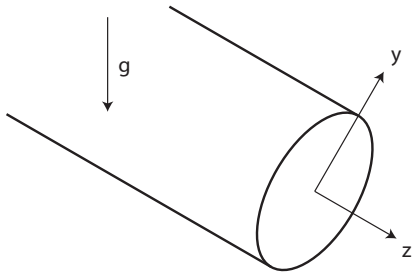
**Homework #9 • MATH 462 • Viscous Flow**

- write-ups are due 12:00 noon on Wednesday 07 April.

**A) Pipe Poiseuille Flow** (4 pages, 15pts) Use the steady Navier-Stokes equations to calculate *gravity-driven* flow through a pipe which is inclined at an angle $\alpha$ from the horizontal. I recommend using cylindrical coordinates with the $\hat{z}$-axis oriented with the pipe axis – hence, gravity will be seem to be *tilted* in the equations.




Show that in the above coordinate geometry

$$\vec{g} = g\left(-\cos\alpha\ (\sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}) + \sin\alpha\,\hat{z}\right)\ .$$

It is not surprising that the axial velocity is $z$-independent, but it turns out also to be $\theta$-independent (like usual pipe Poiseuille flow). Thus $W(r)$ only. The pressure is also $z$-independent, but develops a cross-sectional distribution.

Calculate the mass flux through the pipe. Plot the cross-sectional pressure distribution.

**B) Spinning Sphere** (5 pages, 20pts) Solve the problem as posed by #7.2 in Acheson. Presentation (with discussion) will be graded seriously. Show that you understand both the mathematics and the fluid dynamics. (Presentations with only equations will earn only a 1/4-grade. Key words are: correctness, clarity & conciseness.) The solution strategy for this problem parallels the flow past of a sphere problem in Section 7.2.