Homework #7 • MATH 462 • Surface Waves & Sound Waves

- submit your write-up Friday 19 March.

A) Added Mass Effect (3 pages + plot, 10pts) The d'Alembert paradox refers to the failure of potential flow to describe the drag force of a body moving through a fluid — despite that the lift force is well-represented. Oddly enough, potential flow is capable of describing the *added mass* effect, which is a drag-like force on an object being accelerated in a fluid.

A body moving through a fluid must displace the fluid directly in front of it (and replace it in behind). One might intuit that this dynamical displacement of mass must result in a form of resistance. The d'Alembert paradox is that for constant flow, there is a symmetry in the front and rear pressure fields that exactly cancel. However, for a time-dependent flow, this is not the case, and the effect is often referred to as the *added mass* effect. Note that this is a virtual effect, as there is no actual creation of mass, the terminology derives from the fact the force required to accelerate the body is more than $f = m_{body}\, a$.

(i) Begin by deriving an extra term for the Blasius formula that is obtained by considering a time-dependent velocity potential for a velocity.

(ii) Write down the complex potential, $\Phi(z,t)$, for flow around a cylinder whose u-velocity at infinity is $U(t)$. By Galilean thinking, this will be equivalent to a body moving with velocity $-U(t)$ in a stationary fluid. Then calculate the fluid force, $\vec{f}_{am}$, on the cylinder due to the time-dependent term — check that it is zero for constant flow.

(iii) Plot the surface pressure $p_{surf}(\theta)$ to illustrate the asymmetry of the force on the cylinder.

B) Finite Depth Fluid (3 pages + plot, 10pts) Give a complete discussion for the derivation of the travelling wave solution to the linearized surface wave equations with a bottom boundary (located at $y = -H$). Summarize clearly the formulas for the PDE solutions and the wavespeed. Note that the ratio $c(k,H)/c(k,\infty)$ is only a function of one variable, make a plot and explain what it tells. Quality of presentation will be a significant part of the grade for this problem.

bonus: Research some tsunami data and verify whether the shallow water assumptions are justified, and how well the wavespeed formula applies in the open ocean.