

Phys101 Lectures 20-21

Fluids I

Key points:

- **Pressure and Pascal's Principle**
- **Buoyancy and Archimedes' Principle**

Ref: 10-1,2,3,4,5,6,7.

10-1 Phases of Matter

The three common phases of matter are **solid, liquid, and gas.**

A solid has a definite **shape and size.**

A liquid has a fixed **volume** but can be any shape.

A gas can be any shape and also can be easily **compressed.**

Liquids and gases both **flow**, and are called **fluids.**

10-2 Density and Specific Gravity

The density ρ of a substance is its mass per unit volume:

$$\rho = \frac{m}{V}.$$

The SI unit for density is kg/m^3 . Density is also sometimes given in g/cm^3 . How do you convert g/cm^3 to kg/m^3 ?

$$1 \text{ g} / \text{cm}^3 = 1 \frac{\text{g}}{\text{cm}^3} \frac{1 \text{ kg}}{1000 \text{ g}} \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^3 = \frac{10^6}{10^3} \text{ kg} / \text{m}^3 = 1000 \text{ kg} / \text{m}^3$$

Water at 4°C has a density of $1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$.

The specific gravity of a substance is the ratio of its density to that of water.

10-3 Pressure in Fluids

Pressure is defined as the force per unit area.

$$\text{pressure} = P = \frac{F}{A}.$$

Pressure is a scalar; the units of pressure in the SI system are pascals:

$$1 \text{ Pa} = 1 \text{ N/m}^2.$$

**i-clicker question 20-1:
unit conversion from cm² to m².**

500 cm² is equal to

- A) 50.0 m².**
- B) 5.00 m².**
- C) 0.500 m².**
- D) 0.0500 m².**
- E) 0.00500 m².**

$$500\text{cm}^2 = 500\text{cm}^2 \times \left(\frac{1\text{m}}{100\text{cm}}\right)^2 = \frac{500}{10000}\text{m}^2 = 0.05\text{m}^2$$

Example: Calculating pressure.

The two feet of a 60-kg person cover an area of 500 cm².

(a) Determine the pressure exerted by the two feet on the ground.

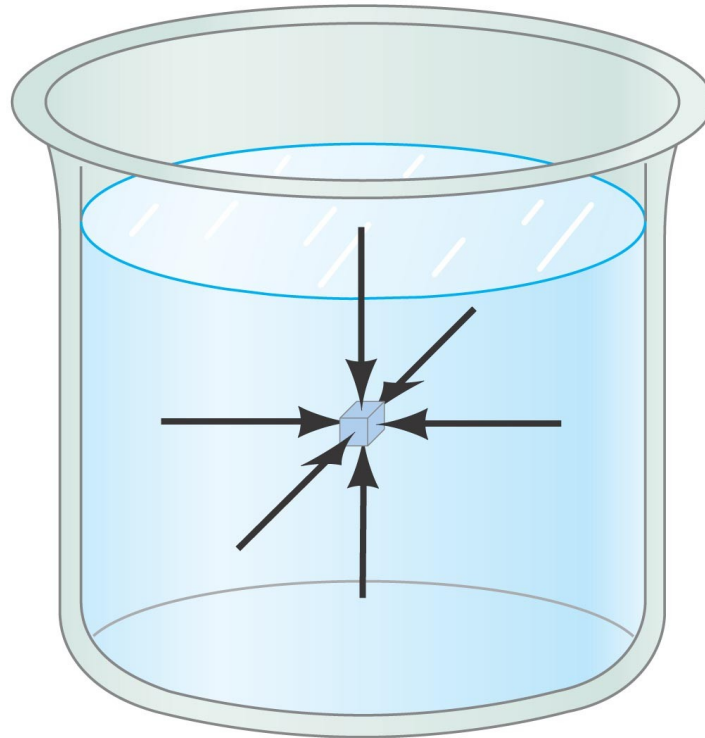
(b) If the person stands on one foot, what will the pressure be under that foot?

$$(a) \quad P = \frac{F}{A} = \frac{mg}{A} = \frac{60 \times 9.8}{0.050} = 1.2 \times 10^4 \text{ N} / \text{m}^2$$

$$(b) \quad P = \frac{F}{A} = \frac{mg}{A} = \frac{60 \times 9.8}{0.025} = 2.4 \times 10^4 \text{ N} / \text{m}^2$$

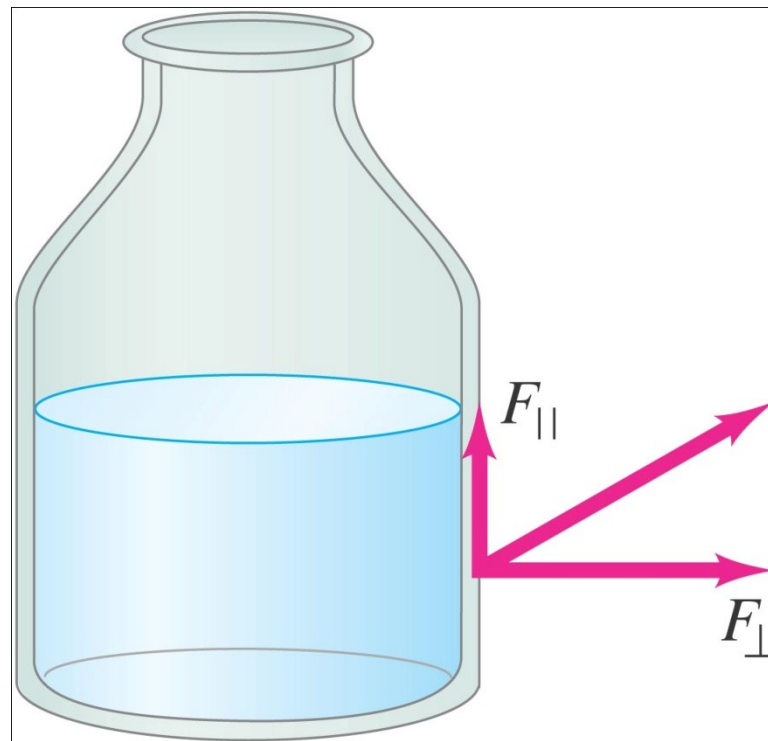
10-3 Pressure in Fluids

Pressure is the same in every direction in a static fluid at a given depth; if it were not, the fluid would flow.



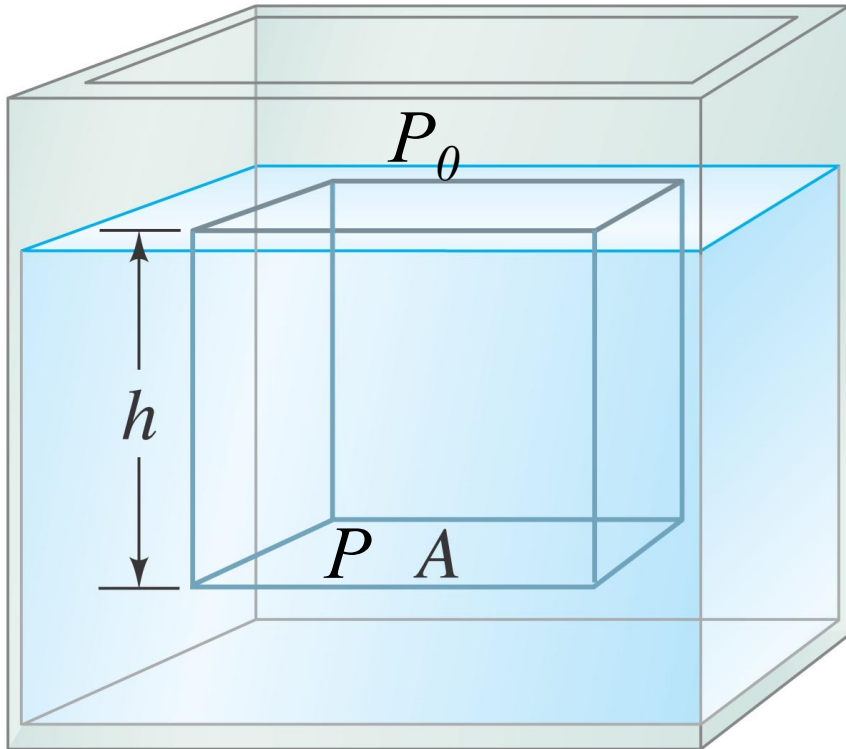
10-3 Pressure in Fluids

For a fluid at rest, there is also no component of force **parallel** to any solid surface—once again, if there were, the fluid would flow.



10-3 Pressure in Fluids

The pressure due to the weight of the liquid at a depth h :



$$P = \frac{F}{A} = \frac{\rho Ahg}{A}$$

$$P = \rho gh.$$

This is the pressure due to the liquid. It equals the pressure on A if what above the liquid surface is vacuum.

However, if there is an external pressure on the surface of the liquid such as the atmospheric pressure P_0 , then the actual pressure on A should be $P = \rho gh + P_0$.

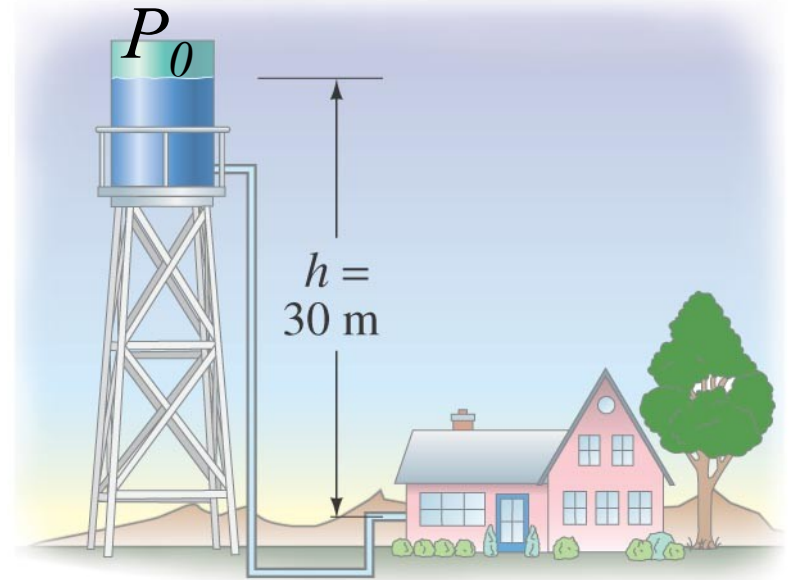
In general,

$$P - P_0 = \Delta P = \rho gh$$

Example: Pressure at a faucet.

The surface of the water in a storage tank is 30 m above a water faucet in the kitchen of a house. Calculate the difference in water pressure between the faucet and the surface of the water in the tank.

$$\begin{aligned}\Delta P &= P - P_0 = \rho gh \\ &= (1.0 \times 10^3 \text{ kg} / \text{m}^3)(9.8 \text{ m} / \text{s}^2)(30 \text{ m}) \\ &= 2.9 \times 10^5 \text{ N} / \text{m}^2 \\ &= 2.9 \times 10^5 \text{ Pa}\end{aligned}$$



10-4 Atmospheric Pressure and Gauge Pressure

At sea level the atmospheric pressure is about $1.013 \times 10^5 \text{ N/m}^2$; this is called 1 atmosphere (atm).

Another unit of pressure is the bar:

$$1 \text{ bar} = 1.00 \times 10^5 \text{ N/m}^2.$$

Standard atmospheric pressure is just over 1 bar.

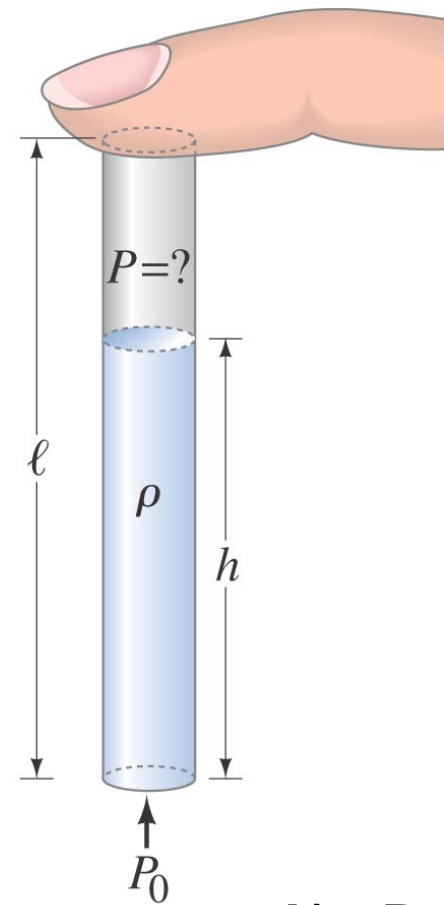
This is 10 times as large as the pressure we apply on our feet ! However, it does not crush us, as our body maintains an internal pressure that balances it.

demo

I-clicker question 24-2

Conceptual Example: Finger holds water in a straw.

You insert a straw of length ℓ into a tall glass of water. You place your finger over the top of the straw, capturing some air above the water but preventing any additional air from getting in or out, and then you lift the straw from the water. You find that the straw retains most of the water. Does the air in the space between your finger and the top of the water have a pressure P that is greater than, equal to, or less than the atmospheric pressure P_0 outside the straw?



demo

- A) $P > P_0$
- B) $P = P_0$
- C) $P < P_0$

If $P=0$, how much water can be held by the atmospheric pressure?

$$P_0 A = mg = \rho g A h, \quad h = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5 \text{ N/m}^2}{(1.0 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 10 \text{ m}$$

10-4 Atmospheric Pressure and Gauge Pressure

Most pressure gauges measure the pressure above the atmospheric pressure—this is called the gauge pressure.

The absolute pressure is the sum of the atmospheric pressure and the gauge pressure.

$$P = P_0 + P_G .$$

i-clicker question 20-2

“The pressure in a flat tire is zero”. Here “the pressure” refers to:

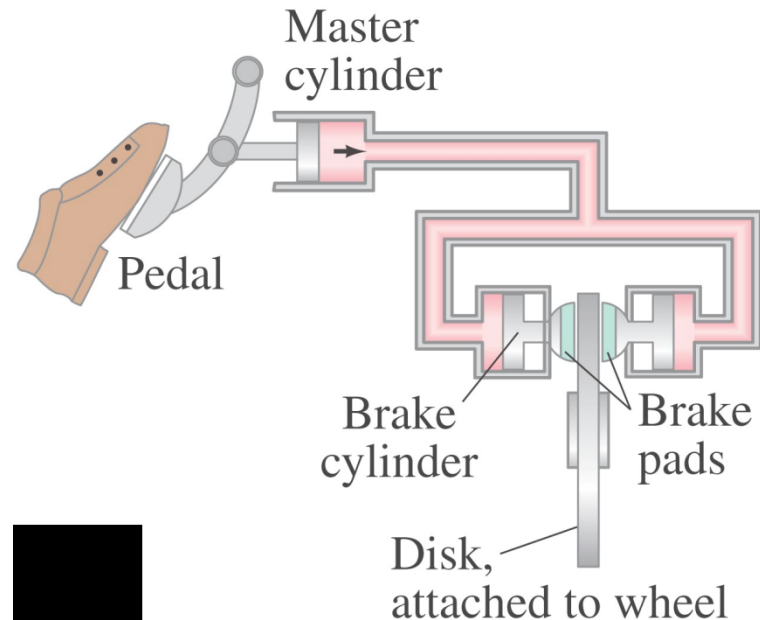
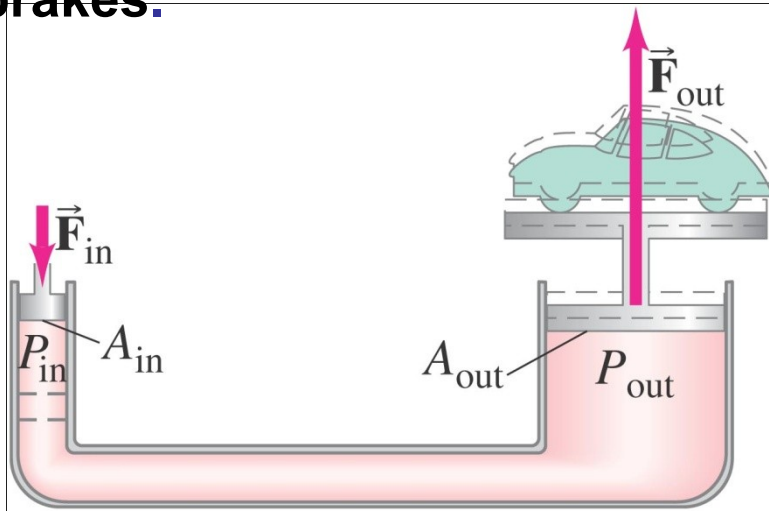
- A. The absolute pressure.**
- B. The gauge pressure.**

Note: The normal pressure in a tire is typically 30 psi (pounds per square inch). 1 psi = 6895 pa. The atmospheric pressure is about 15 psi.

10-5 Pascal's Principle

If an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.

This principle is used, for example, in hydraulic lifts and hydraulic brakes.

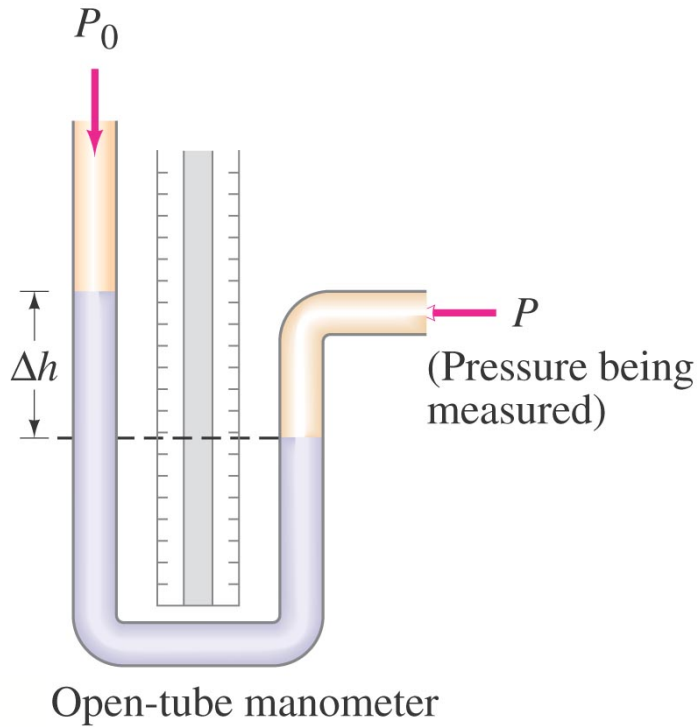


$$P_{in} = P_{out}$$

$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}, \quad F_{out} = \frac{A_{out}}{A_{in}} F_{in}$$

$$\therefore \text{ if } A_{out} \gg A_{in}, \quad F_{out} \gg F_{in}$$

10-6 Measurement of Pressure; Gauges and the Barometer



There are a number of different types of pressure gauges. This one is an open-tube manometer. The pressure in the open end is **atmospheric pressure**; the pressure being measured will cause the fluid to **rise** until the pressures on both sides at the same height are **equal**.

$$P = \rho g \Delta h + P_0$$

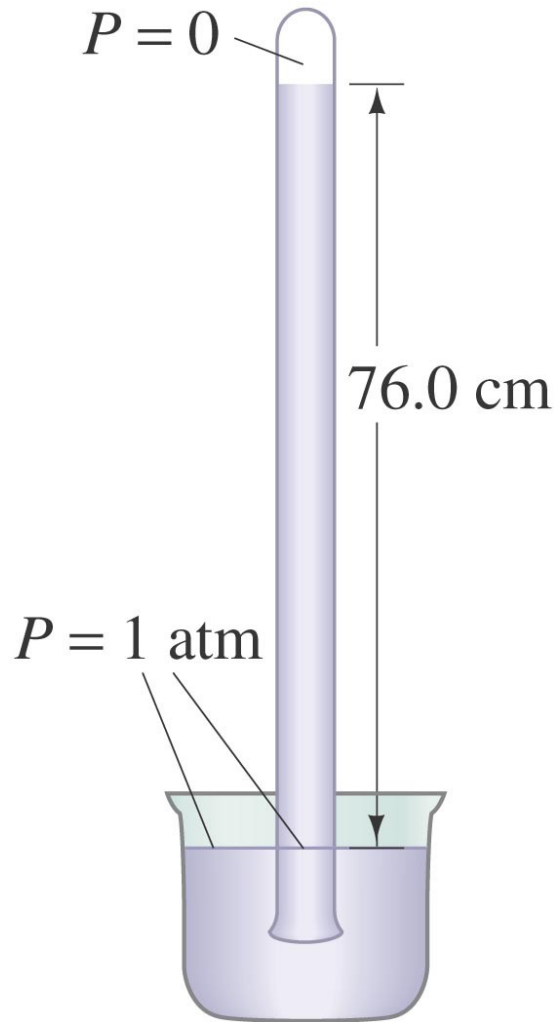
10-6 Measurement of Pressure; Gauges and the Barometer

Pressure is measured in a variety of different units. This table gives the conversion factors.

TABLE 10–2 Conversion Factors Between Different Units of Pressure

In Terms of $1 \text{ Pa} = 1 \text{ N/m}^2$	1 atm in Different Units
$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$
$= 1.013 \times 10^5 \text{ Pa} = 101.3 \text{ kPa}$	
$1 \text{ bar} = 1.000 \times 10^5 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \text{ bar}$
$1 \text{ dyne/cm}^2 = 0.1 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \times 10^6 \text{ dyne/cm}^2$
$1 \text{ lb/in.}^2 = 6.90 \times 10^3 \text{ N/m}^2$	$1 \text{ atm} = 14.7 \text{ lb/in.}^2$
$1 \text{ lb/ft}^2 = 47.9 \text{ N/m}^2$	$1 \text{ atm} = 2.12 \times 10^3 \text{ lb/ft}^2$
$1 \text{ cm-Hg} = 1.33 \times 10^3 \text{ N/m}^2$	$1 \text{ atm} = 76 \text{ cm-Hg}$
$1 \text{ mm-Hg} = 133 \text{ N/m}^2$	$1 \text{ atm} = 760 \text{ mm-Hg}$
$1 \text{ torr} = 133 \text{ N/m}^2$	$1 \text{ atm} = 760 \text{ torr}$
$1 \text{ mm-H}_2\text{O} (4^\circ\text{C}) = 9.81 \text{ N/m}^2$	$1 \text{ atm} = 1.03 \times 10^4 \text{ mm-H}_2\text{O} (4^\circ\text{C})$

10-6 Measurement of Pressure; Gauges and the Barometer



This is a **mercury** barometer, developed by Torricelli to measure **atmospheric pressure**. The **height of the column of mercury** is such that the pressure in the tube at the surface level is **1 atm**.

Therefore, pressure is often quoted in **millimeters (or inches) of mercury**.

$$\begin{aligned} 1 \text{ atm} &= \rho gh \\ &= (13.6 \times 10^3 \text{ kg} / \text{m}^3) (9.8 \text{ m} / \text{s}^2) (0.760 \text{ m}) \\ &= 1.013 \times 10^5 \text{ Pa} \\ &= 760 (\text{mmHg}) \\ &= 760 \text{ torr} \end{aligned}$$

10-6 Measurement of Pressure; Gauges and the Barometer



Any liquid can serve in a Torricelli-style barometer, but the most **dense** ones are the most convenient. This barometer uses **water**.

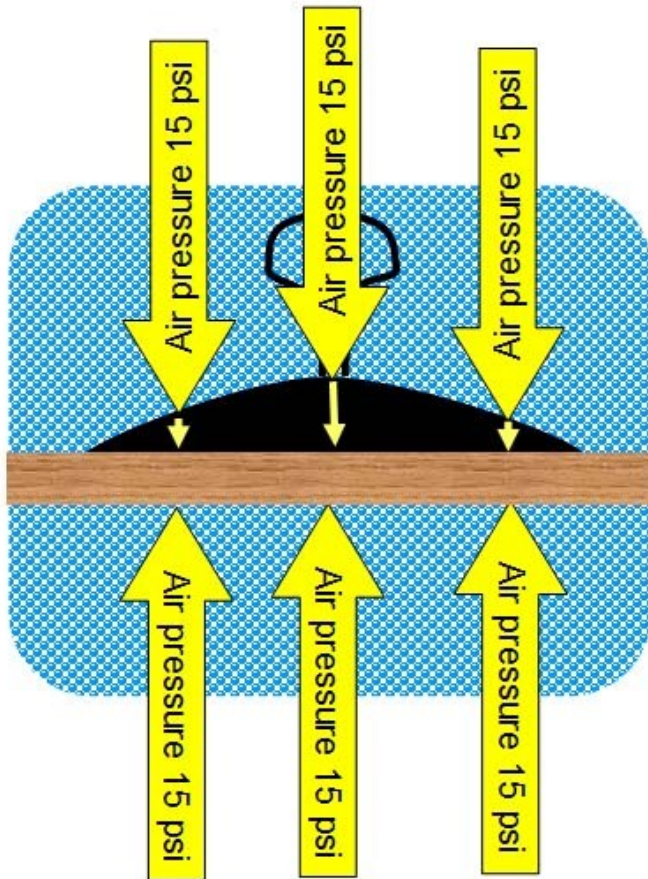
$$P_0 = \rho g h,$$

$$h = \frac{P_0}{\rho g}$$

$$\begin{aligned} &= \frac{1.013 \times 10^5 \text{ N} / \text{m}^2}{(1.0 \times 10^3 \text{ kg} / \text{m}^3)(9.8 \text{ m} / \text{s}^2)} \\ &= 10 \text{ m} \end{aligned}$$

Conceptual Example: Suction.

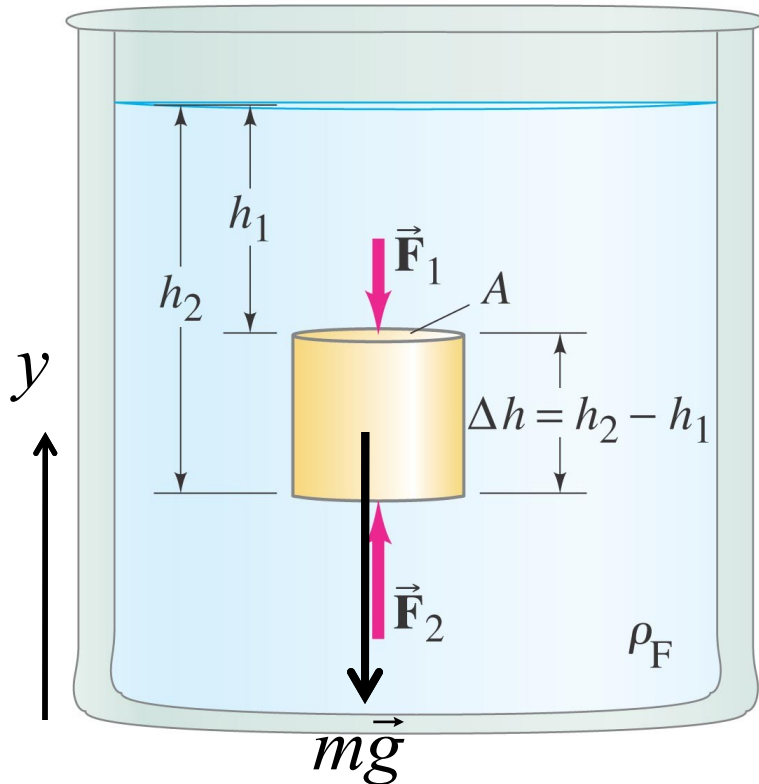
A student suggests suction-cup shoes for Space Shuttle astronauts working on the exterior of a spacecraft. Having just studied this Chapter, you gently remind him of the fallacy of this plan. What is it?



There is no atmosphere in the outer space.

10-7 Buoyancy and Archimedes' Principle

This is an object submerged in a fluid. There is an upward force on the object due to fluid pressure because the pressures at the top and bottom of it are different.



This upward force is called the buoyant force

$$\begin{aligned} F_B &= F_2 - F_1 = \rho_F g A (h_2 - h_1) \\ &= \rho_F g A \Delta h \\ &= \rho_F V g \\ &= m_F g. \end{aligned}$$

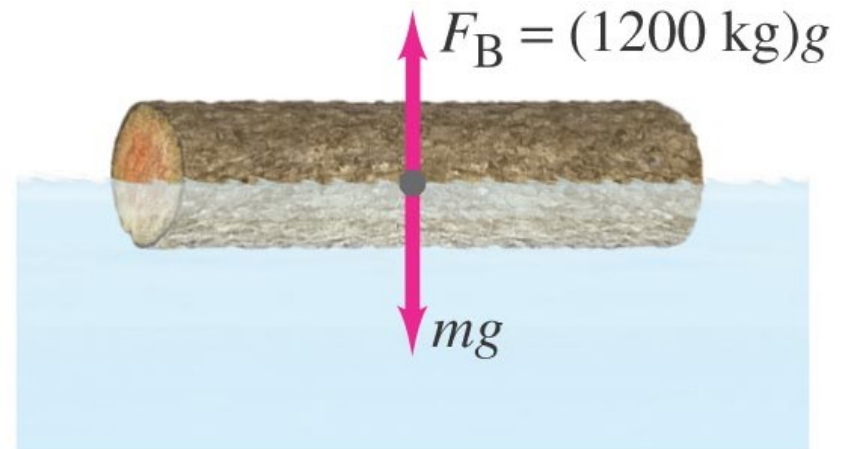
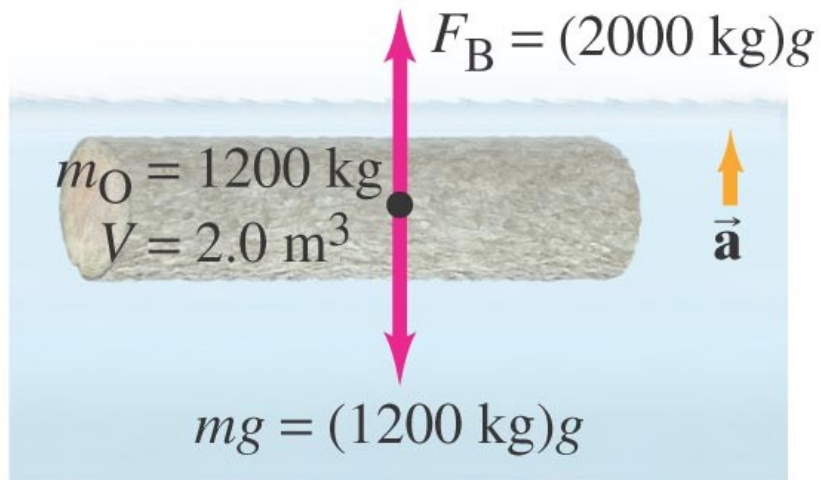
Which is equal to the weight of the fluid that takes up the same volume as the object.

$$\text{y-comp of net force: } F_2 - F_1 - mg = m_F g - mg = Vg(\rho_F - \rho)$$

Therefore, float if $(\rho < \rho_F)$; but sink if $(\rho > \rho_F)$.

10-7 Buoyancy and Archimedes' Principle

If an object's **density** is **less** than that of water, there will be an **upward net force** on it, and it will rise until it is **partially out of the water**.



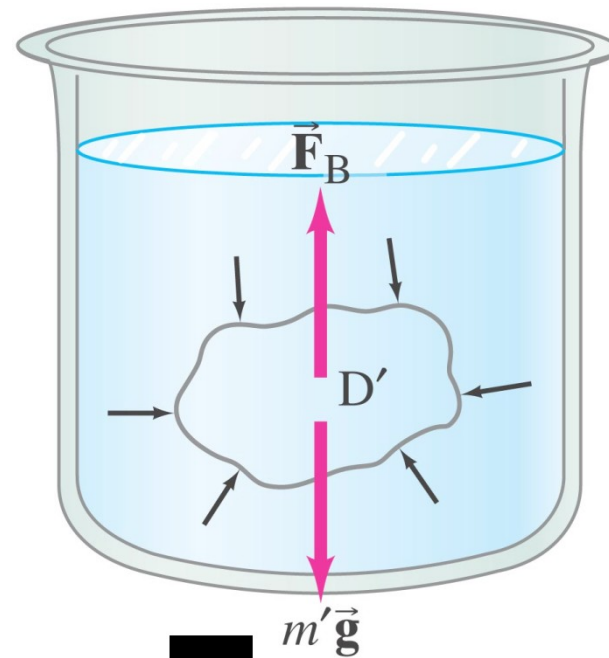
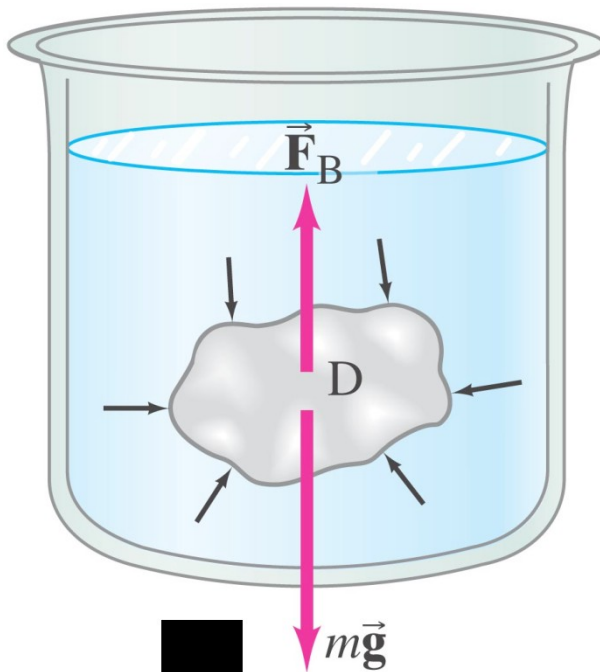
Density of water: 1000 kg/m^3

Density of wood: 600 kg/m^3

10-7 Buoyancy and Archimedes' Principle

Archimedes' principle:

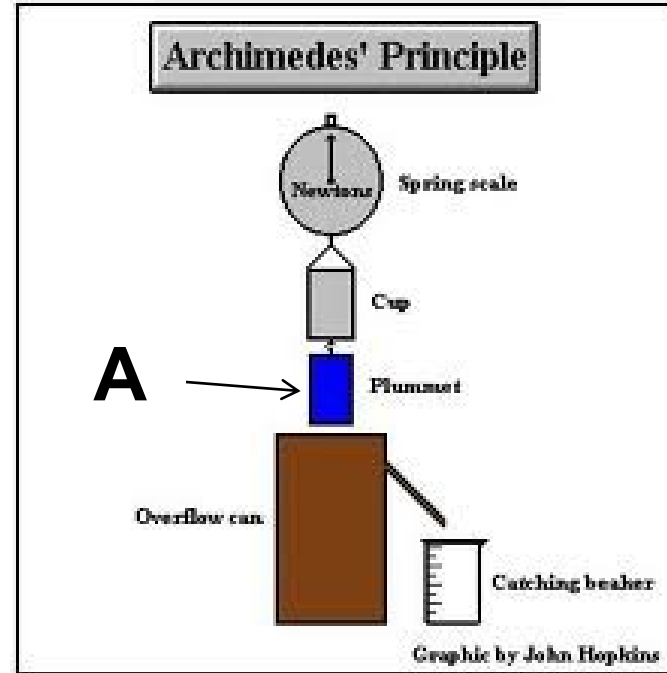
The buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by that object.



Demo

I-clicker question (20-3):
The buoyant force on A is

- A) 15.0 N
- B) 11.4 N
- C) 3.6 N
- D) 0 N



Weight (when A is in air): 15.0N

Apparent weight (when A is in water): 11.4N

Buoyant force: $15.0 - 11.4 = 3.6\text{N}$

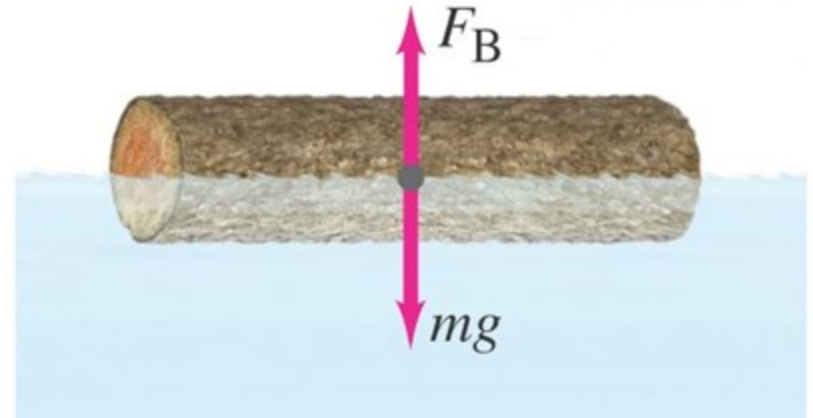
Weight of displaced water: 3.6N

Buoyant force = weight of displaced water.

I-clicker question 20-4: Two pails of water.

Consider two identical pails of water filled to the brim. One pail contains only water, the other has a piece of wood floating in it. Which pail has the greater weight?

- A. The pail with wood.
- B. The pail without wood.
- C. The two pails have the same weight.



The buoyant force on the wood object equals the weight of the wood object (static equilibrium).

Also, the buoyant force is equal to the weight of the spilled (i.e., displaced) water (Archimedes' principle).

In other words, the weight of wood is the same as the weight of the spilled water (both equal to the buoyant force).

Therefore, putting the wood doesn't change the weight of the pail.

Example: Recovering a submerged statue.

A 70-kg ancient statue lies at the bottom of the sea. Its volume is $3.0 \times 10^4 \text{ cm}^3$. How much force is needed to lift it?

If the statue were in the air, the force needed to lift it would be mg . i.e., you need to overcome the gravity.

Now the statue is submerged in water. The buoyant force due to water is helping you. The force you need now is

$$F = mg - F_B$$

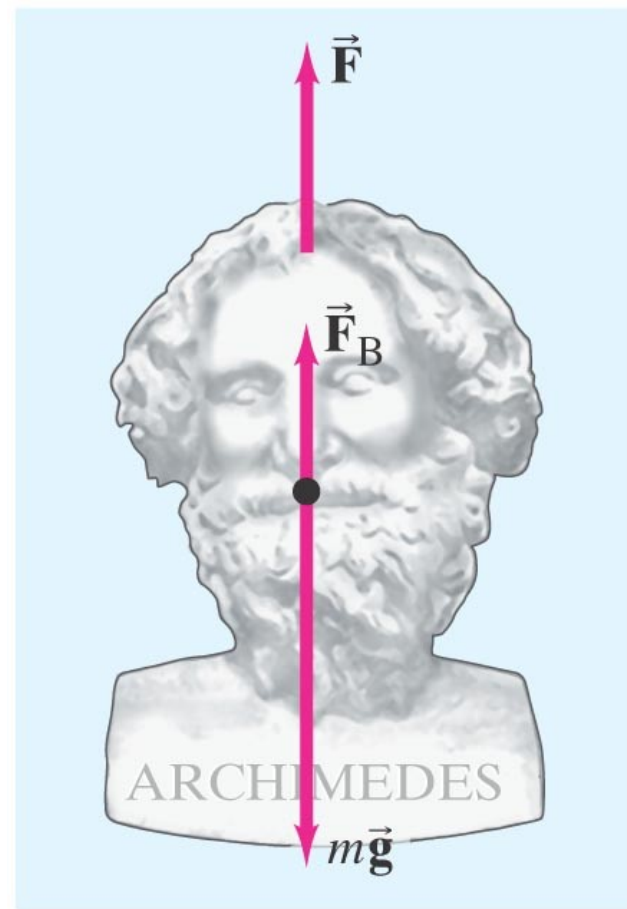
$$\left(\because \vec{F} + m\vec{g} + \vec{F}_B = 0 \Rightarrow F - mg + F_B = 0 \right)$$

$$F = mg - \rho_{\text{water}} V g$$

$$= 70 \times 9.8 - 1000 \times 0.03 \times 9.8 = 390 \text{ N}$$

The apparent weight of the statue is 390N.

demo



Example: Archimedes: Is the crown gold?

When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?

Idea: density of Au: $19.3 \times 10^3 \text{ kg/m}^3$

From the FBDs:

$$F_T - mg = 0$$

$$F'_T + F_B - mg = 0$$

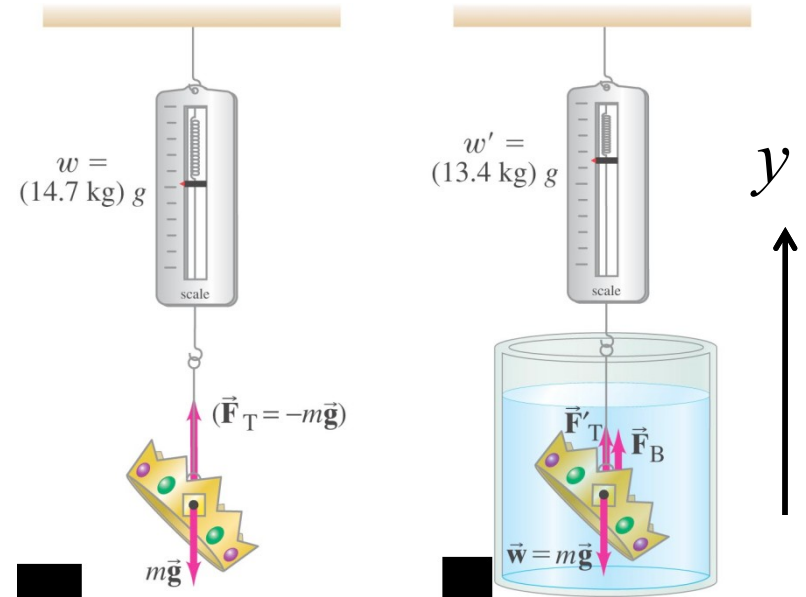
$$W - \rho_C Vg = 0 \quad (1)$$

$$W' + \rho_{\text{water}} Vg - \rho_C Vg = 0 \quad (2)$$

$$\frac{(1)}{(1)-(2)} : \quad \frac{W}{W - W'} = \frac{\rho}{\rho_{\text{Water}}}$$

$$\frac{\rho_C}{\rho_{\text{water}}} = \frac{W}{W - W'} = \frac{14.7}{14.7 - 13.4} = 11.3$$

The density of the crown is $11.3 \times 10^3 \text{ kg/m}^3$. It's made of lead (or some alloy) !



The story:

Hiero, king of Syracuse in the third century B.C., ordered a jeweler to make a bar of gold into a crown. When the jeweler delivered the crown to the king, the king believed that the jeweler had cheated him and substituted some less precious metal for the gold. Hiero asked Archimedes, to find a way to demonstrate that the crown was not pure gold.

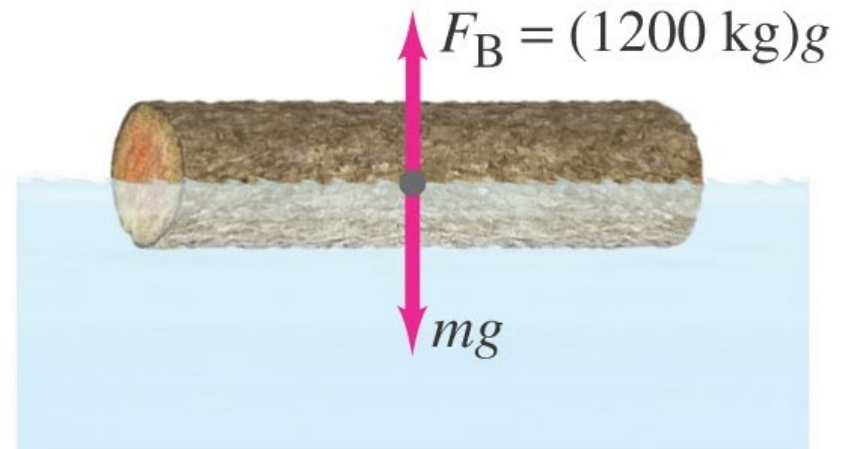
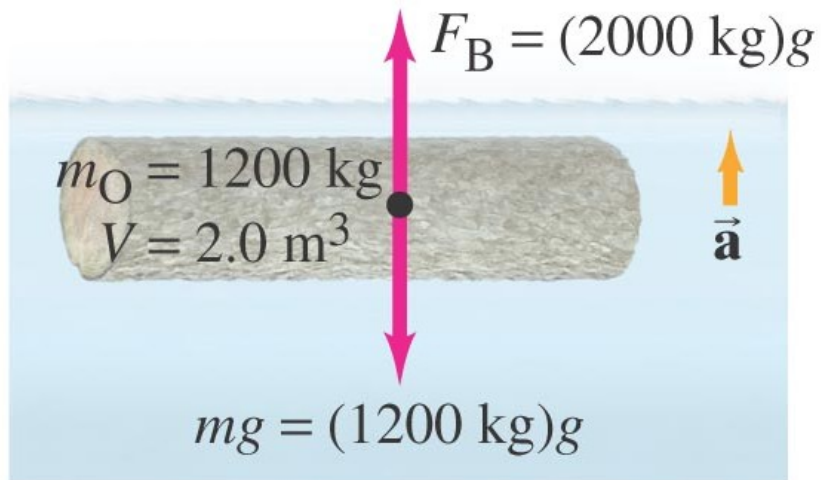
Archimedes struggled with the problem for a while. Then, one day, as he stepped into a bathtub filled with water, he saw that the water overflowed. He realized that he he had found the answer and ran from the bathhouse crying, “Eureka!” (“I have found it!”)



1 lb of silver has a larger volume than 1 lb of gold and therefore would displace more water.

10-7 Buoyancy and Archimedes' Principle

If an object's density is less than that of water, there will be an upward net force on it, and it will rise until it is partially out of the water.



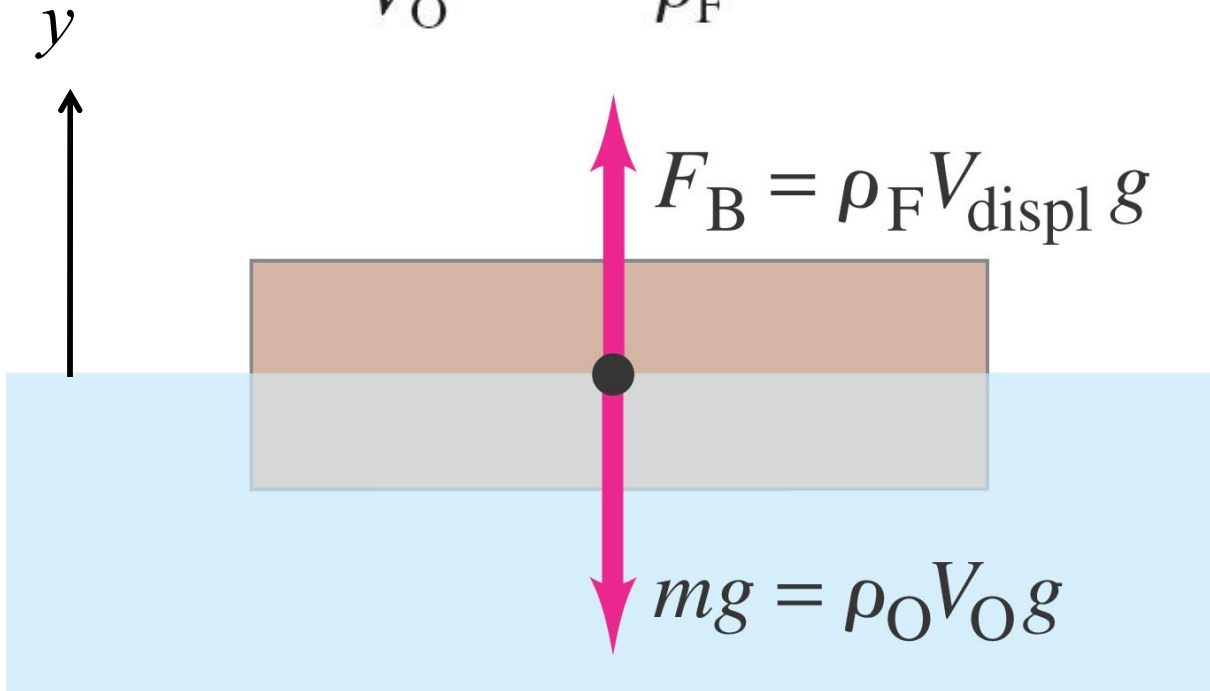
Density of water: 1000 kg/m^3

Density of wood: 600 kg/m^3

10-7 Buoyancy and Archimedes' Principle

For a floating object, the fraction that is submerged is given by the ratio of the object's density to that of the fluid.

$$\frac{V_{\text{displ}}}{V_O} = \frac{\rho_O}{\rho_F}.$$



$$F_B - mg = 0$$

$$F_B = mg$$

$$\rho_F V_{\text{displ}} g = \rho_O V_O g$$

$$\frac{V_{\text{displ}}}{V_O} = \frac{\rho_O}{\rho_F}$$

Example: Hydrometer calibration.

A hydrometer is a simple instrument used to measure the specific gravity of a liquid by indicating how deeply the instrument sinks in the liquid. This hydrometer consists of a glass tube, weighted at the bottom, which is 25.0 cm long and 2.00 cm² in cross-sectional area, and has a mass of 45.0 g. How far from the end should the 1.000 mark be placed?

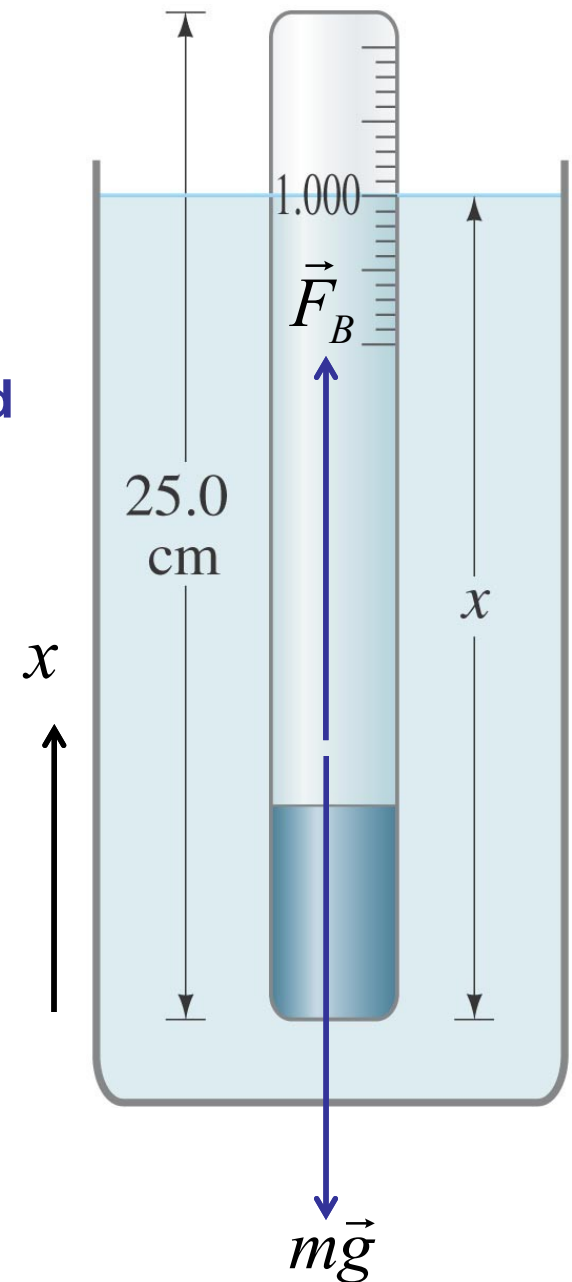
$$F_B - mg = 0, \quad \rho_F A x g - mg = 0$$

$$x = \frac{mg}{\rho_F A g} = \frac{m}{\rho_F A}$$

When the specific gravity is 1.000,

$$\rho_F = \rho_{\text{Water}} = 1.000 \times 10^3 \text{ kg} / \text{m}^3$$

$$x = \frac{45.0 \times 10^{-3}}{1.000 \times 10^3 \times 2.00 \times 10^{-4}} = 0.225 \text{ m}$$



Example: Helium balloon.

What volume V of helium is needed if a balloon is to lift a load of 180 kg (including the weight of the empty balloon)?

$$F_B - m_{load}g - m_{He}g = 0$$

$$F_B = m_{load}g + m_{He}g$$

$$\rho_{air}Vg = m_{load}g + \rho_{He}Vg$$

Here we ignore the volume of the load.

$$\rho_{air}V = m_{load} + \rho_{He}V$$

$$V = \frac{m_{load}}{(\rho_{air} - \rho_{He})} = \frac{180}{1.29 - 0.179} = 160m^3$$

