A Uniform PV Framework for Balanced Dynamics

- ▷ vertical structure of the troposphere
- ▷ surface quasigeostrophic models

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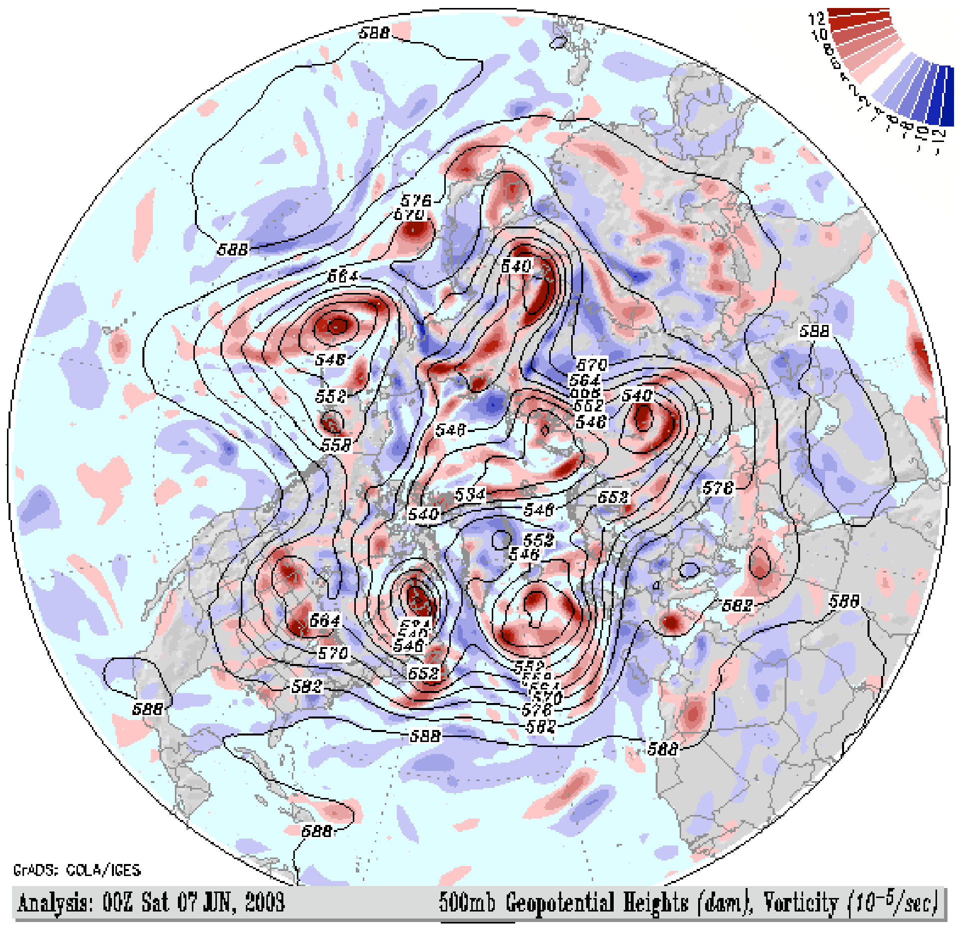

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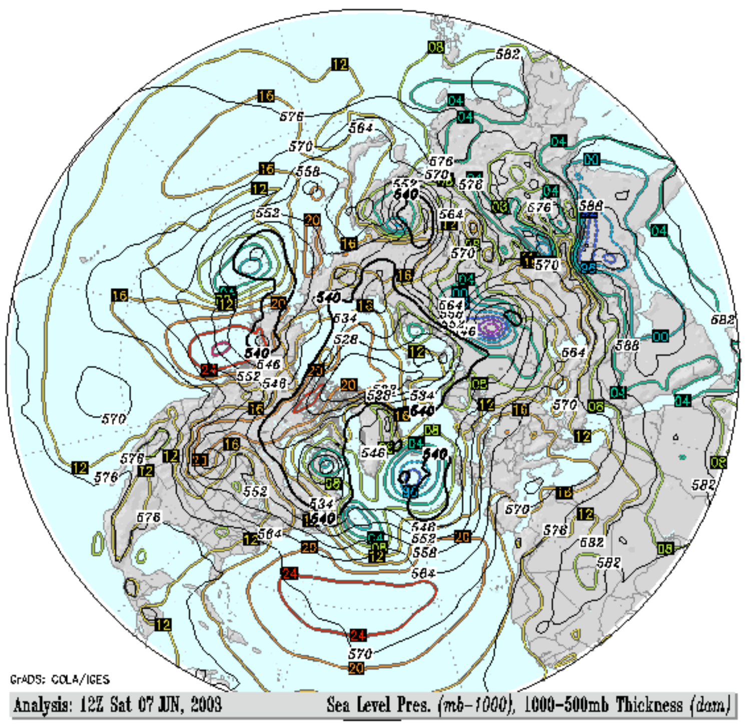
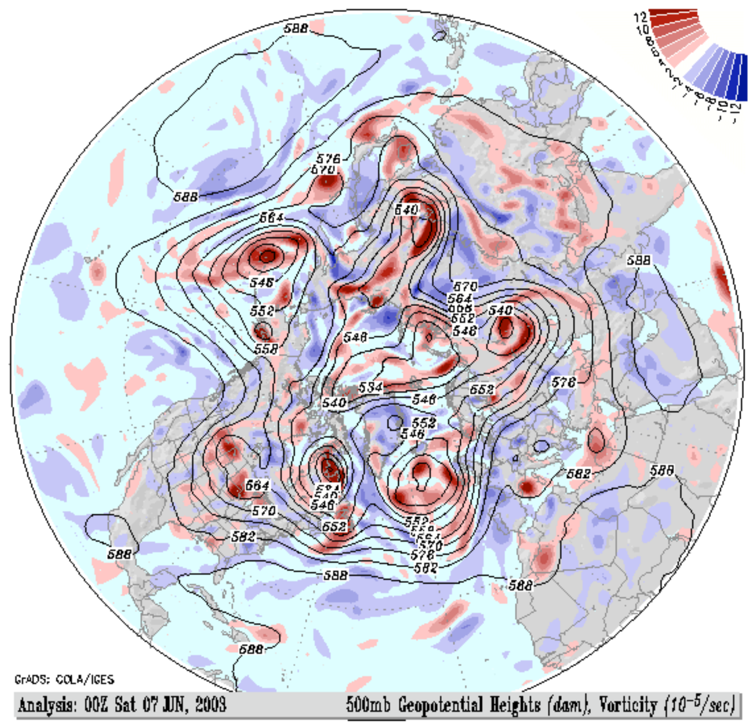
Dynamical Structures of the Troposphere

Two-Dimensional Idealizations

- 2D barotropic vorticity: rotating dynamics
- shallow water: balanced and gravity wave dynamics & vertical displacement
- surface QG: balanced dynamics & uniform PV with vertical structure

- surface pressure & thickness vis-à-vis 500 hPa vorticity & geopotential





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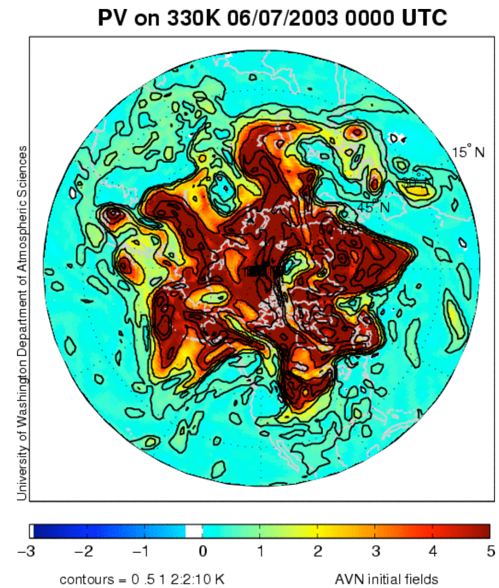
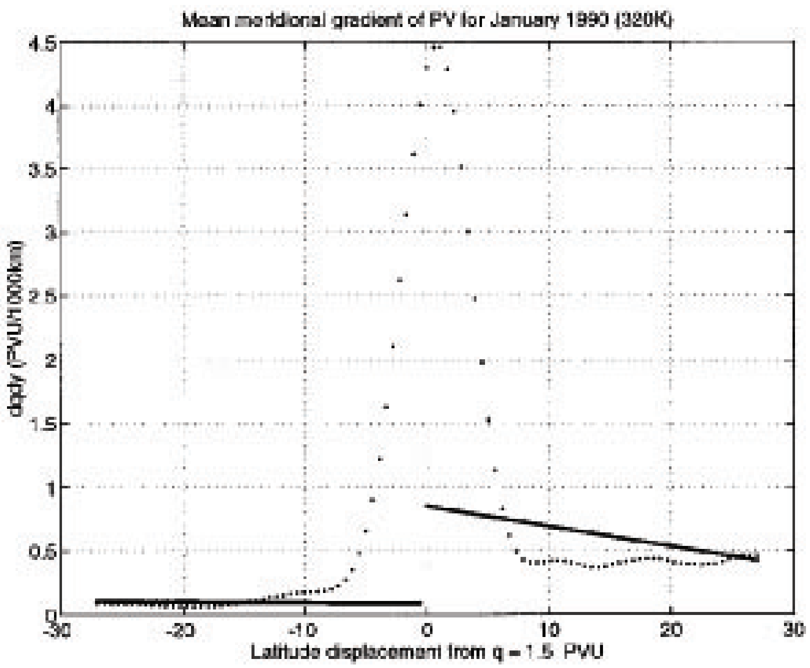
PV Structure of the Lower Atmosphere

Well-Mixed Troposphere

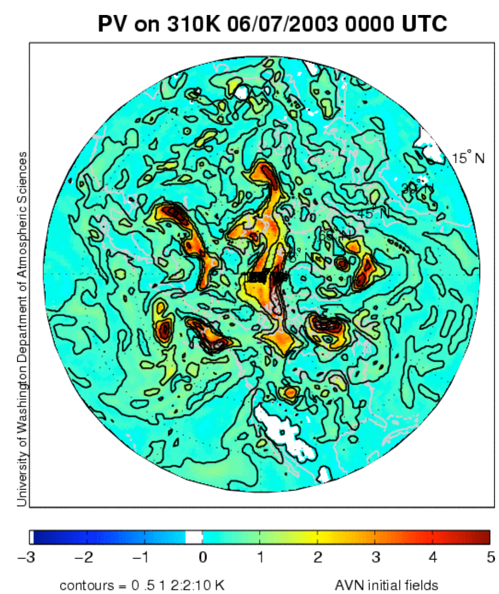
- PV gradients are relatively weak in troposphere
- dominant PV influence from tropopause displacement

- PV on 330K & 310K surfaces, mean PV gradient





morgan & nielsen-gammon (1999)



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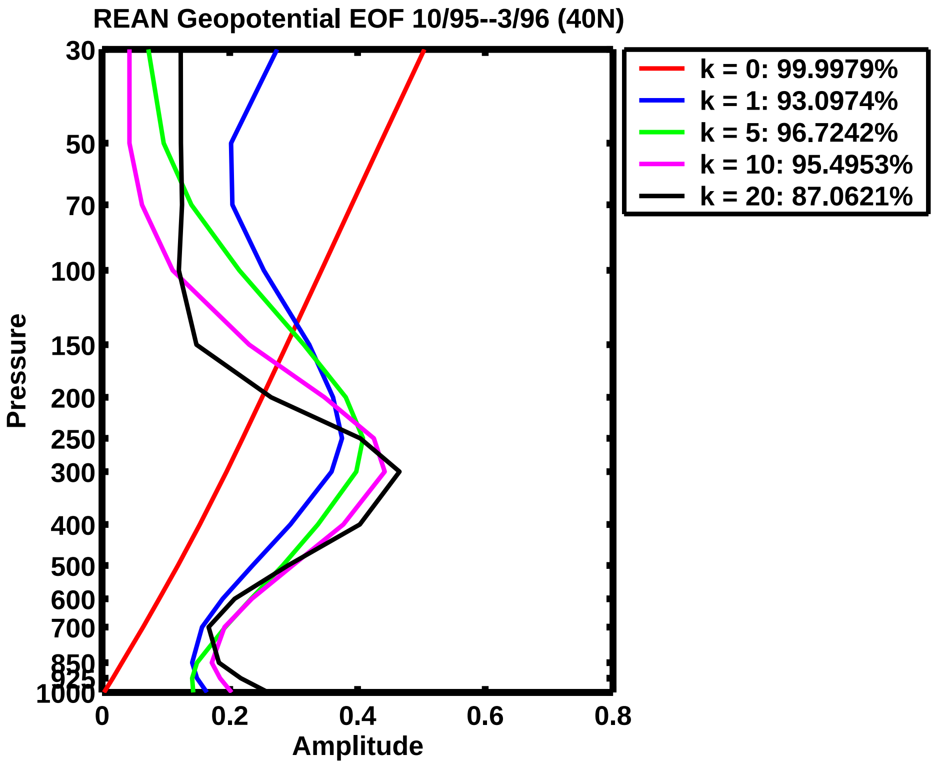
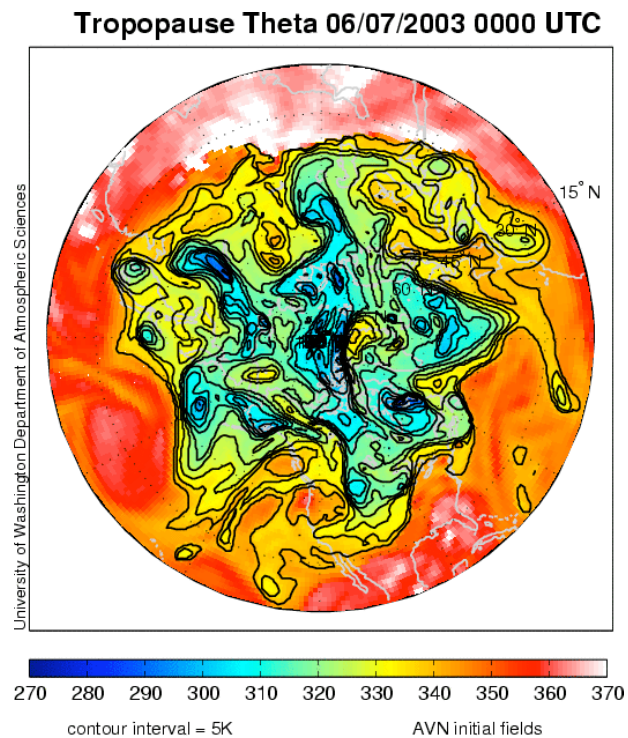
Vertical Structure of the Lower Atmosphere

Upper-Level Disturbances

- ▷ disturbance amplitudes peaked at tropopause level, decrease in troposphere

- ▷ vertical structure of geopotential (zonal fourier amplitudes) at 40°N
- ▷ tropopause map of potential temperature





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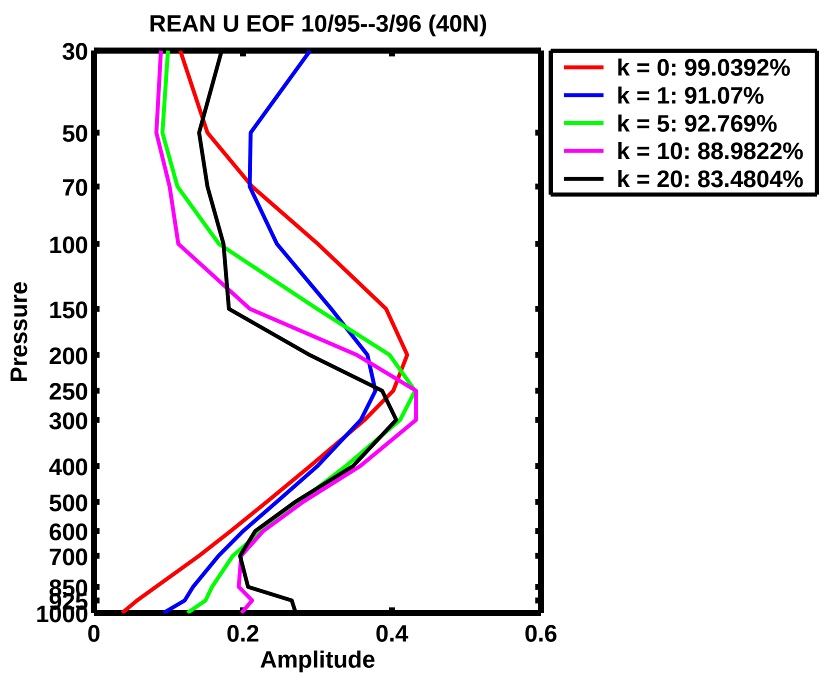
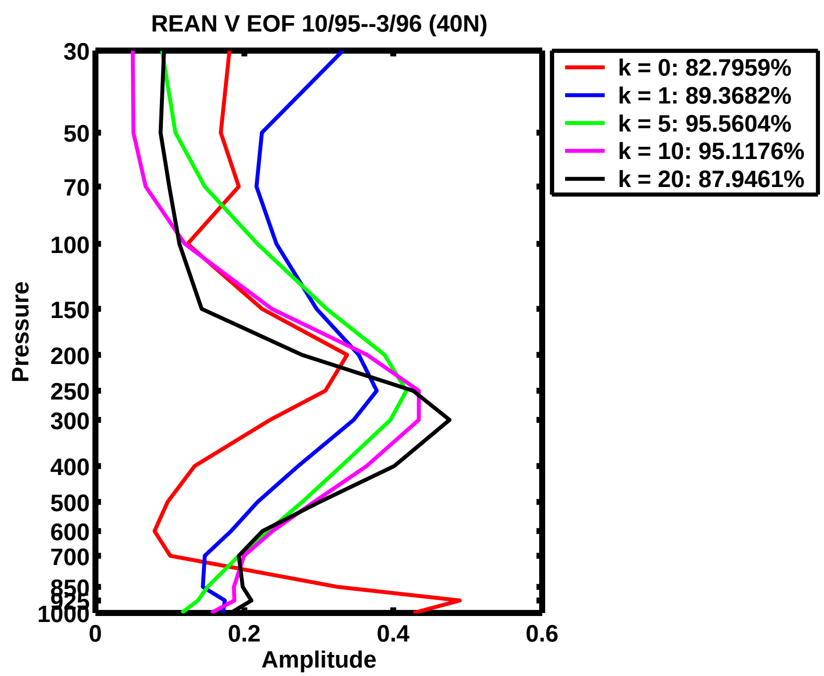
- ▷ tropopause potential temperature as key dynamical variable for simple tropospheric model

Vertical Structure of the Lower Atmosphere II

Upper-Level Disturbances

- ▷ disturbance amplitudes peaked at tropopause level, decrease in troposphere
- ▷ vertical structure of winds (zonal fourier amplitudes) at 40°N





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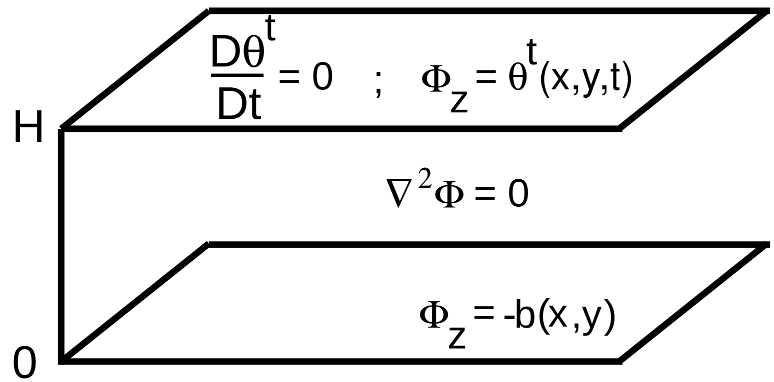
Zero PV Quasigeostrophic Dynamics

Well-Mixed Troposphere $\Rightarrow q \equiv 0$

- ▷ dynamically active, rigid tropopause ($z = H$) & passive, topographic lower surface ($z = 0$)



Surface QG for a Finite Depth Troposphere (HsQG)

- ▷ 3D PV inversion from tropopause & topographic BCs:

$$\nabla^2 \Phi = 0 \quad ; \quad \Phi_z(z = H) = \theta^t(x, y, t) \quad ; \quad \Phi_z(z = 0) = -b(x, y)$$

- ▷ 2D advection of tropopause potential temperature, θ^t:

$$\frac{D\theta^t}{Dt} = \theta^t_t + u^t \theta^t_x + v^t \theta^t_y = 0 \quad ; \quad u^t = -\Phi_y(z = H) \quad ; \quad v^t = \Phi_x(z = H)$$

- ▷ sQG *interface* as model for tropopause: Rivest, et.al. (1992); Juckes (1994)

Zero PV Fourier Inversion

- fourier transform of streamfunction ($m = \sqrt{k^2 + l^2}$)

$$\hat{\Phi}(k.l; z, t) = \left\{ \frac{\cosh mz}{m \sinh mH} \right\} \hat{\theta^t}(k, l; t) \quad + \quad \left\{ \frac{\cosh m(z - H)}{m \sinh mH} \right\} \hat{b}(k, l)$$

- vertical decay away from boundaries

  $\longrightarrow$ small scales are more localized to tropopause/surface ($mH \gg 1$)

  $\longrightarrow$ larger scales extend deeper into troposphere ($mH \ll 1$)

Small & Large-Scale Dynamics

- fourier transform of tropopause-level streamfunction (w/o topography)

$$\hat{\Phi^t}(x, y; H, t) = \left\{ \frac{\coth mH}{m} \right\} \hat{\theta^t}(k, l; t) \sim \begin{cases} \dfrac{\hat{\theta^t}}{m} & mH \gg 1 \\ \dfrac{\hat{\theta^t}/H}{m^2} & mH \ll 1 \end{cases}$$

  $\longrightarrow$ small horizontal scales (relative to depth) invert as sQG ($mH \gg 1$)

  $\longrightarrow$ larger horizontal scales large invert as barotropic vorticity ($mH \ll 1$)

- on the large scales, potential temperature gradient $(-\theta^t/H)$ evolves as barotropic vorticity ζ

Topographic Flows

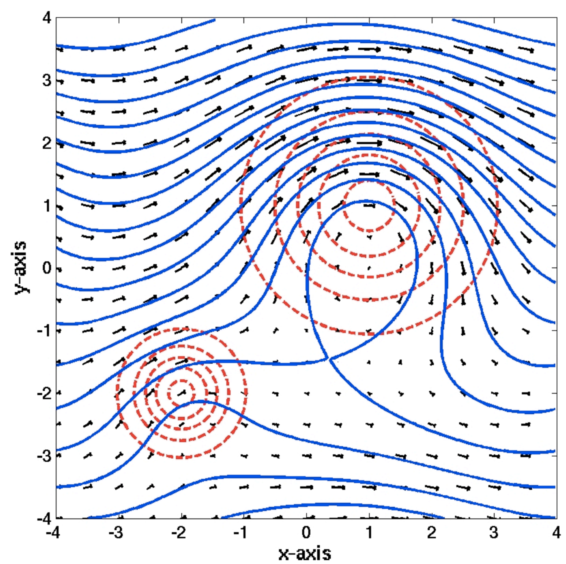
Sloping Bottom Topography ($b = \sigma y$)

- illustration of dynamic similarity between horizontal PV gradient & sloping bottom
- streamfunction: Eady shear with simple travelling fourier mode ($m = \sqrt{k^2 + l^2}$)

$$\Phi(x, y, z, t) = -\sigma y z + A \left\{ \frac{\cosh mz}{m \sinh mH} \right\} \cos k(x - c t)\ \cos ly$$

- topographic Rossby wave dispersion relation, analogous to Rhines (1970)

$$c = \sigma H \left\{ 1 - \frac{\coth mH}{mH} \right\}$$



Flow over Bottom Topography

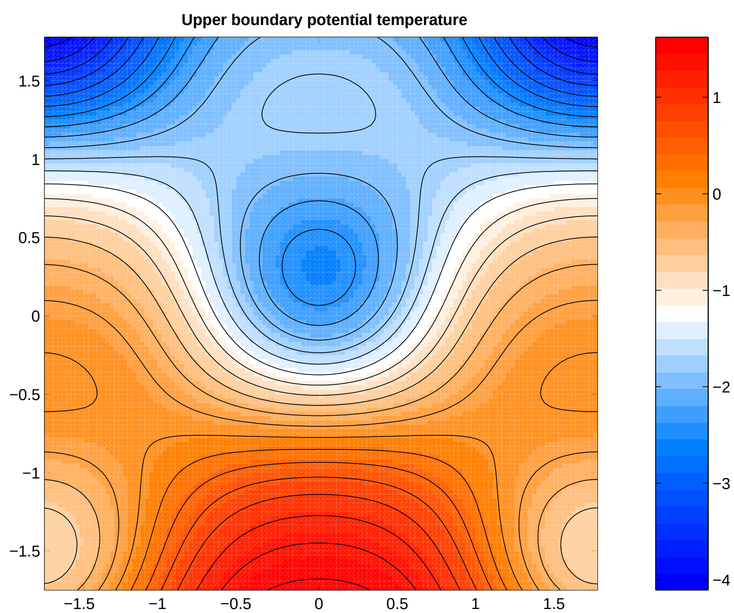
- tropopause streamfunction for gaussian topography, $b = e^{-\alpha r^2}$

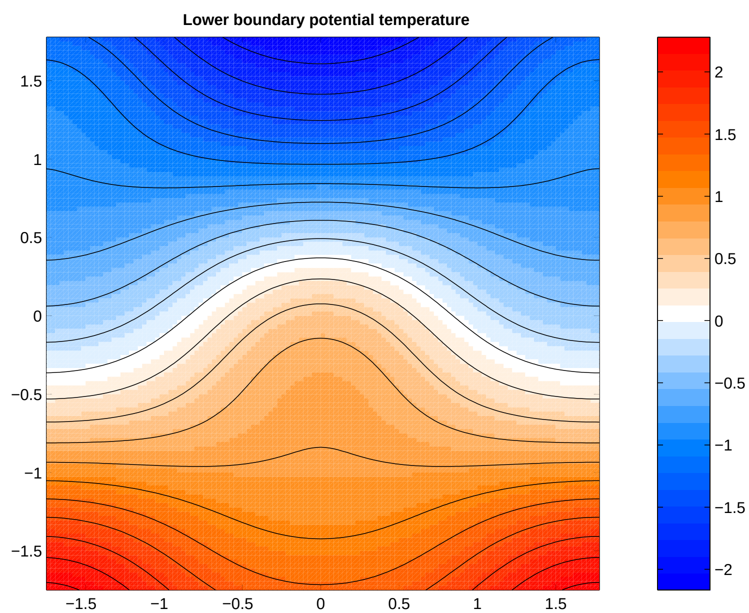
$$\Phi(x, y, H, t) = -\,U^{\infty} y - \int_0^{\infty} \frac{J_0(mr)}{\sinh mH} \frac{e^{-m^2/4\alpha}}{2\alpha}\, dm$$

Two-Surface Edge Wave

Finite Rossby Number Corrections

- nonlinear edge wave solution with simple Eady shear, correct to $O(\mathcal{R})$
- square wave $k = l = 1$, vertical mode number $m = \sqrt{k^2 + l^2} = 2.5$
- beyond short-wave stability criterion: $m > m_c \approx 2.399$
- upper-level cyclone asymmetry for $\mathcal{R} = 0.1$
- nonlinear wavespeed same as neutral linear edge waves





Free-Surface Dynamics

Uniform PV Inversion (in progress, R Tulloch)

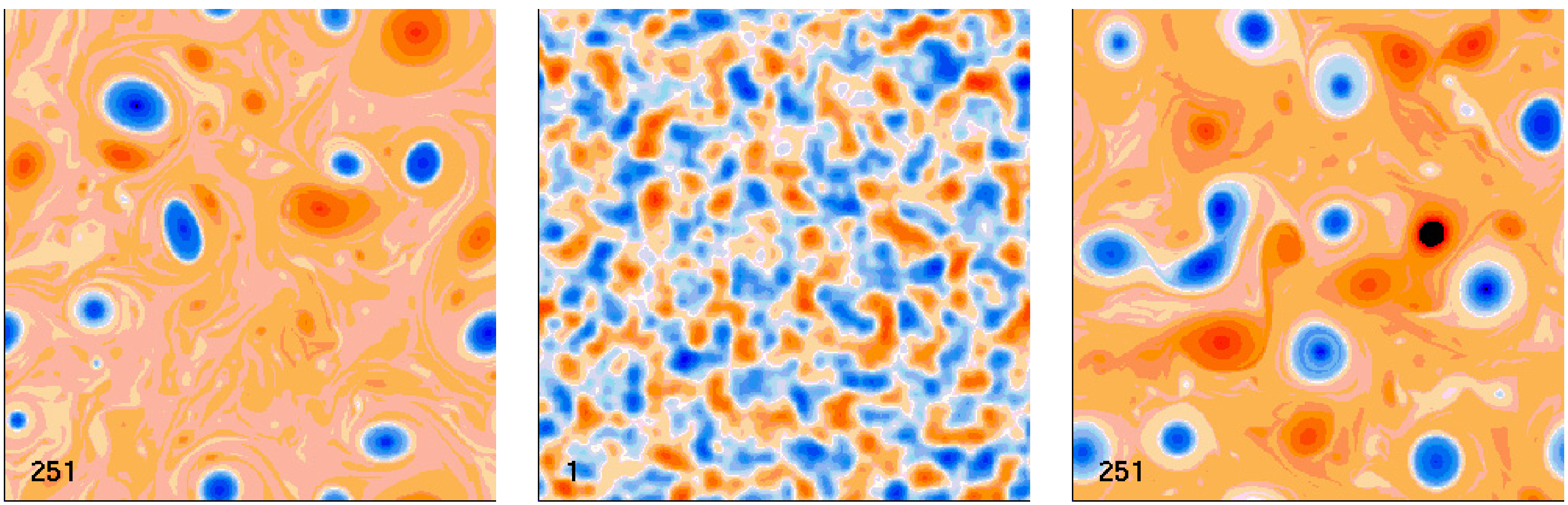
- moving free-surface at $z = \mathcal{R}h(x, y; t)$
- total surface potential temperature, $\theta^s(x, y; t) = h(x, y; t) + \theta(x, y, \mathcal{R}h(x, t; t), t)$
- surface BCs: kinematic conditions with continuity of potential temperature and pressure
- Fourier solution of the 3D streamfunction ($m = \sqrt{k^2 + l^2}$)

$$\Phi(x, y, z; t) = \int_{-\infty}^{+\infty} \hat{\theta}^s(k, l; t) \left\{ \frac{1}{m + \sigma^{-1}} \right\} e^{i(kx+ly)} \, dk \, dl$$

$\longrightarrow$ surface value of potential temperature is $-\sigma$

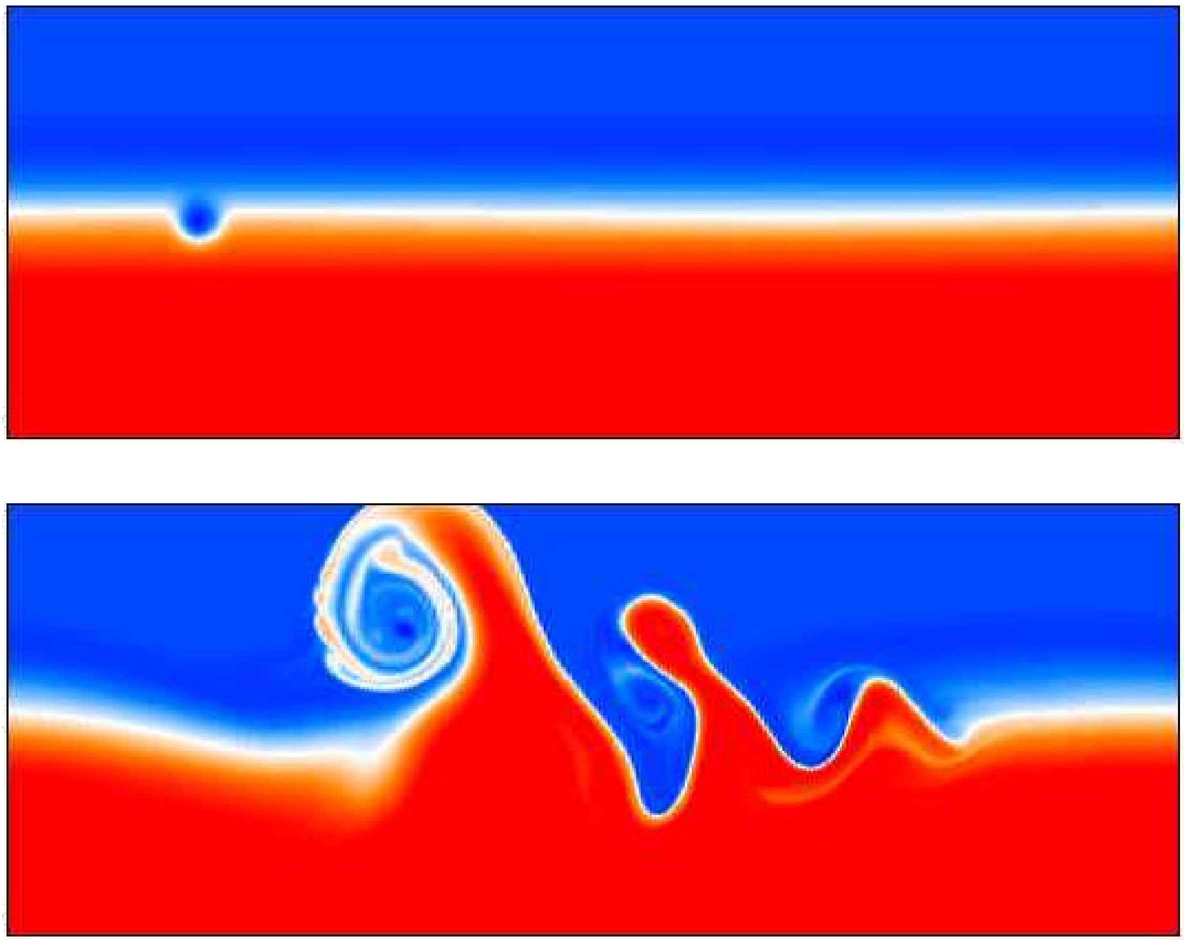
- freely decaying vortex organizations

sQG^{+1} $\longleftarrow$ $\longleftarrow$ random IVs $\longrightarrow$ $\longrightarrow$ fsQG^{+1}



$2sQG^{+1}$: Downstream Development

- tropopause baroclinic wave



Uniform PV Thinking

Dynamics of Uniform PV Layers

- ▷ significant part of tropospheric dynamics are strongly influenced by tropopause & ground
- ▷ simple formulation for understanding rotating, stratified flows dominated by surfaces/boundaries
- ▷ surface dynamics embeds large & small-scale limits:
  - $\longrightarrow$ large-scale barotropic vorticity dynamics
  - $\longrightarrow$ small-scale surface-trapped dynamics
- ▷ moving interface formulations:
  - $\longrightarrow$ free-surface dynamics, as a continuously-stratified shallow-water analog
  - $\longrightarrow$ tropopause dynamics

Computational Efficiency of sQG Fourier Inversion

- ▷ <u>resolution of vertical structure is exact for given horizontal discretization</u>
- ▷ only 2D FFTs required to evolve 3D tropospheric flow
- ▷ finite Rossby number corrections also computed with 2D efficiencies

Exact Reformulation of PE

- three-potential representation: Φ, F, G

$$
\begin{aligned}
v &= \Phi_x - G_z \\
-u &= \Phi_y + F_z \\
\theta &= \Phi_z + G_x - F_y \\
\mathcal{R}\, w &= F_x + G_y
\end{aligned}
$$

- potential inversions

$$
\begin{aligned}
\nabla^2 \Phi &= q - \mathcal{R} \left\{ \nabla \cdot \left[\theta \, (\nabla \times \vec{\mathbf{u}}_H) \right] \right\} \\
\nabla^2 F &= \mathcal{R} \left\{ -\left(\frac{D\theta}{Dt} \right)_x + \left(\frac{Dv}{Dt} \right)_z \right\} \\
\nabla^2 G &= \mathcal{R} \left\{ -\left(\frac{D\theta}{Dt} \right)_y - \left(\frac{Du}{Dt} \right)_z \right\}
\end{aligned}
$$

- surface boundary conditions

$$\mathcal{R}\, w^s = (F_x + G_y)^s \quad ; \quad \theta^t = (\Phi_z + G_x - F_y)^s$$

- advection dynamics (interior & surface)

$$\frac{Dq}{Dt} = 0 \quad ; \quad \frac{D\theta^t}{Dt} + w^s = 0$$