Math 821 – Combinatorics
Homework Assignment #4
14/3/2007

To be handed by 21/3/2007

The questions are sorted roughly according to their difficulty, the number of marks for them will correspond to this.

Question 1: Show that a strongly regular graph is extremal in the following sense. Let G be a graph with v vertices, each of degree at most k. Suppose that any two adjacent vertices, respectively nonadjacent vertices, have at least λ, respectively μ, common neighbors. Then

$$k(k-1-\lambda) \geq \mu(v-k-1)$$

and equality implies that G is strongly regular.

Question 2: (left from class) An incidence structure $S = (P, B, I)$ (we call elements of P points and elements of B lines) is a *generalized quadrangle* if

1. For any two different points p, q, there is at most one line incident with both of them.
2. If a point p and a line L are not incident, there is exactly one point q and line L' such that L' is incident with both p and q, and q is incident with L.
3. There are two distinct points p, q such that no line is incident with both of them.
4. There are two distinct lines L, M such that no point is incident with both of them.

Prove that there are distinct lines L_1, L_2, L_3, L_4 and distinct points p_1, p_2, p_3, p_4 such that p_i is incident with L_i and L_{i+1} (indices modulo 4) for $i = 1, 2, 3, 4$.

Question 3: Show that a SRG$(28, 9, 0, 4)$ does not exist. Use only combinatorial arguments (no eigenvalues, algebra, ...).

Question 4: Let $\mathcal{D}$ be a $2-(v, k, \lambda)$ design such that any two distinct blocks of $\mathcal{D}$ have exactly l_1 or exactly l_2 points in common. Let G be the graph with the blocks of $\mathcal{D}$ as vertices, and with two blocks adjacent iff they have exactly l_1 points in common. Suppose G is connected.

1. Find eigenvalues of G.
2. Show that G is strongly regular and find its parameters.

Hint: You may use the following result: If a connected regular graph has exactly three distinct eigenvalues then it is strongly regular.