Phys101 Lectures 18, 19

Statics

Key points:

- **The Conditions for static equilibrium**
- **Solving statics problems**
- **Stress and strain**

Ref: 9-1,2,3,4,5.

The Conditions for Static Equilibrium

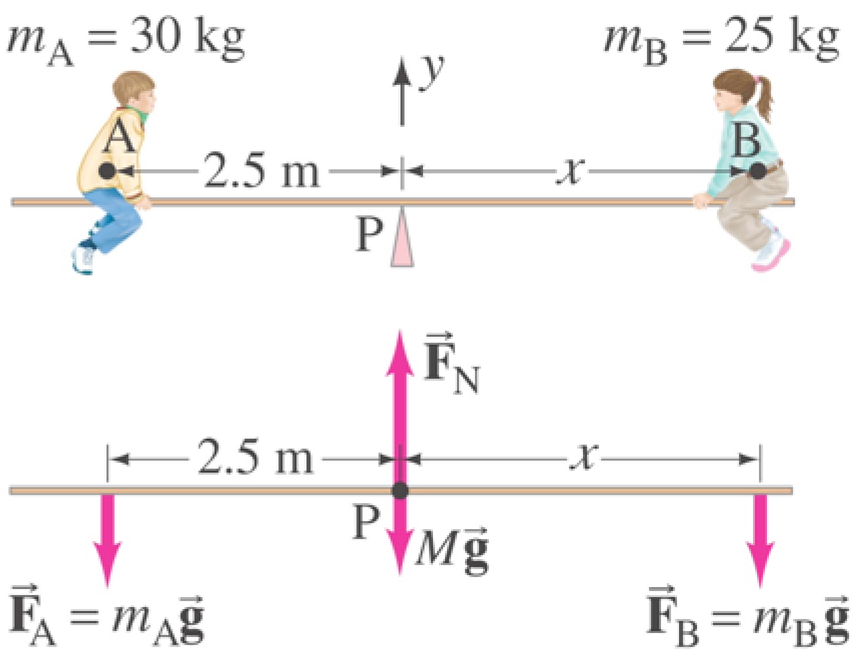
An object in static equilibrium must satisfy two conditions:

1. The net force acting on the object is zero;

$$\sum \vec{F} = 0 \quad \textit{i.e.,} \quad \sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0.$$

2. The net torque about ***any*** axis is zero.

$$\sum \tau = 0$$



Example: A board of mass $M = 2.0$ kg serves as a seesaw for two children. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, P (his center of mass is 2.5 m from the pivot). At what distance x from the pivot must child B, of mass 25 kg, place herself to balance the seesaw? Assume the board is uniform and centered over the pivot.

Note: Center of Gravity = Canter of mass

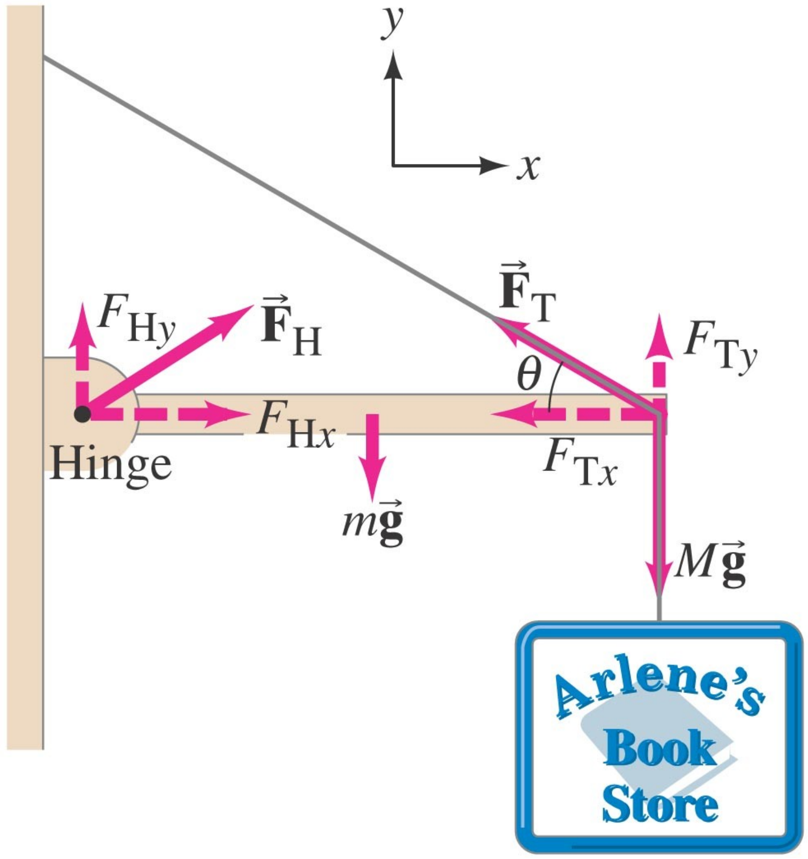
$$\sum \vec{F} = 0 \quad \Rightarrow \quad F_N - m_A g - m_B g - Mg = 0$$

$$\sum \tau = 0 \quad \Rightarrow \quad m_A g(2.5) - m_B g\, x = 0$$

$$\text{solve for } x: \quad x = \frac{m_A}{m_B}(2.5) = \frac{30}{25}(2.5) = 3.0\, m$$

Example: Hinged beam and cable.

A uniform beam, 2.20 m long with mass m = 25.0 kg, is mounted by a small hinge on a wall. The beam is held in a horizontal position by a cable that makes an angle θ = 30.0°. The beam supports a sign of mass M = 28.0 kg suspended from its end. Determine the components of the force $\vec{F}_H$ that the (smooth) hinge exerts on the beam, and the tension F_T in the supporting cable.



First, FBD of the beam. Then,

$$\sum \vec{F} = 0 \quad \Rightarrow \quad F_{Hx} - F_T \cos\theta = 0$$

$$F_{Hy} - mg - Mg + F_T \sin\theta = 0$$

$$\sum \tau = 0 \quad \Rightarrow \quad F_T l \sin\theta - Mgl - mg\frac{l}{2} = 0$$

$$\text{solve for } F_T: \quad F_T = \frac{2M + m}{2\sin\theta} g = 794\,N$$

$$F_{Hx} = F_T \cos\theta = 687\,N$$

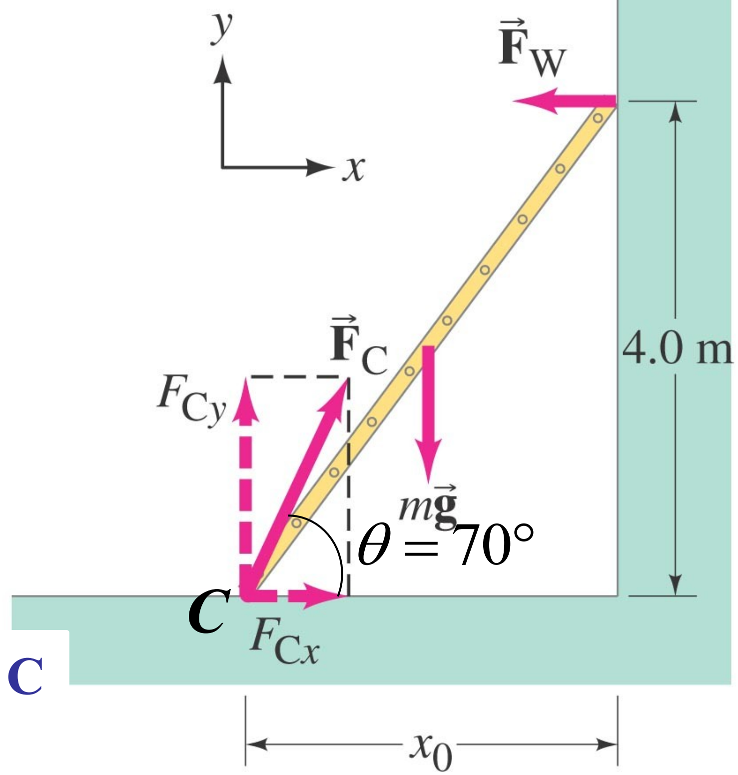
$$F_{Hy} = mg + Mg - F_T \sin\theta$$

$$= 123\,N$$

Example: A 5.0-m-long ladder leans against a smooth wall at a point 4.0 m above a cement floor. The ladder is uniform and has mass *m* = 12.0 kg. Assuming the wall is frictionless (but the floor is not), determine the forces exerted on the ladder by the floor and by the wall.

$$l = 5.0m, \quad h = 4.0m,$$

$$x_0 = \sqrt{5^2 - 4^2} = 3.0m$$



First, FBD of the ladder. Then,

$$\sum \vec{F} = 0 \quad \Rightarrow \quad F_{Cx} - F_W = 0$$

$$F_{Cy} - mg = 0$$

$$\sum \tau = 0 \quad \Rightarrow \quad F_W h - mg\frac{x_0}{2} = 0$$

← **About C**

$$F_W = \frac{mg\,x_0}{2h} = \frac{12\times 9.8\times 3.0}{2\times 4.0} = 44\,N$$

$$F_{Cx} = F_W = 44\,N$$

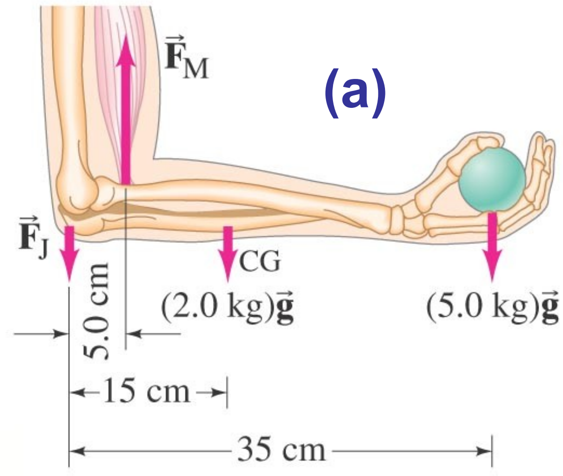
$$F_{Cy} = mg = 12\times 9.8 = 118\,N$$

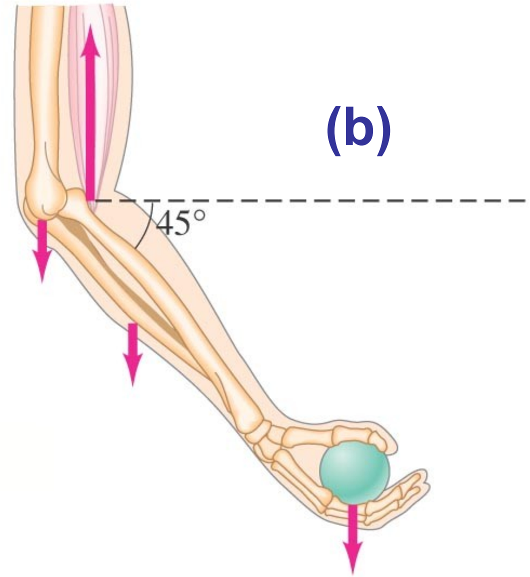
$$F_C = \sqrt{F_{Cx}^2 + F_{Cy}^2} = 130N$$

$$\theta = tan^{-1}\frac{F_{Cy}}{F_{Cx}} = tan^{-1}\frac{118}{44} = 70°$$

Example: Force exerted by biceps muscle.

How much force must the biceps muscle exert when a 5.0-kg ball is held in the hand (a) with the arm horizontal, and (b) when the arm is at a 45° angle? The biceps muscle is connected to the forearm by a tendon attached 5.0 cm from the elbow joint. Assume that the mass of forearm and hand together is 2.0 kg and their CG is as shown.





(a)

$$\sum \tau = 0$$

$$F_M(0.05) - (2.0)g(0.15) - (5.0)g(0.35) = 0$$

$$F_M = \frac{0.30 + 1.75}{0.05} g = 41g = 400\,N$$

(b)

$$\sum \tau = 0$$

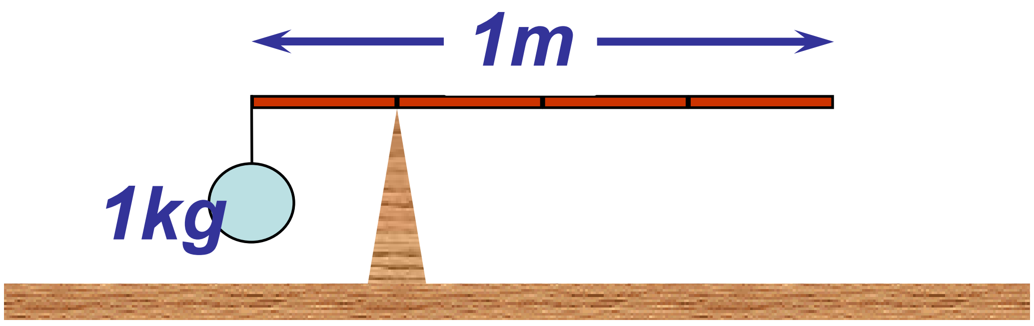
$$F_M(0.05)\sin 45° - (2.0)g(0.15)\sin 45° - (5.0)g(0.35)\sin 45° = 0$$

$$F_M = 400\,N \quad \text{same as (a).}$$

I-clicker question 18-1

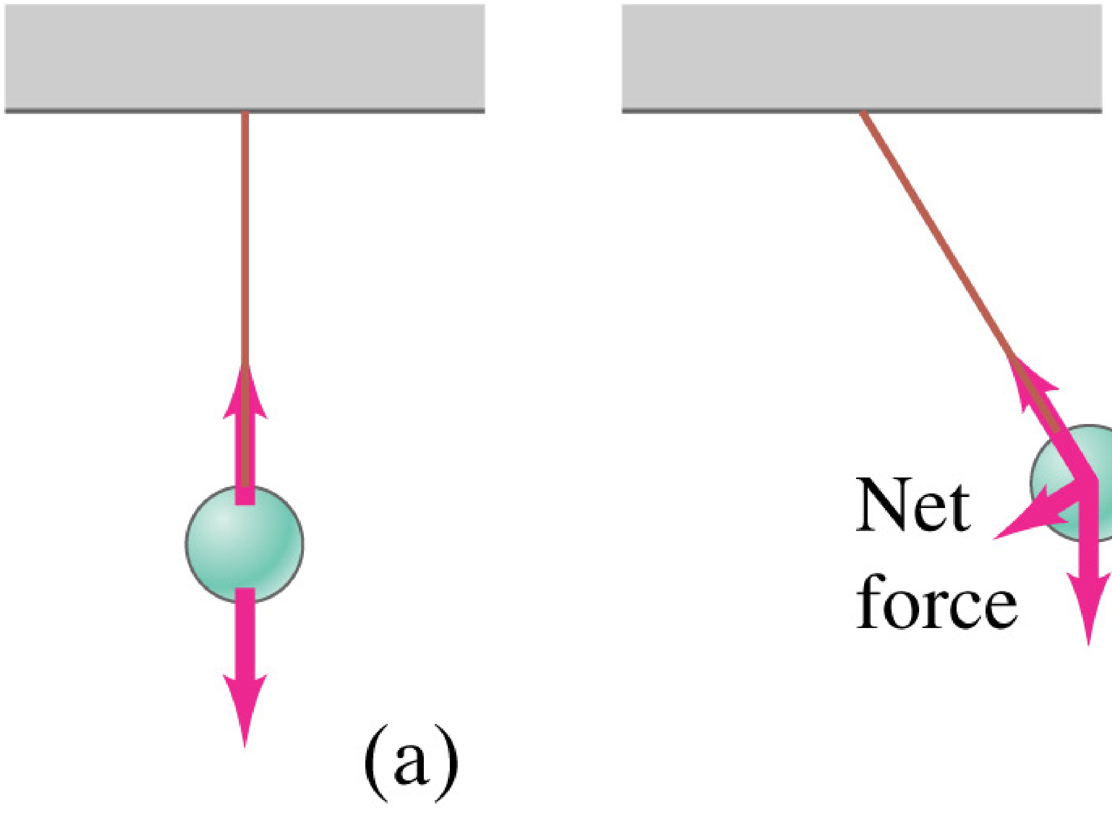
A 1-kg ball is hung at the end of a rod 1-m long. If the system balances at a point on the rod 0.25 m from the end holding the mass, what is the mass of the rod?

1) 0.25 kg
2) 0.50 kg
3) 1 kg
4) 2 kg
5) 4 kg



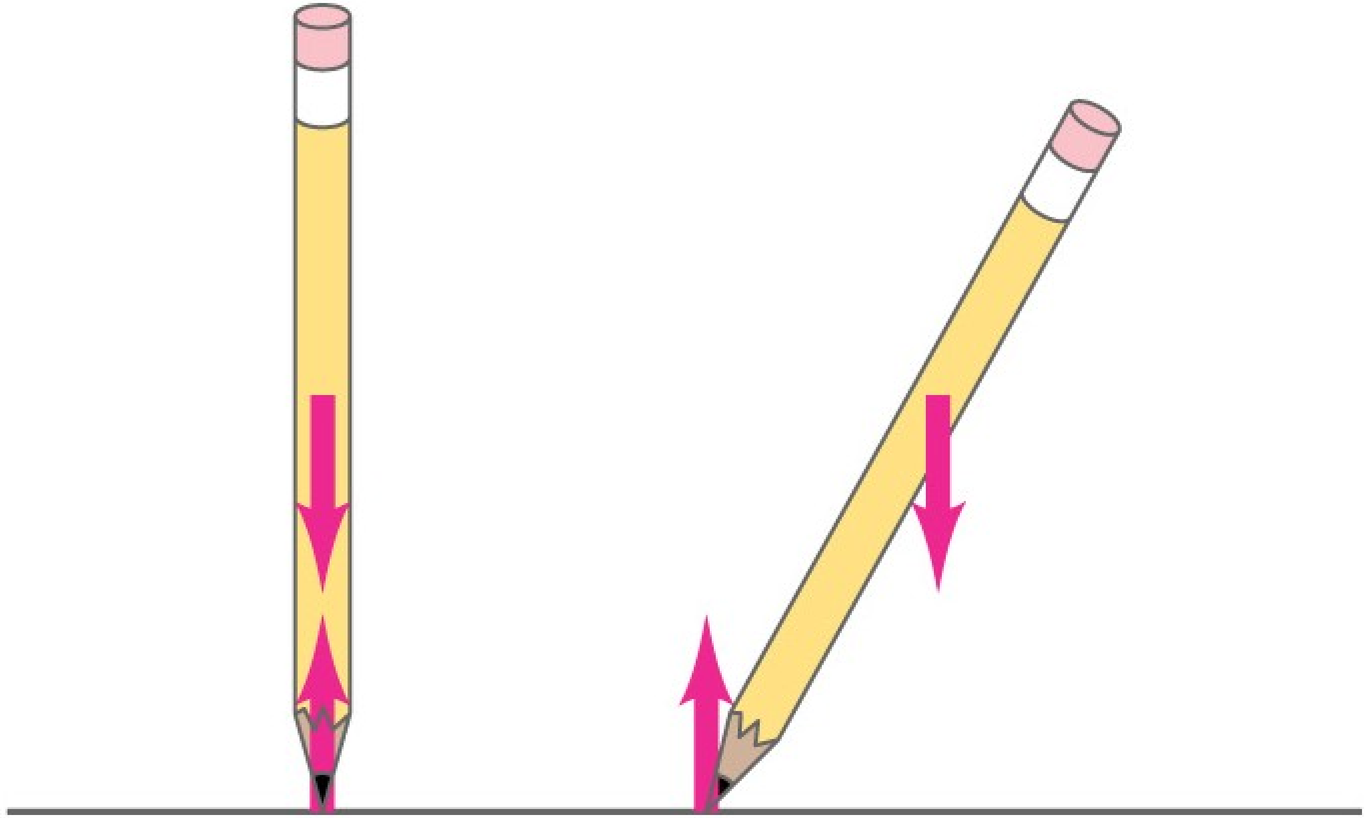
Stability and Balance

If the forces on an object are such that they tend to **return** it to its equilibrium position, it is said to be in **stable** equilibrium.



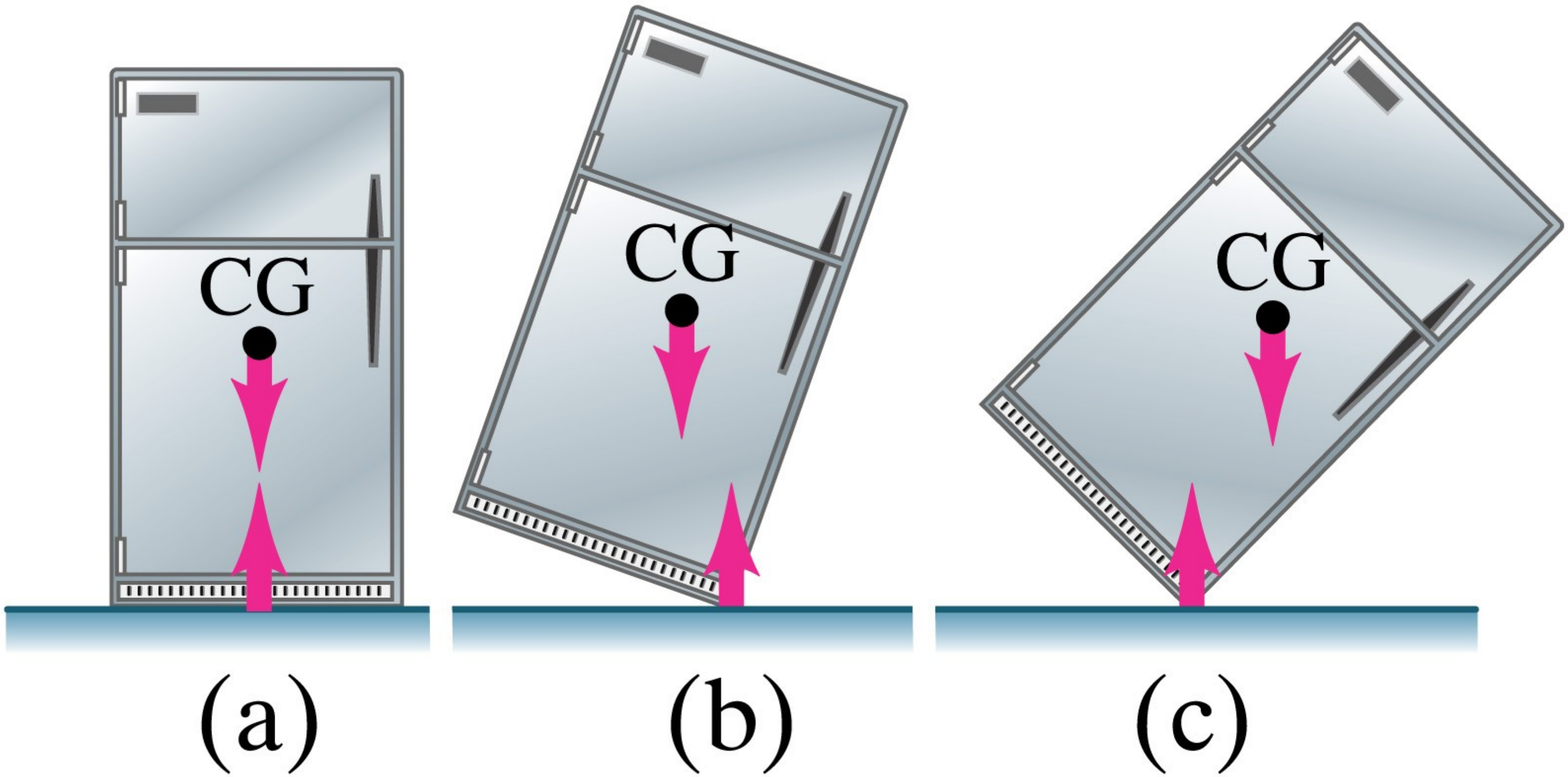
Stability and Balance

If, however, the forces tend to move it **away** from its equilibrium point, it is said to be in **unstable** equilibrium.



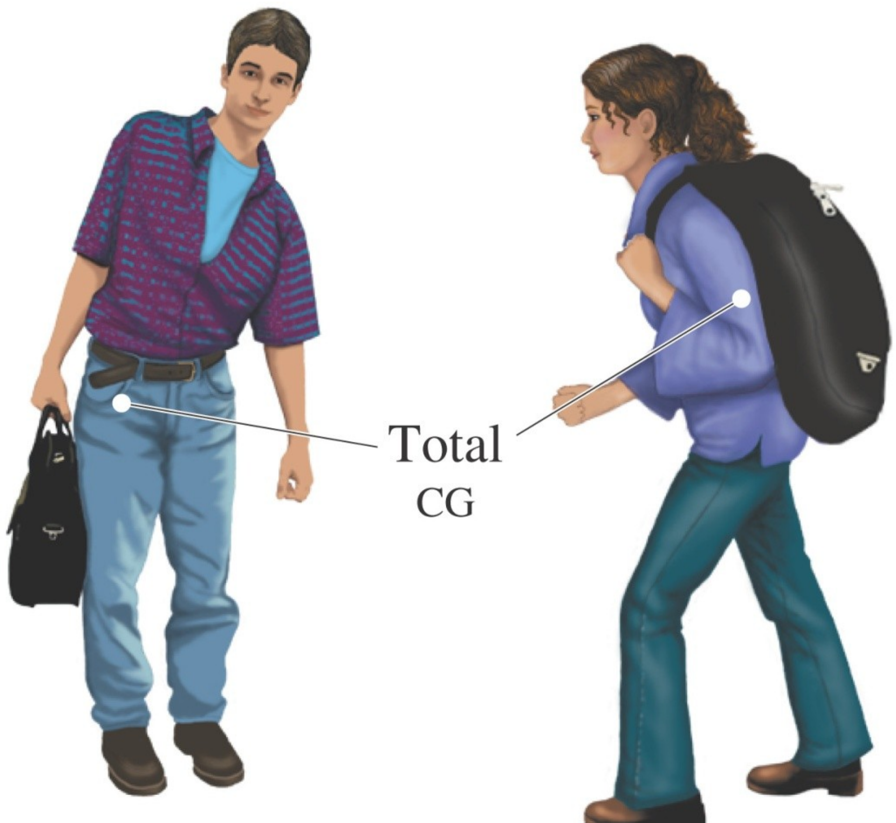
9-4 Stability and Balance

An object in **stable** equilibrium may become **unstable** if it is **tipped** so that its center of gravity is **outside** the pivot point. Of course, it will be stable again once it lands!

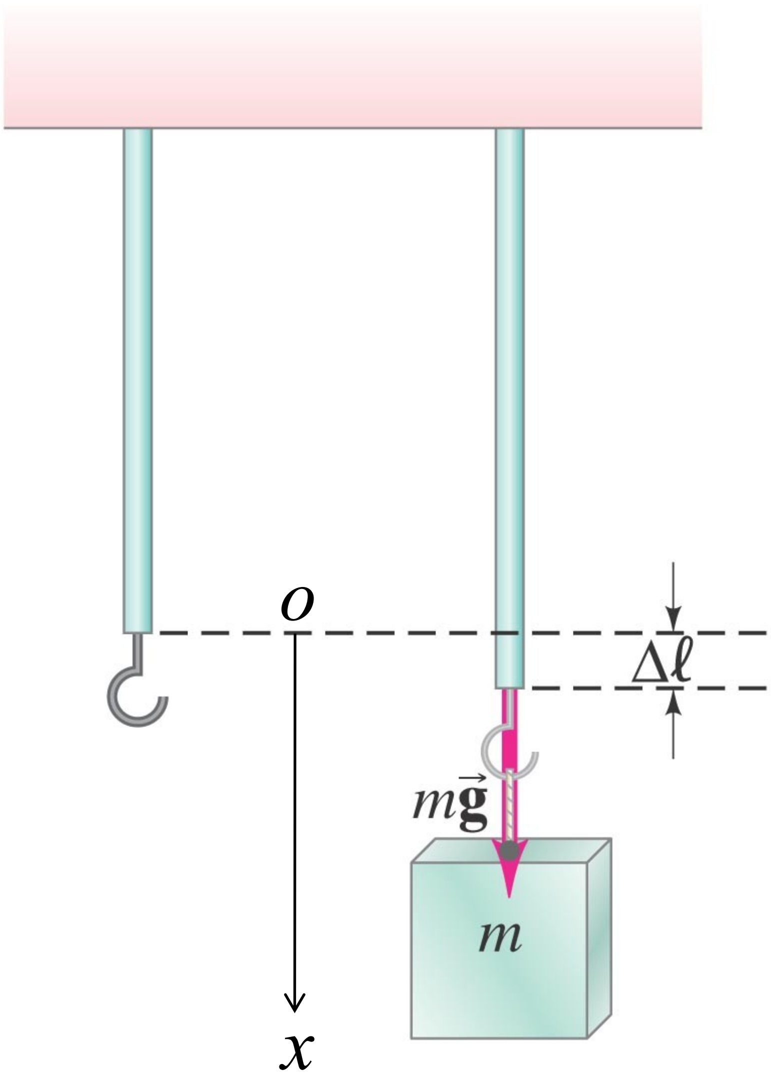


Stability and Balance

People carrying heavy loads automatically **adjust** their posture so their **center of mass** is over their **feet**. This can lead to injury if the contortion is too great.



Elasticity; Stress and Strain



Hooke's law: **the change in length is proportional to the applied force.**

$$F = k\,\Delta\ell$$

(As far as the magnitude is concerned).

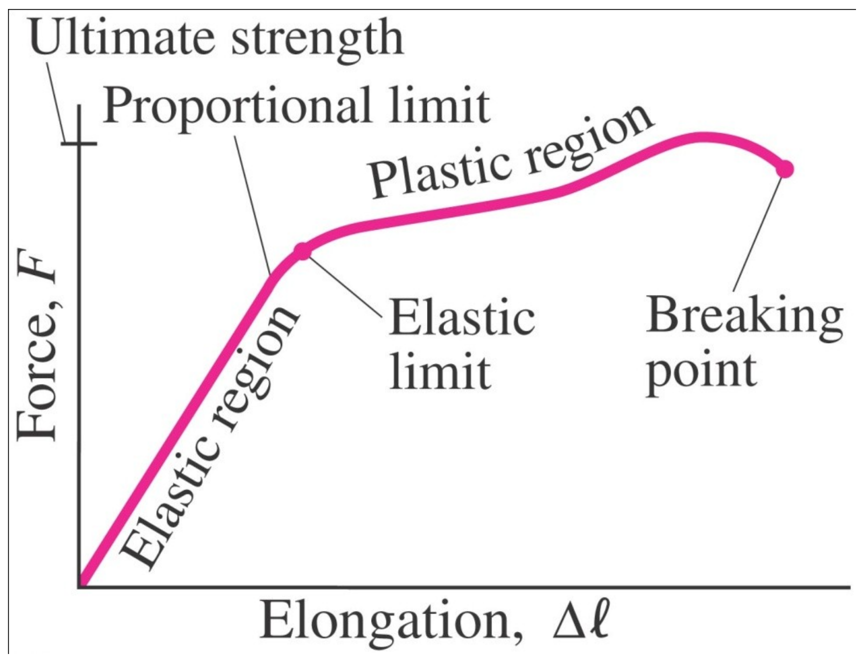
If we choose x=0 as the equilibrium position,

$$F_x = -k\,\Delta x = -k\,x$$

k **– depends on the size, shape and material property. It represents the stiffness of the rod.**

Elasticity; Stress and Strain

This proportionality holds until the force reaches the proportional limit. Beyond that, the object will still return to its original shape up to the elastic limit. Beyond the elastic limit, the material is permanently deformed, and it breaks at the breaking point.



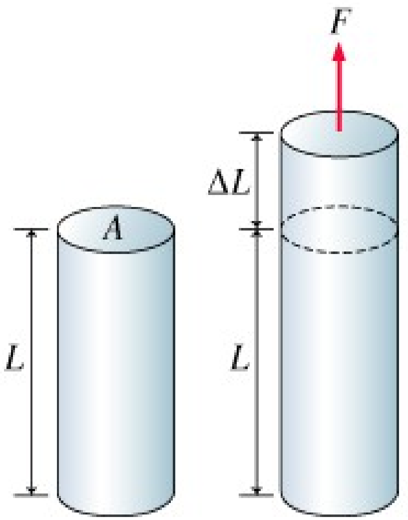
The **change** in length of a stretched object depends not only on the applied **force**, but also on its **length**, cross-sectional **area** and the **material** from which it is made.

$$\Delta \ell = \frac{1}{E}\frac{F}{A}\ell_0.$$

The material factor, E, is called the elastic modulus or **Young's modulus**, and it has been measured for many materials.

The force per unit area is called stress:

$$Stress = \frac{F}{A}$$



The ratio of the change in length to the original length is called the strain:

$$Strain = \frac{\Delta l}{l}$$

Therefore, the elastic modulus is equal to the stress divided by the strain:

$$E = \frac{Stress}{Strain}$$

Example: Tension in piano wire.

A 1.60-m-long steel piano wire has a diameter of 0.20 cm. How great is the tension in the wire if it stretches 0.25 cm when tightened?

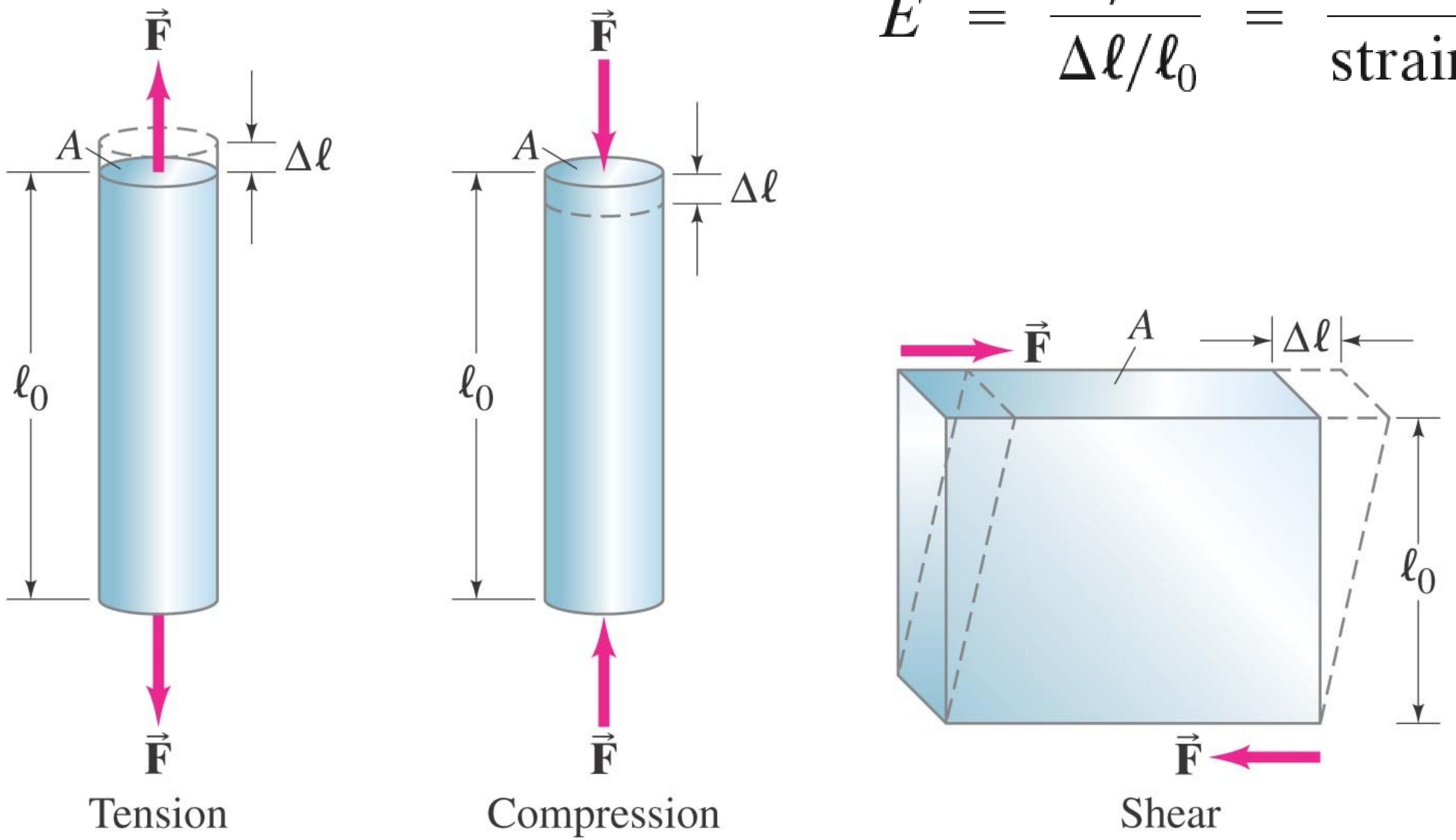
Look up the textbook: The Young's modulus of steel is $2.0\text{x}10^{11}$ N/m^2.

$$E = \frac{Stress}{Strain} = \frac{F / A}{\Delta l / l}$$

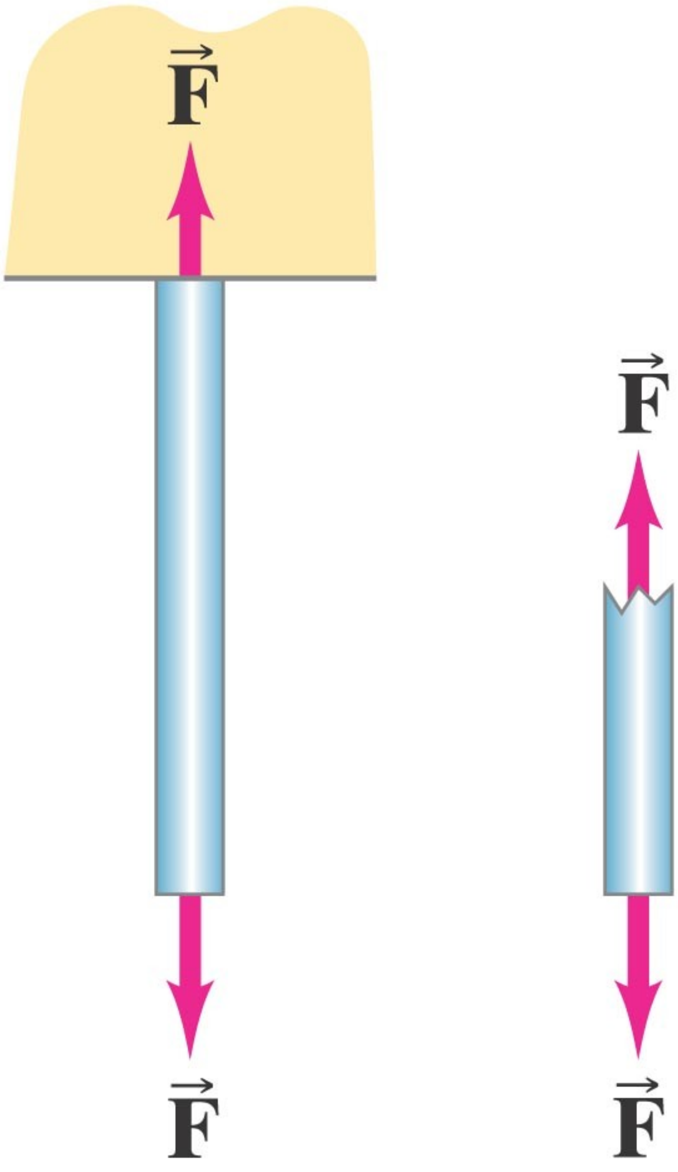
$$F = \frac{A\,E\,\Delta l}{l} = \frac{\pi\left(\frac{0.002}{2}\right)^2 \left(2.0\times10^{11}\right)\left(0.0025\right)}{1.60} = 980\,N$$

The three types of stress for rigid objects:

$$E = \frac{F/A}{\Delta\ell/\ell_0} = \frac{\text{stress}}{\text{strain}}.$$



Elasticity; Stress and Strain



In tensile stress, forces tend to stretch the object.

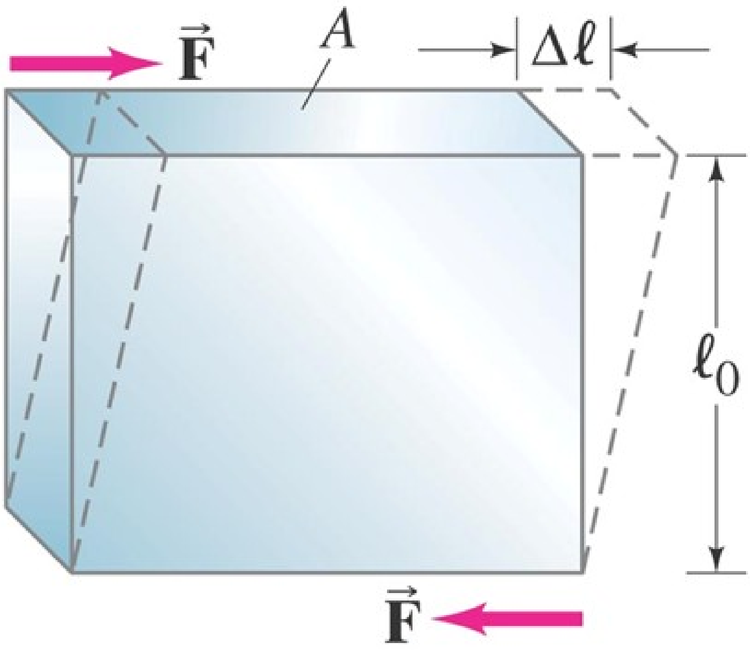
Elasticity; Stress and Strain

Compressional stress is exactly the opposite of tensional stress. These columns are under **compression**.

The shear strain, where G is the shear modulus:

$$\Delta \ell = \frac{1}{G}\frac{F}{A}\ell_0.$$

$$G = \frac{F/A}{\Delta \ell/\ell_0} = \frac{\text{stress}}{\text{strain}}.$$



Elasticity; Stress and Strain

TABLE 12–1 Elastic Moduli

Material	Young's Modulus, E (N/m^2)	Shear Modulus, G (N/m^2)	Bulk Modulus, B (N/m^2)
Solids			
Iron, cast	100×10^9	40×10^9	90×10^9
Steel	200×10^9	80×10^9	140×10^9
Brass	100×10^9	35×10^9	80×10^9
Aluminum	70×10^9	25×10^9	70×10^9
Concrete	20×10^9		
Brick	14×10^9		
Marble	50×10^9		70×10^9
Granite	45×10^9		45×10^9
Wood (pine) (parallel to grain)	10×10^9		
(perpendicular to grain)	1×10^9		
Nylon	5×10^9		
Bone (limb)	15×10^9	80×10^9	
Liquids			
Water			2.0×10^9
Alcohol (ethyl)			1.0×10^9
Mercury			2.5×10^9
Gases†			
Air, H_2, He, CO_2			1.01×10^5

†At normal atmospheric pressure; no variation in temperature during process.

Volume Change and Bulk Modulus

If an object is subjected to inward forces on all sides, its volume changes depending on its bulk modulus. This is the only deformation that applies to fluids.

$$\frac{\Delta V}{V_0} = -\frac{1}{B}\Delta P$$

or

$$B = -\frac{\Delta P}{\Delta V/V_0}.$$