Phys101 Lectures 30, 31

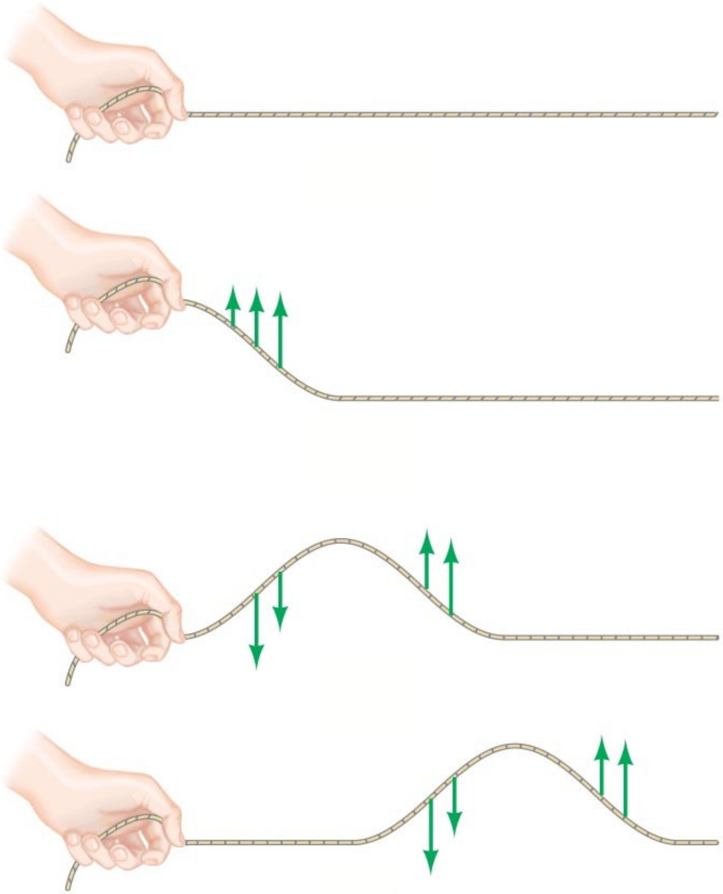
Wave Motion

Key points:

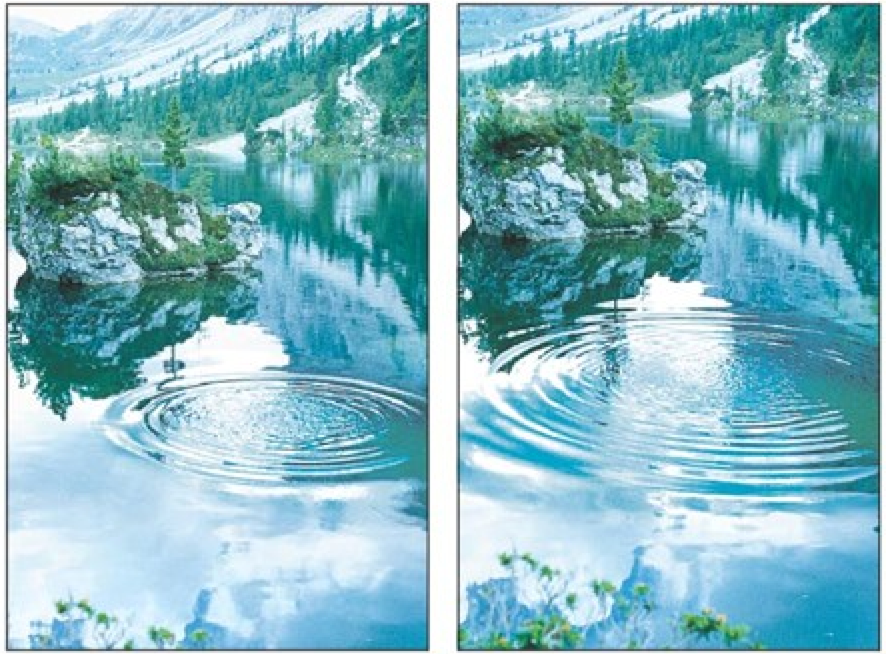
- **Types of Waves: Transverse and Longitudinal**
- **Mathematical Representation of a Traveling Wave**
- **The Principle of Superposition**
- **Standing Waves; Resonance**

Ref: 11-7,8,9,10,11,16,12,13,16.

Characteristics of Wave Motion

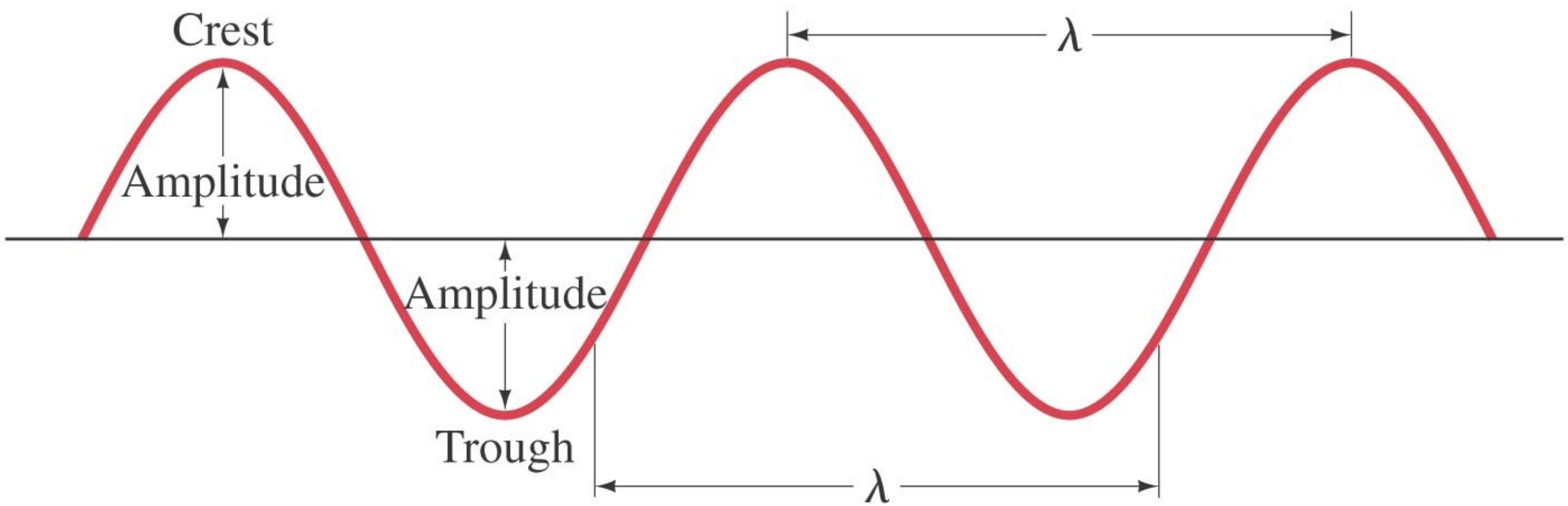


When a vibration propagates, we have a wave.

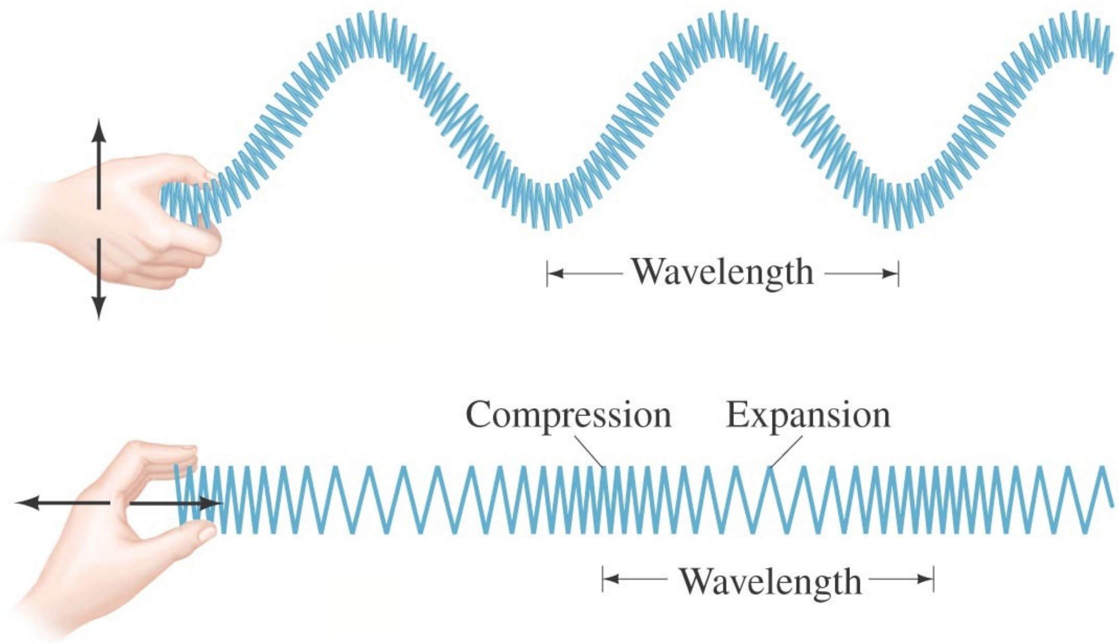


Characteristics of Wave Motion

- **Amplitude**, A
- **Wavelength**, λ
- **Frequency**, f and **period**, T $\quad f = \dfrac{1}{T}$
- Wave **velocity**, $v = \lambda f$



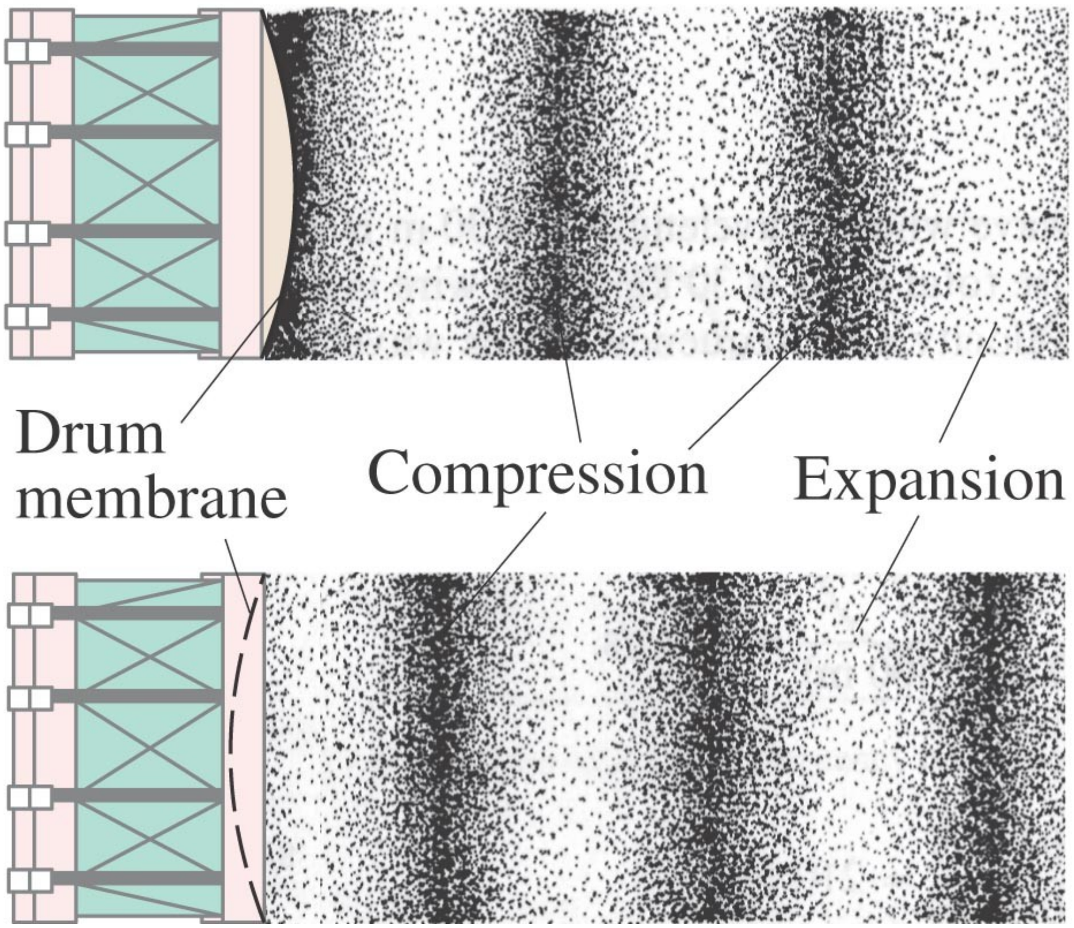
Types of Waves: Transverse and Longitudinal



The motion of particles in a wave can be either **perpendicular** to the wave direction (**transverse**) or **parallel** to it (**longitudinal**).

Types of Waves: Transverse and Longitudinal

Sound waves are **longitudinal** waves:



Types of Waves: Transverse and Longitudinal

The velocity of a transverse wave on a cord is given by:

$$v = \sqrt{\frac{F_{\mathrm{T}}}{\mu}}.$$

Where F_T is the tension in the cord and μ is the linear density (mass per unit length, μ=m/L).

The velocity increases when the tension increases, and decreases when the mass increases.

Types of Waves: Transverse and Longitudinal

The velocity of a longitudinal wave depends on the elastic restoring force of the medium and on the mass density.

$$v = \sqrt{\frac{E}{\rho}}$$ **For a solid rod.**

or

$$v = \sqrt{\frac{B}{\rho}}.$$ **For fluids**

E – Young's Modulus;
B – Bulk modulus.

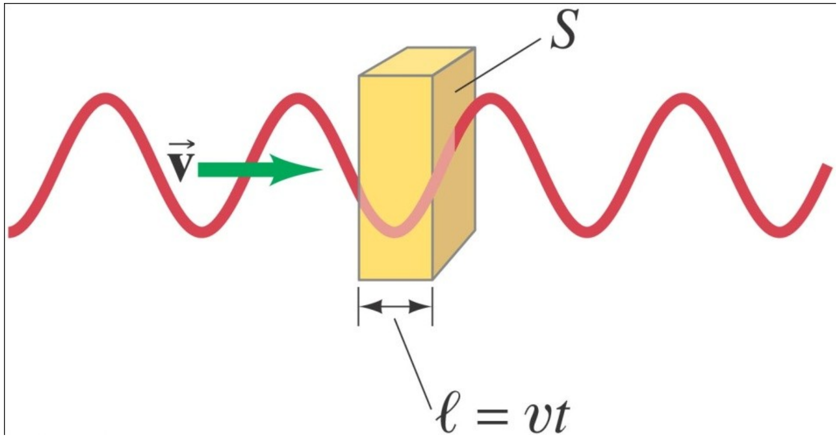
Example: Echolocation.

Echolocation is a form of sensory perception used by animals such as bats, toothed whales, and dolphins. The animal emits a pulse of sound (a longitudinal wave) which, after reflection from objects, returns and is detected by the animal. Echolocation waves can have frequencies of about 100,000 Hz. (a) Estimate the wavelength of a sea animal's echolocation wave. (b) If an obstacle is 100 m from the animal, how long after the animal emits a wave is its reflection detected?

(a)

(b)

Energy Transported by Waves



By looking at the **energy** of a particle of matter in the medium of a wave, we find:

$$E = \tfrac{1}{2}kA^2 = 2\pi^2 m f^2 A^2.$$

Then, assuming the **entire** medium has the same **density**, the power (energy through S per unit time) is

$$\overline{P} = 2\pi^2(\rho S l) f^2 A^2 / t = 2\pi^2 \rho S v f^2 A^2$$

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

$$k = 4\pi^2 f^2 m$$

The intensity (power per unit area) is

$$I = \frac{\overline{P}}{S} = 2\pi^2 v \rho f^2 A^2.$$

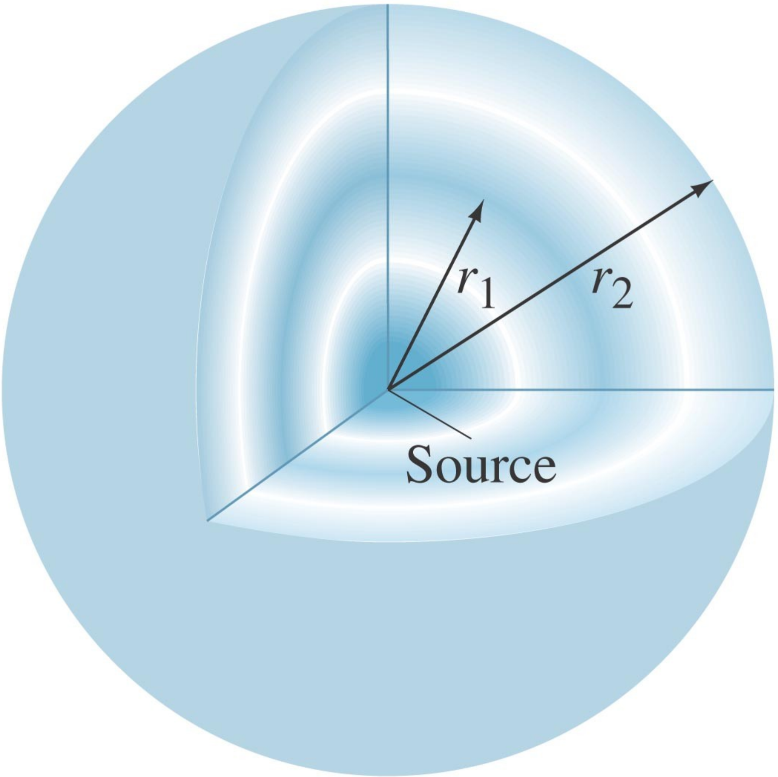
The **intensity** is proportional to the square of the **frequency** and to the square of the **amplitude**.

Energy Transported by Waves

If a wave is able to spread out three-dimensionally from its source, and the medium is uniform, the wave is **spherical**.

Just from geometrical considerations, as long as the power output is **constant**, we see:

$$I \propto \frac{1}{r^2}.$$



$$\text{Total power at } r_1 \text{ and } r_2 : \; P_1 = P_2 = P_0$$

$$\text{Intensity at } r_1 : \; I_1 = \frac{P_1}{4\pi r_1^2} = \frac{P_0}{4\pi r_1^2}$$

$$\text{Intensity at } r_2 : \; I_2 = \frac{P_2}{4\pi r_2^2} = \frac{P_0}{4\pi r_2^2}$$

$$\therefore \text{ Intensity at } r :$$

$$I = \frac{P_0}{4\pi r^2} \propto \frac{1}{r^2}$$

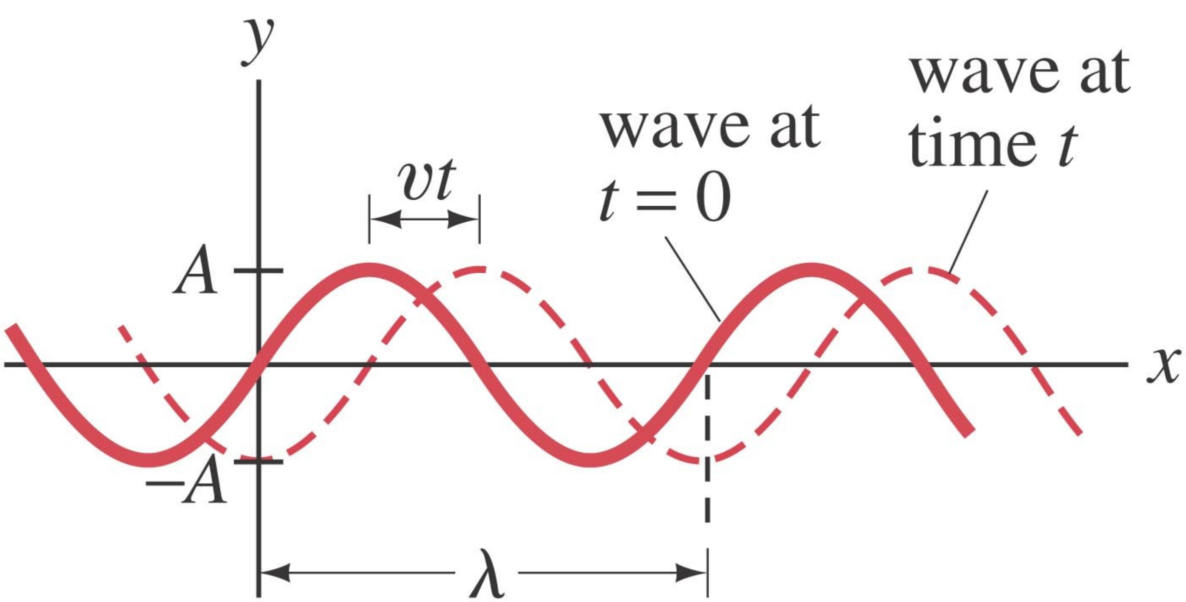
Example: Earthquake intensity.

The intensity of an earthquake P wave traveling through the Earth and detected 100 km from the source is 1.0 x 10^6 W/m^2. What is the intensity of that wave if detected 400 km from the source?

Mathematical Representation of a Traveling Wave

Suppose the shape of a wave is given by:

$$D(x) = A \sin \frac{2\pi}{\lambda} x.$$



Spacial dependence

$$k = \frac{2\pi}{\lambda}.$$

$$D(x) = A\,sin(kx)$$

Time dependence

$$D(t) = A\,sin(\omega t + \phi)$$

$$\omega = \frac{2\pi}{T}$$

Mathematical Representation of a Traveling Wave

After a time t, the wave crest has traveled a distance vt, so we write:

$$D(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right].$$

Or: $$D(x,t) = A\sin(kx - \omega t),$$

with $\omega = 2\pi f$, $k = \frac{2\pi}{\lambda}$. (wavenumber)

$$\omega = kv \quad \text{or} \quad v = \frac{\omega}{k}$$

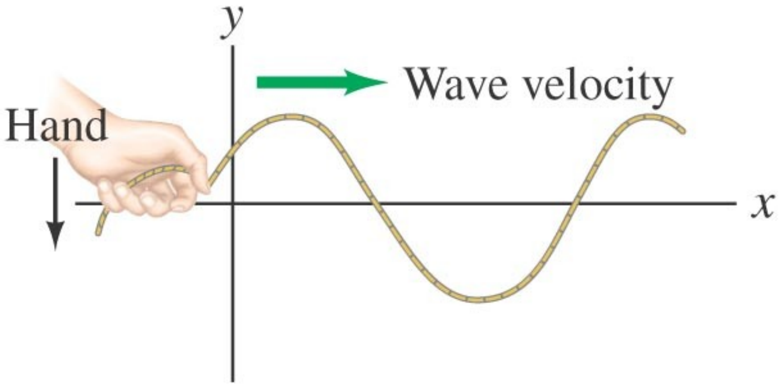
In general, $$D(x,t) = A\,sin(kx - \omega t + \phi)$$

or $$D(x,t) = A\,cos(kx - \omega t + \phi)$$

Example: A traveling wave.

The left-hand end of a long horizontal stretched cord oscillates transversely in SHM with frequency f = 250 Hz and amplitude 2.6 cm. The cord is under a tension of 140 N and has a linear density μ = 0.12 kg/m. At t = 0, the end of the cord has an upward displacement of 1.6 cm and is falling. Determine (a) the wavelength of waves produced and (b) the equation for the traveling wave.

(a)

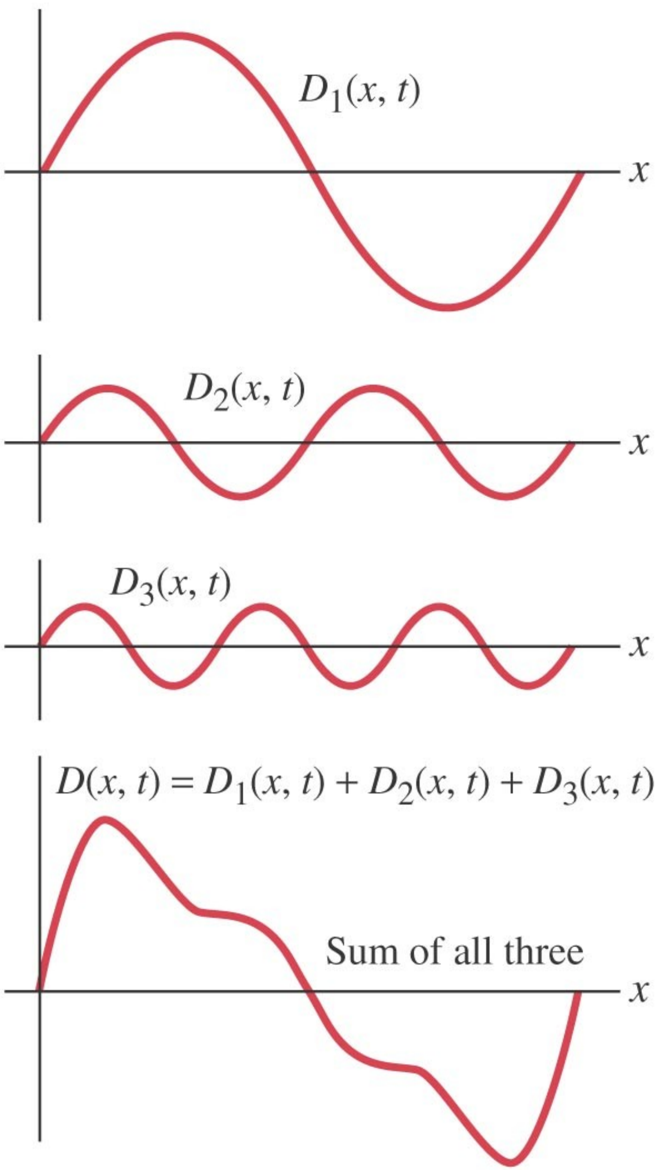


(b)

The Principle of Superposition

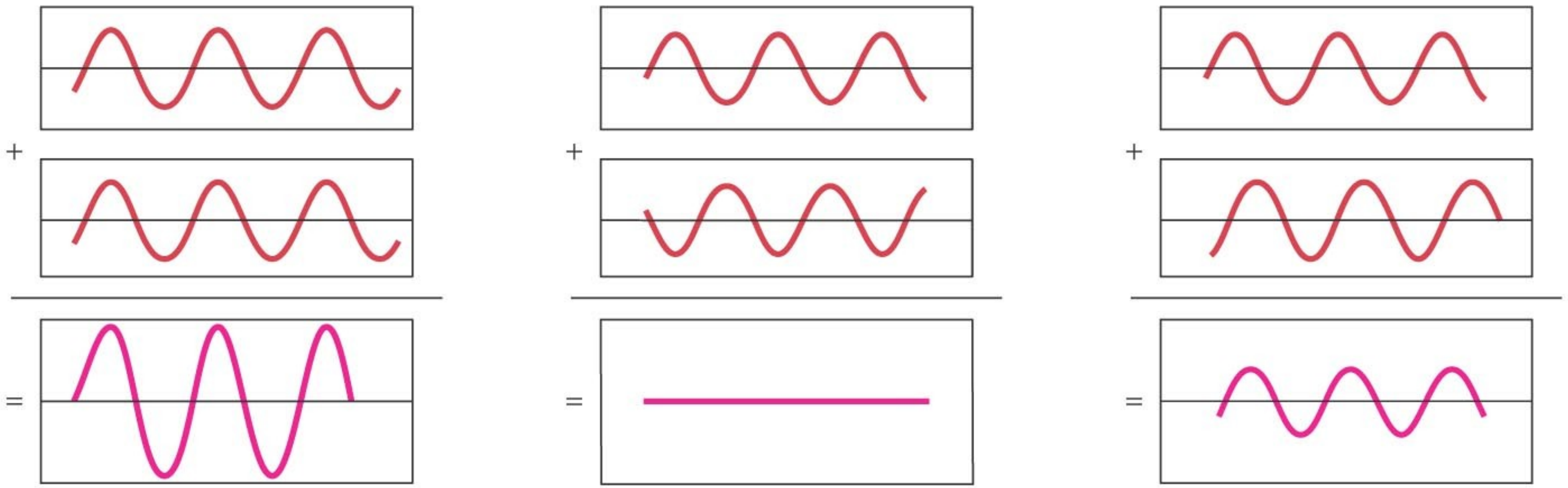
Superposition: The displacement at any point is the vector sum of the displacements of all waves passing through that point at that instant.

Fourier's theorem: Any complex periodic wave can be written as the sum of sinusoidal waves of different amplitudes, frequencies, and phases.

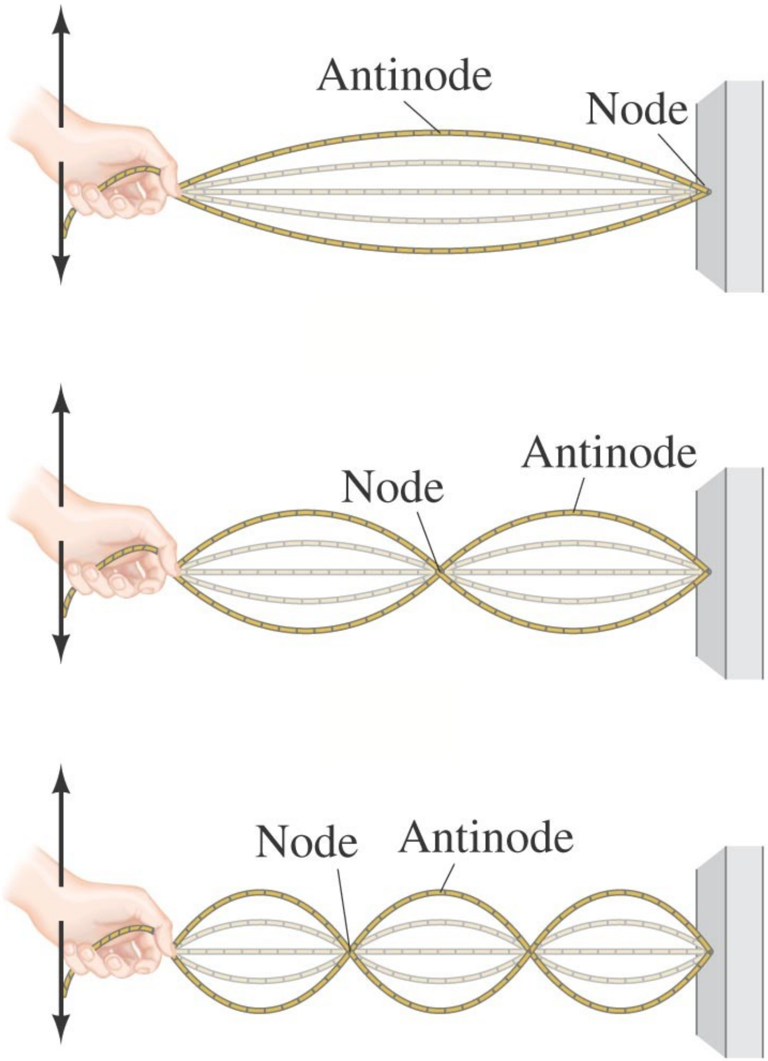


Interference

These graphs show the sum of two waves. In (a) they add constructively; in (b) they add destructively; and in (c) they add partially destructively.



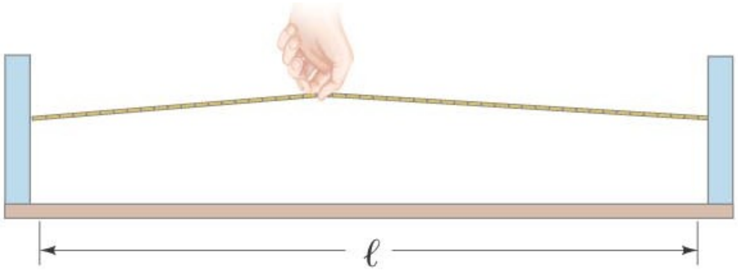
Standing Waves; Resonance

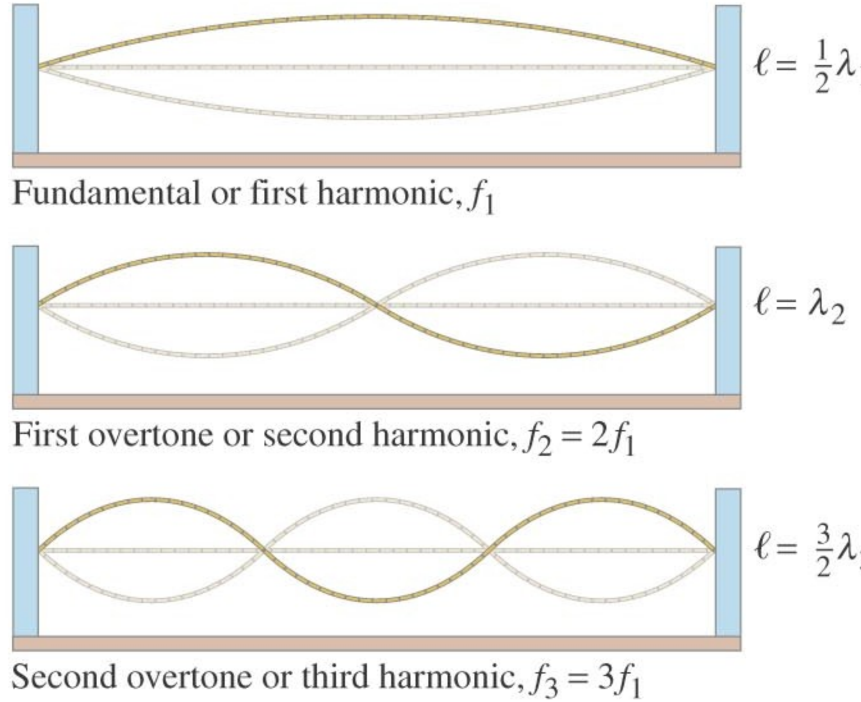


Standing waves occur when both ends of a string are **fixed**. In that case, only waves which are **motionless** at the ends of the string can persist. There are **nodes**, where the amplitude is always zero, and **antinodes**, where the amplitude reaches the maximum value.

Standing Waves; Resonance

$$l = \frac{\lambda}{2} n$$





The **frequencies** of the standing waves on a particular string are called **resonant** frequencies.

They are also referred to as the **fundamental** and **harmonics**.

The **wavelengths** and **frequencies** of standing waves are:

$$\lambda_n = \frac{2\ell}{n}, \qquad n = 1, 2, 3, \cdots,$$

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2\ell} = nf_1, \qquad n = 1, 2, 3, \cdots$$

Example: Piano string.

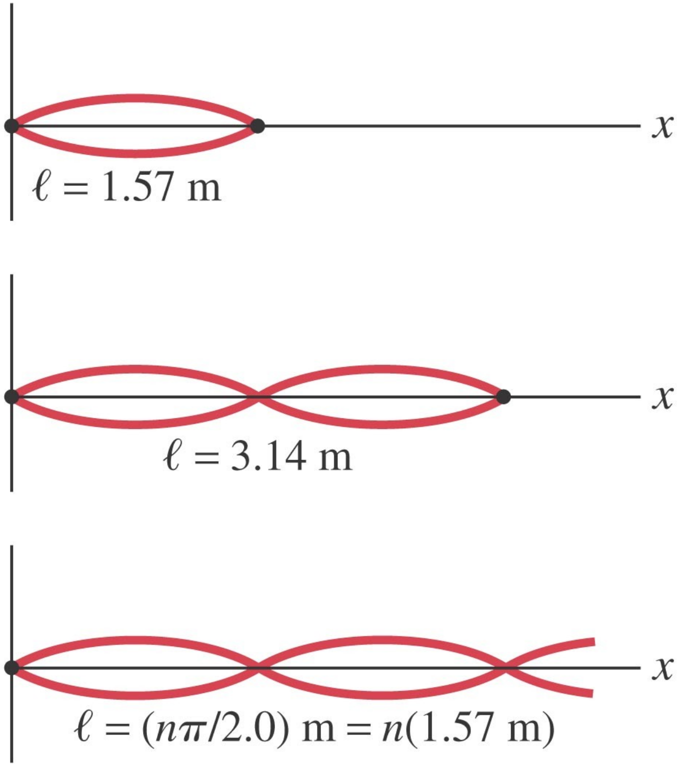
A piano string is 1.10 m long and has a mass of 9.00 g. (a) How much tension must the string be under if it is to vibrate at a fundamental frequency of 131 Hz? (b) What are the frequencies of the first four harmonics?

Example: Wave forms.

Two waves traveling in opposite directions on a string fixed at $x = 0$ are described by the functions

$D_1 = (0.20\ \text{m})\sin(2.0x - 4.0t)$ and $D_2 = (0.20\text{m})\sin(2.0x + 4.0t)$

(where x is in m, t is in s), and they produce a standing wave pattern. Determine (a) the function for the standing wave, (b) the maximum amplitude at $x = 0.45$ m, (c) where the other end is fixed ($x > 0$), (d) the maximum amplitude, and where it occurs.



I-clicker question 30-1

If the amplitude of a simple harmonic oscillator is doubled, which of the following quantities will change the most?

A) frequency
B) period
C) maximum speed
D) maximum acceleration
E) total mechanical energy

I-clicker question 30-2

Pendulum A has a length of L_A and period of T_A. Pendulum B has a length of L_B and period of T_B. $L_B = 4L_A$. Compare their periods.

A) $T_B = 4T_A$

B) $T_B = 2T_A$

C) $T_B = T_A$

D) $T_B = T_B/2$

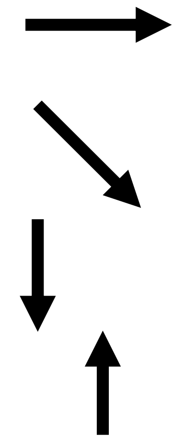
E) $T_A = 16T_B$

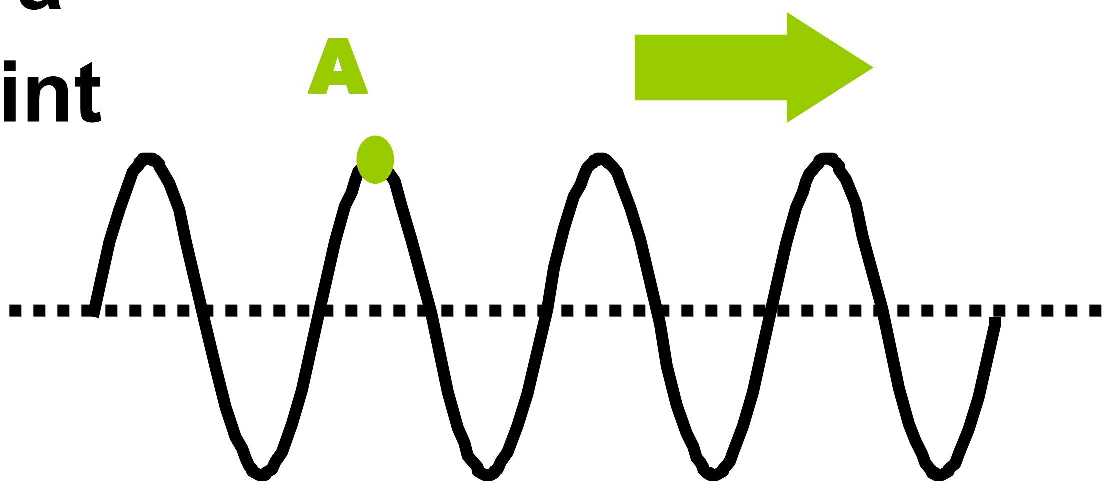
I-clicker question 30-3

Consider a wave on a string moving to the right, as shown below.

What is the direction of the velocity of a particle at the point labeled A?

A)
B)
C)
D)
E) zero





I-clicker question 30-4

Consider a wave on a string moving to the right, as shown below.

What is the direction of the velocity of a particle at the point labeled **B**?

A) →

B) ↘

C) ↓

D) ↑

E) zero