Homework #2 • Numerical Analysis II (math 416) • Nonlinear BVPs

- due Wednesday 27 September.
- please indicate any collaborations, or acknowledge any useful e-mails.
- remember that the class e-mail is open for discussion.
- *annotate* all plots (can be handwritten on plots). — page limits include annotated plots.

A) (1 page) This problem is based on Heath, p263, #8.33 as well as sections 8.7-8. On page 257,

> By using function values at additional points, $x \pm 2h$, $x \pm 3h$, ..., we can derive similar finite difference approximations with still higher accuracy ...

Show how the next-order finite-difference approximations are easily derived using the idea of Richardson extrapolation that combines the low-order h and $2h$-discretizations. Address the issues raised by question #8.33. Don't show needless algebra, but do explain the essence of any calculations.

B) (2 pages) Download the script *lect07.m* for solving the nonlinear 2-point BVP

$$y'' + y - \beta\, y^3 = 0 \qquad \text{on} \quad 0 \le x \le L \qquad \text{with} \quad y(0) = y(L) = 0$$

and clean up the script to make it a strictly sparse implementation. Investigate how λ varies as a function of $A = \max\{y(x)\}$ and β. Note that the above relationship is the reverse of the script, but this is really just an exchange of plot axes. Find the simplest expression you can for $\lambda(A, \beta)$ when λ is small. Remember that your numbers are going to be N-dependent, but the result of your investigations should be based upon converged runs with a sufficiently large N.

When $L < 0$, note that the Newton iteration always seems to converge to the zero solution. But when $L > 2\pi$, one can find a completely new solution to the above problem. The class challenge problem is to find computationally this new solution – you may post to class e-mail vague hints prior to monday's lecture, after which you will be required to post your methodology, or acknowledge any assistance.

Make brief mention in your write-up of the code verifications you have performed.

C) (2 pages) Make the major modifications to the *lect07.m* script to solve either of the 2-point linear BVPs (no Newton solves are needed)

$$y_n'' = -y_o + \beta\, y_o^3 \qquad \text{on} \quad 0 \le x \le L \qquad \text{with} \quad y_n(0) = y_n(L) = 0$$

or

$$y_n'' + y_n = \beta\, y_o^3 \qquad \text{on} \quad 0 \le x \le L \qquad \text{with} \quad y_n(0) = y_n(L) = 0 \ .$$

Quantify the convergence of this naïve iteration to one of your Newton-converged solutions of part B). You are encouraged to post quant strategies, pseudocode and code snippets to the class e-mail on this problem. Also, I would like that class split in half over this choice, so please post to class e-mail to organize amongst yourselves.