

Phys101 Lecture 6

Circular Motion

Key points:

- Centripetal acceleration
- Uniform Circular Motion - dynamics

Ref: 5-1,2,3,5,6.

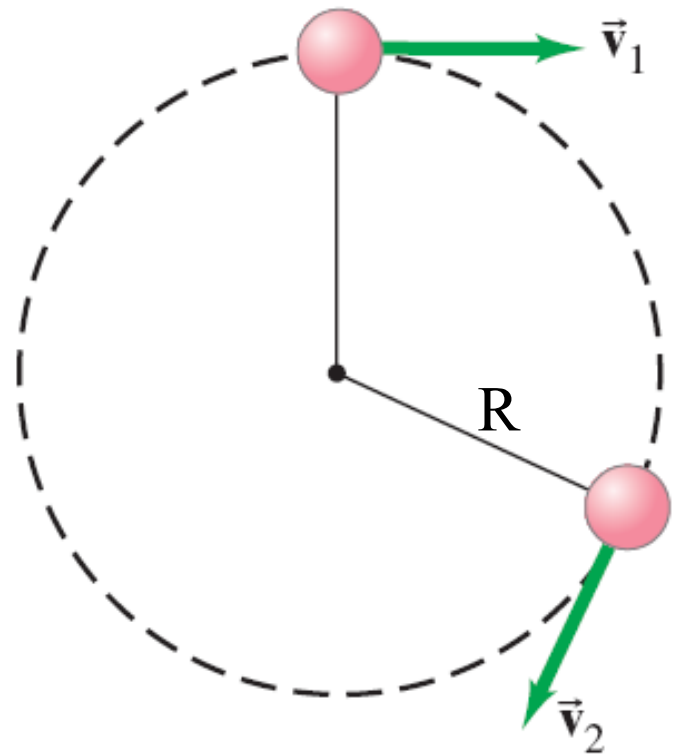
Uniform Circular Motion—Kinematics

Uniform circular motion: motion in a circle at constant speed

Instantaneous velocity is always tangent to the circle.

The magnitude of the velocity is constant:

$$v_1 = v_2 = v$$

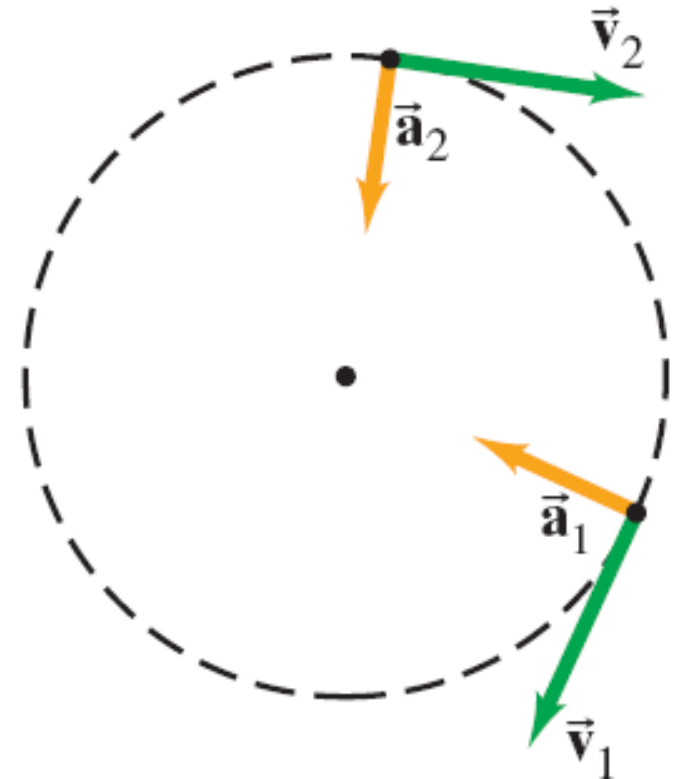


Centripetal acceleration

The acceleration, called the **centripetal acceleration**, points toward the center of the circle.

The magnitude of centripetal acceleration is:

$$a_R = \frac{v^2}{R}$$

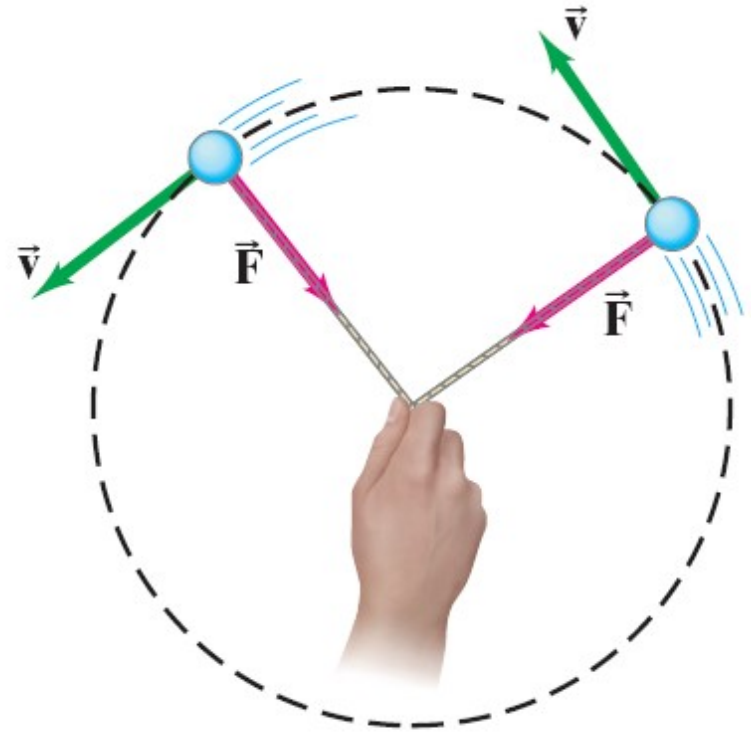


Dynamics of Uniform Circular Motion

For an object to be in uniform circular motion, Newton's 2nd law requires a net force acting on it. This net force is called centripetal force:

$$\Sigma F_R = ma_R = m \frac{v^2}{r}.$$

Physically, the centripetal force can be the tension in a string, the gravity on a satellite, the normal force of a ring, etc.



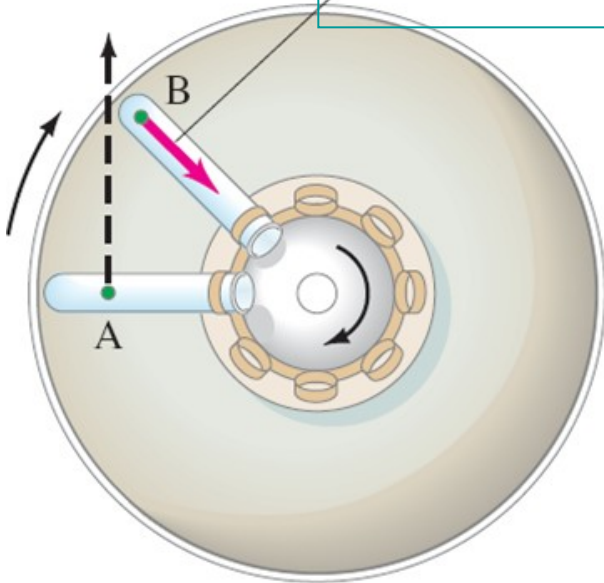
Note: Don't count the centripetal force as an additional force in the free-body-diagram! It refers to the required net force for circular motion.

Centrifuge

A centrifuge works by spinning very fast. An object in the tube requires a large centripetal force. When the liquid can't provide such a large force, the object will move (sink) to the end of the tube.

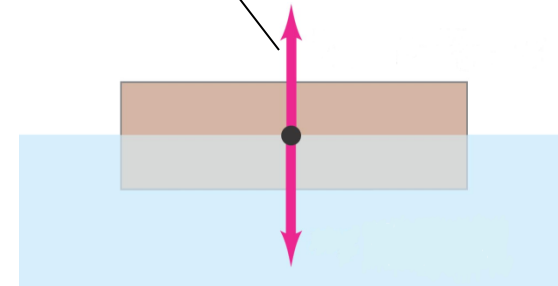
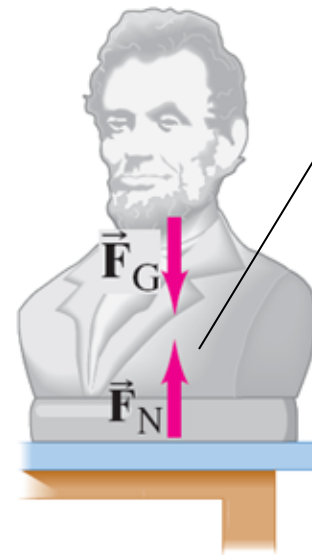
Force required for circular motion:

$$F_R = ma_R = \frac{mv^2}{R}$$



Force required for staying in position:

$$F_N = mg$$



Example: Ultracentrifuge.

The rotor of an ultracentrifuge rotates at 50,000 rpm (revolutions per minute). A particle at the top of a test tube is 6.00 cm from the rotation axis. Calculate its centripetal acceleration, in “g’s.”

$$\text{Radius : } R = 6.00\text{cm} = 0.0600\text{m},$$

$$\text{Period : } T = \frac{1\text{min}}{50000\text{rev}} = \frac{60\text{sec}}{50000\text{rev}} = 1.2 \times 10^{-3} \text{ sec},$$

$$\text{Speed : } v = \frac{2\pi R}{T} = \frac{2\pi(0.06)}{1.2 \times 10^{-3}} = 314\text{m} / \text{s},$$

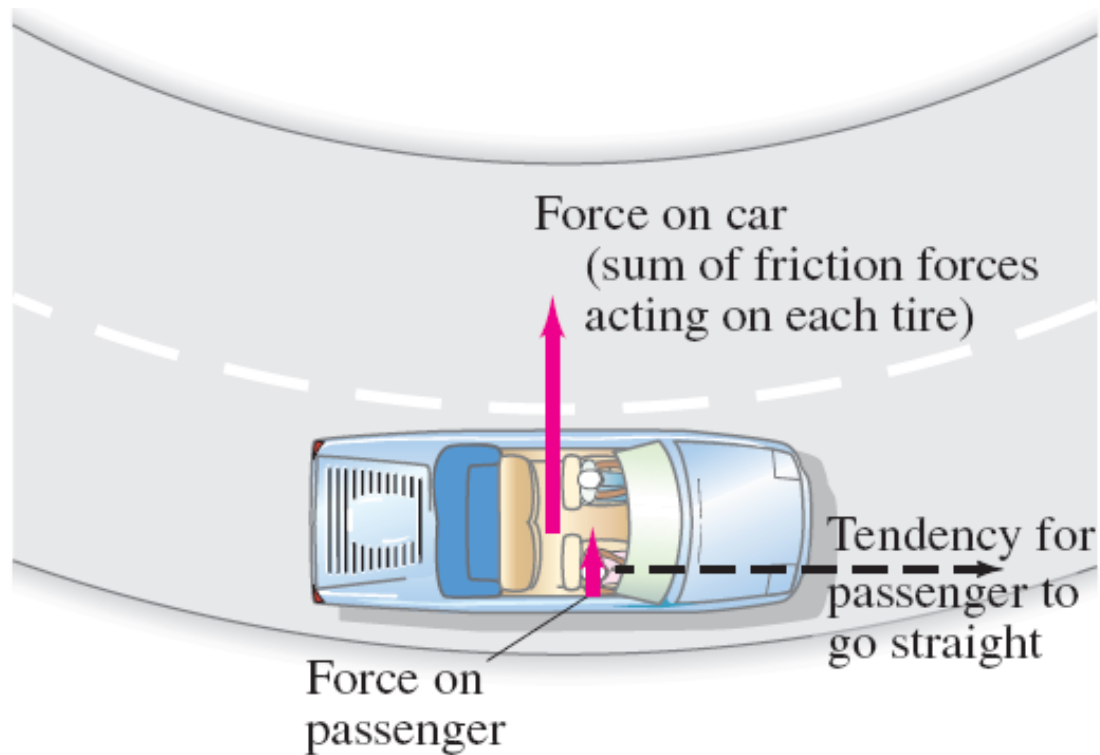
i-clicker question 6-1

An object is in a uniform circular motion. Which of the following statements must be true?

- A. The net force acting on the object is zero.
- B. The velocity of the object is constant.
- C. The speed of the object is constant.
- D. The acceleration of the object is constant.

Highway Curves: Banked and Unbanked

When a car goes around a **curve**, there must be a net force toward the center of the circle of which the curve is an arc. If the road is flat, that force is supplied by **friction**.

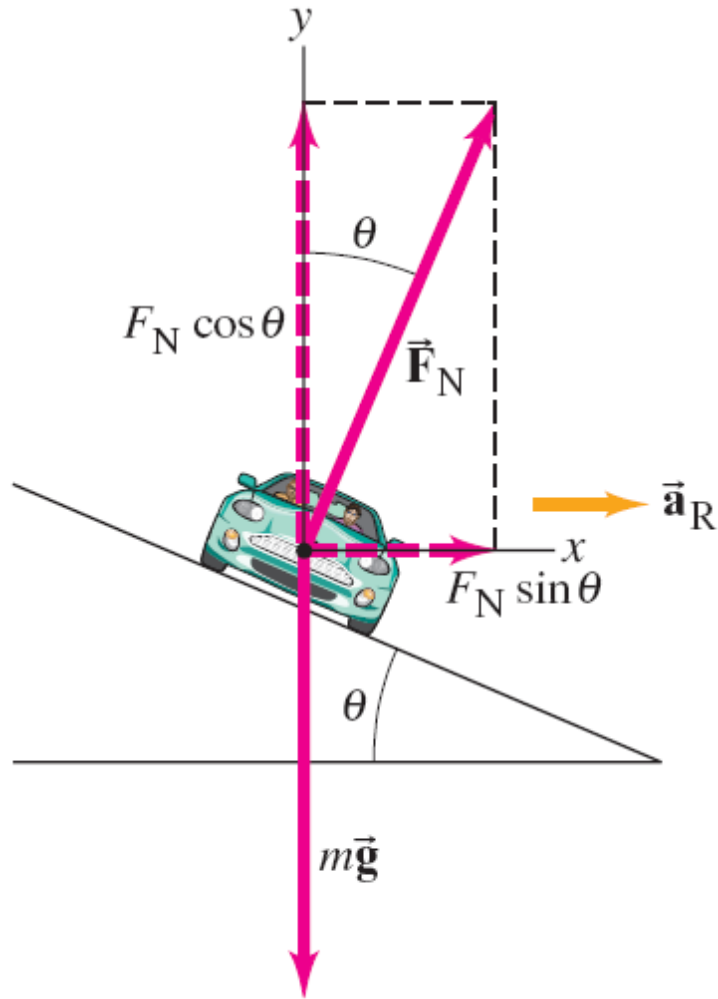


Highway Curves: Banked and Unbanked



If the frictional force is insufficient, the car will tend to move more nearly in a straight line, as the skid marks show.

Highway Curves: Banked and Unbanked



Banking the curve can help keep cars from skidding. When the curve is banked, the centripetal force can be supplied by the horizontal component of the **normal** force. In fact, for every banked curve, there is one speed at which the entire centripetal force is supplied by the horizontal component of the **normal** force, and no friction is required.

Example: Banking angle.

- (a) For a car traveling with speed v around a curve of radius r , determine a formula for the angle at which a road should be banked so that no friction is required.
- (b) What is this angle for an expressway off-ramp curve of radius 50 m at a design speed of 50 km/h?

$$(a) \quad F_N \sin \theta = m \frac{v^2}{R}$$

$$F_N \cos \theta - mg = 0$$

$$\tan \theta = \frac{v^2}{Rg}$$

$$(b) \quad R = 50\text{m}, \quad v = 50\text{km/h} = 13.89\text{m/s}$$

$$\theta = \tan^{-1} \frac{v^2}{Rg} = \tan^{-1} \frac{13.89^2}{50g} = 22^\circ$$

