

Homework #6 • MATH 462 • Last Questions for Potential Flow

- optional problems

A*) Vortex Near the Ground (optional) Use the method of images to construct the complex potential for a line vortex (at $z = +i$, with circulation $\Gamma = 1$) in the vicinity of a ground plane (at $\text{Im}z = 0$). Make a sketch of the flow contours — they are related to Figure 4.2 in Acheson.

Express $\Phi'(z)$ as a Laurent series about $z = i$. Next, apply the Blasius force integral for a suitable body imbedded in your flow (indicate on your sketch). Can you use this result to explain why the vortex in the airplane landing video moved as it did?

bonus: Does incorporating a second vortex line account for the vertical motion?

B*) Plotting an Airfoil (optional) Following Acheson, section 4.9

In view of Figs 4.5 and 4.6 it will come as no surprise that if we use the mapping (4.56) on a circle in the z -plane which passes through $z = a$ but which encloses $z = -a$, we obtain an aerofoil with a rounded nose, but a sharp trailing edge . . .

Verify this statement by plotting an asymmetric airfoil using the Zhukovsky transformation

$$Z = z + \frac{a^2}{z} .$$

The plotting command *axis equal* may be needed to get the image to look right. Give a formula for the angle which the trailing edge makes with the horizontal, and use your plot to verify its correctness.

