Exact & Asymptotic Methods for Differential Equations

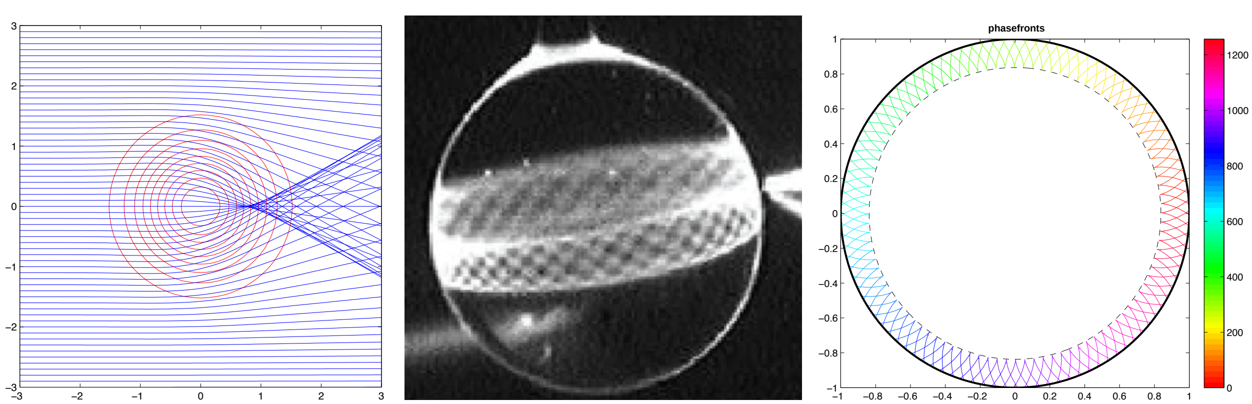
The **APMA 900** course includes an overview of classes of ordinary and partial differential equations (ODEs & PDEs) that are solved by exact methods. The idea of Fourier series on finite domains is extended to integral solution methods on infinite domains; these include the Fourier and Laplace transforms. Investigation of this solution perspective establishes the close connection between complex variable theory and DEs. Development of other solution approaches naturally lead to an introduction to Sturm-Liouville eigenfunctions, function (Hilbert) spaces and special functions.

But many ODEs and PDEs encountered in applications are not amenable to exact solution. Nonetheless, a variety of so-called asymptotic methods are available for extracting analytical understanding. These approximate methods can also be useful in benchmarking numerically-computed solutions, and even decoding exact solutions whose formula complexity defies interpretation. Perturbation theory analyzes problems that are "nearby" to those with known exact properties. This perspective also gives mathematical insight into the consequences of approximations which neglect complicating effects in the development of model equations. Yet other types of asymptotic methods address singular situations where small changes to DE problems have a large impact on the solution. Examples include techniques known as: multiple-scale, averaging, WKB (geometrical optics) and boundary-layer methods.

Lectures will be based upon a case-study approach of ODE & PDE examples. Computational illustration will be an important tool for the lectures and assigned work. Computer visualization and numerical computing will involve the use and modification of Matlab scripts.

Course prerequisites: Undergraduate introduction to ODEs and linear PDEs. Other useful background includes real & complex analysis, elementary numerical analysis &/or scientific computing. SFU undergraduates with Math 418 credit are welcome to inquire about enrollment.

Further information & updates: www.math.sfu.ca/∼muraki



These images are visual representations of PDE behaviours associated with optical phenomena. A ray tracing describing the focussing by a lens that results from the WKB method applied to the wave equation. Laser light trapped as a whispering gallery mode in a 300-μm glass sphere illuminates the principles of wave propagation & refraction. A plot derived from a 2D version of the whispering gallery wave obtained through the analysis of a Bessel function.