Homework #6 • Numerical Analysis II (math 416) • Stability for Numerical PDEs

- due Wednesday 08 November.
- please indicate any collaborations, or acknowledge any useful e-mails.

A) (4 pages) Present a Von Neumann analysis for the centered difference scheme

$$u_j^{k+1} = u_j^k - \frac{c\,\Delta t}{2\,\Delta x}\left(u_{j+1}^k - u_{j-1}^k\right)$$

which confirms that it is (weakly) unconditionally unstable. In your derivation, use the fact that

$$u_{j+\bar{j}}^{k+\bar{k}} = e^{i\alpha\,\bar{j}\Delta x}\; e^{-i\omega\,\bar{k}\Delta t}\; u_j^k$$

to reduce the notational work. Verify that the amplification $|Z(n)|$ plot is correctly implemented in the script *lect22a.m*.

As a basis for understanding the meaning of the amplification plot, follow this recipe for making two plots. First, the script *lect18.m* can be modified to run the centered difference scheme – set the spatial and temporal discretization to $M = N = 128$. The final numerical solution at $T = 2\pi$ is clearly corrupted by an instability. Note the period/wavenumber of the spatial oscillation. Second, run the script *lect22.a* for the above case $M = N = 128$. For the value of n/N which has the maximum growth rate, make a plot of the Von Neumann solution (real part) at some fixed time. Discuss what you learn from comparing the two plots.

B) (3 pages) Use the upwind difference scheme to solve the one-way wave problem for $u(x,t)$

$$u_t + c\,u_x = 0 \qquad \text{periodic on} \qquad 0 \le x \le 2\pi$$

$$\text{with IVs}: \quad u(x,0) = e^{i\,4x}\ .$$

Compute the amplitude and phase error relative to the exact solution. Verify the results with the output of the error script *lect22a.m*.

bonus: You may rework one problem from the midterm for extra credit. Please turn this in separately from the homework during an office hour (or other appointed time).