Homework #3 • Numerical Analysis II (math 416) • Fast Fourier Transform

- due Wednesday 04 October.
- please indicate any collaborations, or acknowledge any useful e-mails.
- remember that the class e-mail is open for discussion.

A) (2 pages) The attached page comes from my Schaum's outline *Math Handbook*. On the class webpage is a script *hw03a.m* which investigates the periodic version of a zigzag wave (23.8). Using this script, you can show that the convergence of the FFT coefficients to the exact values is not spectrally accurate (as shown in class for $e^{\cos x}$), but has the power law scaling $O(h^2)$.

Show how to derive the Fourier coefficients that are given for the periodic square wave (23.7). Then, modify the script to analyze the Fourier coefficients for this case. Note that the convergence has yet a different power law scaling from the zigzag wave! (**Challenge:** give a conjecture for the different convergence behaviors of these waves.)

B) (2 pages) Download the data file *data03* which gives a 9-point curve that has a period interval on $0 \leq x \leq 2\pi$. The Matlab command *load data03* will input the numbers into an array called *data03* where you can then manipulate them. Use Matlab's FFT commands to perform a spectral interpolation of this data onto a 65-point curve. Note that the last point is the periodic point of the first & should be discarded for the FFT analysis, but is nice to include in the plots. The Matlab command *zeros* will prove helpful.

Given that the original data came from the periodic function

$$f(x) = \sqrt{3 + \cos(x) - \sin(2x - \pi/4) - \cos(3x - \pi/5)} \ ,$$

can you account for the differences between the original curve and your interpolation? That is, some of the interpolated values seem to be better than others, why?

C) (2 pages) Modify the script from class *lect11.m* to calculate the spectral derivative of my mystery function

$$g(x) = e^{\cos x} \ .$$

Show that the FFT-calculated values of $g'(x)$ are spectrally accurate with the exact derivative. (You do not need to use the exact Fourier coefficients in the script – you may delete those lines.) Note also, that a handy vector of n-values is defined in the script, and the imaginary number "i" is just the symbol i.