Math 821 – Combinatorics
Homework Assignment #2
7/2/2007

To be handed by 15/2/2007

1. Find some specific values (and possibly an infinite sequence of integers for which:
   (a) Theorem BRC cannot be applied.
   (b) Theorem BRC can be applied and this case does not follow by the corollary that v is the sum of two squares. (If you cannot find such examples, just discuss possible values of $v \leq 35$.)

2. Prove that every projective plane of order 7 is isomorphic to the Fano plane. Find (the smallest integer) v for which there exist two nonisomorphic STS(v).

3. We say that a projective plane $(\mathcal{P}', \mathcal{B}', \mathcal{I}')$ of order m is a subplane of a projective plane $(\mathcal{P}, \mathcal{B}, \mathcal{I})$ of order n if $\mathcal{P}' \subseteq \mathcal{P}$, every line $B' \in \mathcal{B}'$ is contained in some line in $B \in \mathcal{B}$, and the incidence I' is induced by the incidence relation I (with respect to the correspondence $B' \mapsto B$).

   Prove that $m^2 \leq n$ and give an example where $m^2 = n$.

   Show that if $m^2 = n$, the following holds:
   (a) For every point $p \in \mathcal{P} \backslash \mathcal{P}'$ there exists a unique line $\ell \in \mathcal{B}'$ through p.
   (b) For every line $\ell \in \mathcal{B} \setminus \mathcal{B}'$ there exists a unique point $p \in \mathcal{P}'$ on ℓ.