Reflections #2 • Math 495 / Phys 484 • Fast/Slow Dynamics

- due date is Monday 05 February.
- remember that webct is an open forum for discussion.
- please acknowledge collaborations & assistance from colleagues – these are encouraged.
- the instructions from the first assignment apply (please review). Pay special attention to producing *annotated* plots.

A) Slow Manifold I: (1 page + annotated plots) Recall the slaving ODE example

$$\begin{aligned} u_1' &= r\,u_1 + u_1 u_2 \\ u_2' &= -u_2 - u_1^2 \end{aligned}$$

where r is the growth rate of u_1. Show that there is an $r \ll 1$ rescaling of the above equations which produces the fast/slow dynamical ODEs

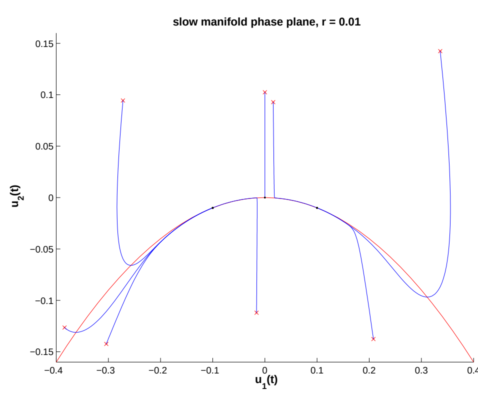
$$\begin{aligned} \frac{dy_1}{d\tau} &= (\tilde{r} + y_2)\,y_1 \\ \frac{dy_2}{d\tau} &= -\frac{1}{\epsilon}\,(y_2 + y_1^2) \end{aligned}$$

where all quantities are $O(1)$. Begin by introducing the scaling $r = \tilde{r}\epsilon$.

One interpretation of this result is that, for the small-r limit, the fast/slow dynamics is an asymptotically valid theory in a restricted area of the (u_1, u_2)-phase plane. The size of this area scales with the smallness of r. Make small modifications to the *manif.m* script which:

a) computes the slaved ODE system, and

b) rescales the phase plane coordinates, and timestep on r, to focus *only* on the regime of fast/slow behaviour. How small does r seem to have do be, for the approximation to be a good one?



B) Slow Manifold II: (3 pages + annotated plots) This should be considered a group-oriented exercise. The goal is for the class members to design a computational experiment for evaluating how well the fast/slow dimensional reduction works in practice.

Points to consider:

a) how well do computed ODE trajectories (u_1, u_2) follow the leading-order slow manifold? The first correction? (Quantify error as a function of ϵ.)

b) how well do the computed $u_1(t)$ solutions follow the leading-order $u_1(t)$ from the first-order reduced dynamics. (Again, quantify errors.)

C) Swift-Hohenberg-like: (3-4 pages + annotated plots) Consider the modification to the Swift-Hohenberg PDE

$$u_t = [r - (\nabla^2 + q^2)^2]\, u - u_x\, u_y$$

on a 2D square domain of size $L_x = L_y = \pi$. Following the weakly nonlinear analysis as presented in lecture, determine the fate of the centre modes, and all other Fourier modes excited by the largest nonlinear terms. In the end, you will be considering the Fourier truncation

$$u(\vec{x}, t) = \epsilon\, a_{1,1}(t)\, S_{1,1} + A_{2,2}(t)\, S_{2,2} + A_{3,3}(t)\, S_{3,3} + \tilde{u}_2$$

where the $S_{M,N} = \sin k_M x\, \sin l_N y$. For $\epsilon \ll 1$ and $a_{1,1}(t) = O(1)$, determine the remaining scalings so that the bifurcation equations obtained are exactly that as the fast/slow dynamical ODEs of part **B)**.

Use the given Swift-Hohenberg solver to produce graphical support for your theoretical work. Note that the solver is for a periodic domain, however, solutions initialized as a Fourier sine series remain as such for all time. You are strongly encouraged to discuss the design of experiments and graphics with colleagues and instructors.