar graphs of minimum degree 4 and with precisely eight triangular faces.
4. Read the proof of Theorem 2.5.2 (and the corresponding Lemma 2.5.3) on p.40-41 (supplied on the course web page).
5. Go to Ken Stephenson's (or some other) web page and find some nice drawings of circle packings.