Phys101 Lectures 31, 32

Sound, Decibels, Doppler Effect

Key points:

- **Intensity of Sound: Decibels**

- **Doppler Effect**

Ref: 12-1,2,7.

Characteristics of Sound

Sound can travel through any kind of matter, but not through a vacuum.

TABLE 12–1 Speed of Sound in Various Materials (20°C and 1 atm)	
Material	**Speed (m/s)**
Air	343
Air (0°C)	331
Helium	1005
Hydrogen	1300
Water	1440
Sea water	1560
Iron and steel	≈ 5000
Glass	≈ 4500
Aluminum	≈ 5100
Hardwood	≈ 4000
Concrete	≈ 3000

The speed of sound is different in different materials; in general, it is slowest in gases, faster in liquids, and fastest in solids.

The speed depends somewhat on temperature, especially for gases.

Characteristics of Sound

Loudness: related to intensity of the sound wave

Pitch: related to frequency

Audible range: about 20 Hz to 20,000 Hz; upper limit decreases with age

Ultrasound: above 20,000 Hz;

Infrasound: below 20 Hz

Intensity of Sound: Decibels

TABLE 12–2 Intensity of Various Sounds

Source of the Sound	Sound Level (dB)	Intensity (W/m^2)
Jet plane at 30 m	140	100
Threshold of pain	120	1
Loud rock concert	120	1
Siren at 30 m	100	1×10^{-2}
Auto interior, at 90 km/h	75	3×10^{-5}
Busy street traffic	70	1×10^{-5}
Talk, at 50 cm	65	3×10^{-6}
Quiet radio	40	1×10^{-8}
Whisper	20	1×10^{-10}
Rustle of leaves	10	1×10^{-11}
Threshold of hearing	0	1×10^{-12}

The intensity of a wave is the energy transported per unit time across a unit area.

The human ear can detect sounds with an intensity as low as 10^{-12} W/m^2 and as high as 1 W/m^2.

Perceived loudness, however, is not proportional to the intensity.

Sound Level: Decibels

The loudness of a sound is much more closely related to the logarithm of the intensity.

Sound level is measured in decibels (dB) and is defined as:

$$\beta \,(\text{in dB}) \;=\; 10 \log \frac{I}{I_0}.$$

I_0 is taken to be the threshold of hearing:

$$I_0 = 1.0 \times 10^{-12} \,\text{W/m}^2.$$

Example: Sound intensity on the street.

At a busy street corner, the sound level is 75 dB. What is the intensity of sound there?

$$\beta \ (\text{in dB}) \ = \ 10 \log \frac{I}{I_0}.$$

$$75 = 10 \log \frac{I}{I_0}$$

$$\log \frac{I}{I_0} = 7.5$$

$$\frac{I}{I_0} = 10^{7.5}$$

$$I = 10^{7.5} I_0 = 10^{7.5} \times 10^{-12} = 10^{-4.5} = 3.16 \times 10^{-5} \ W/m^2$$

Example: Loudspeaker response.

A high-quality loudspeaker is advertised to reproduce, at full volume, frequencies from 30 Hz to 18,000 Hz with uniform sound level ± 3 dB. That is, over this frequency range, the sound level output does not vary by more than 3 dB for a given input level. By what factor does the intensity change for the maximum change of 3 dB in output sound level?

$$\text{Given } \Delta\beta = \beta_2 - \beta_1 = 3dB, \quad \text{find}: \frac{I_2}{I_1}$$

$$\beta_2 - \beta_1 = 10\,log\,\frac{I_2}{I_0} - 10\,log\,\frac{I_1}{I_0}$$

$$\beta_2 - \beta_1 = 10\,log\,\frac{I_2}{I_1}$$

$$3 = 10\,log\,\frac{I_2}{I_1}$$

$$\frac{I_2}{I_1} = 10^{0.3} = 2.0$$

3 dB means 2 times in intensity (or energy, power etc.)

Conceptual Example: Trumpet players.

A trumpeter plays at a sound level of 75 dB. Three equally loud trumpet players join in. What is the new sound level?

$$\beta_1 = 10 \, log \, \frac{I_1}{I_0}$$

$$\beta_4 = 10 \, log \, \frac{4I_1}{I_0}$$

$$\beta_4 = 10 \left(log \, 4 + log \, \frac{I_1}{I_0} \right)$$

$$= 10 \, log \, 4 + 10 \, log \, \frac{I_1}{I_0}$$

$$= 6 + \beta_1$$

$$= 6 + 75$$

$$= 81 \, dB$$

Example: Airplane roar.

The sound level measured 30 m from a jet plane is 140 dB. What is the sound level at 300 m? (Ignore reflections from the ground.)

At 30 m: Sound level : β_1

$$\beta_1 = 10 \, log \, \frac{I_1}{I_0} = 140 \, dB$$

Sound intensity I_1 :

$$log \, \frac{I_1}{I_0} = 14; \quad I_1 = I_0 \, 10^{14} = 100 \, W/m^2$$

At 300 m: Sound intensity : I_2

$$I \propto \frac{1}{r^2} \quad \Rightarrow \quad \frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}, \quad I_2 = \frac{r_1^2}{r_2^2} I_1 = \left(\frac{30}{300}\right)^2 I_1 = 1.0 \, W/m^2$$

$$\beta_2 = 10 \, log \, \frac{I_2}{I_0} = 10 \, log\left(\frac{1.0}{1.0 \times 10^{-12}}\right) = 10 \times 12 = 120 dB$$

Example: How tiny the displacement is.

Calculate the displacement of air molecules for a sound having a frequency of 1000 Hz at the threshold of hearing.

[Solution]

$$I = \frac{\overline{P}}{S} = 2\pi^2 (\rho S l) f^2 A^2 / t = 2\pi^2 \rho v f^2 A^2$$
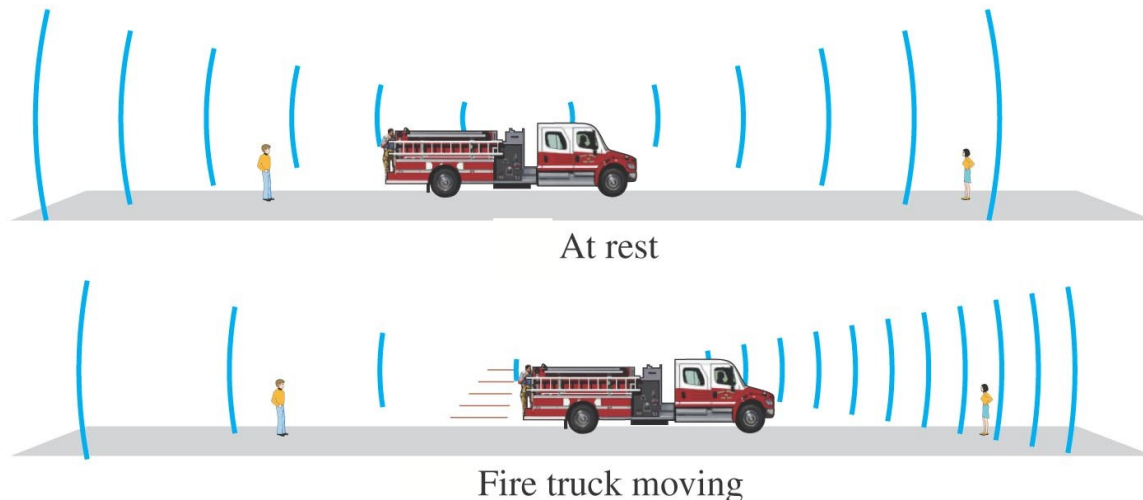
$$A^2 = \frac{I}{2\pi^2 \rho v f^2}$$

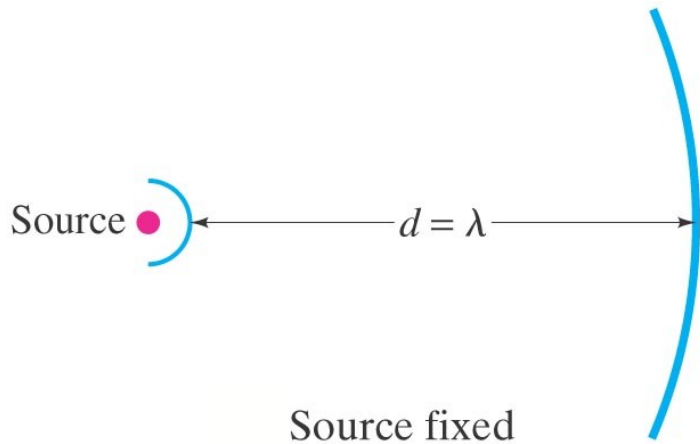$$A = \frac{1}{\pi f} \sqrt{\frac{I}{2\rho v}}$$

$$A = \frac{1}{1000\pi} \sqrt{\frac{10^{-12}}{2 \times 1.29 \times 343}} = 1.1 \times 10^{-11} m$$
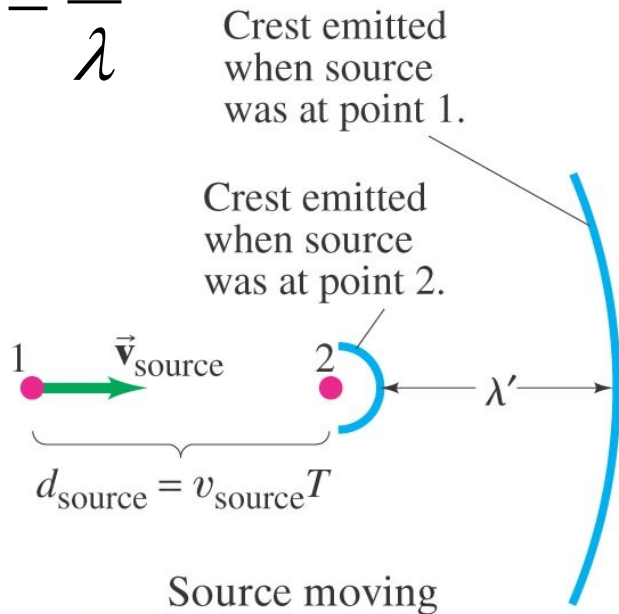
Doppler Effect

The Doppler effect occurs when a source of sound is moving with respect to an observer.

A source moving toward an observer appears to have a higher frequency and shorter wavelength; a source moving away from an observer appears to have a lower frequency and longer wavelength.

At rest

Fire truck moving

Source ●)———— $d = \lambda$ ————→

Source fixed

$$f = \frac{v}{\lambda}$$

Crest emitted when source was at point 1.

Crest emitted when source was at point 2.

1 ●——→ $\vec{v}_{source}$ 2 ●)←— λ' —→

$\underbrace{\qquad\qquad}_{d_{source} = v_{source}T}$

Source moving

$v -$ velocity of sound

If we can figure out what the change in the wavelength is, we also know the change in the frequency.

When the source is approaching, the observed wavelength is shorter:

$$\lambda' = \lambda - v_{source}T$$

Then the observed frequency is:

$$f' = \frac{v}{\lambda'} = \frac{v}{\lambda - v_{source}T}$$

$$= \frac{v}{\lambda\left(1 - \dfrac{v_{source}T}{\lambda}\right)} = \frac{f}{1 - \dfrac{v_{source}T}{\lambda}}$$

$$= \frac{f}{1 - \dfrac{v_{source}}{v}} > f$$

Doppler Effect

The change in the **frequency** is given by:

$$f' = \frac{f}{\left(1 - \dfrac{v_{source}}{v_{snd}}\right)}.$$

Source approaching:
Higher freq.
f'>f

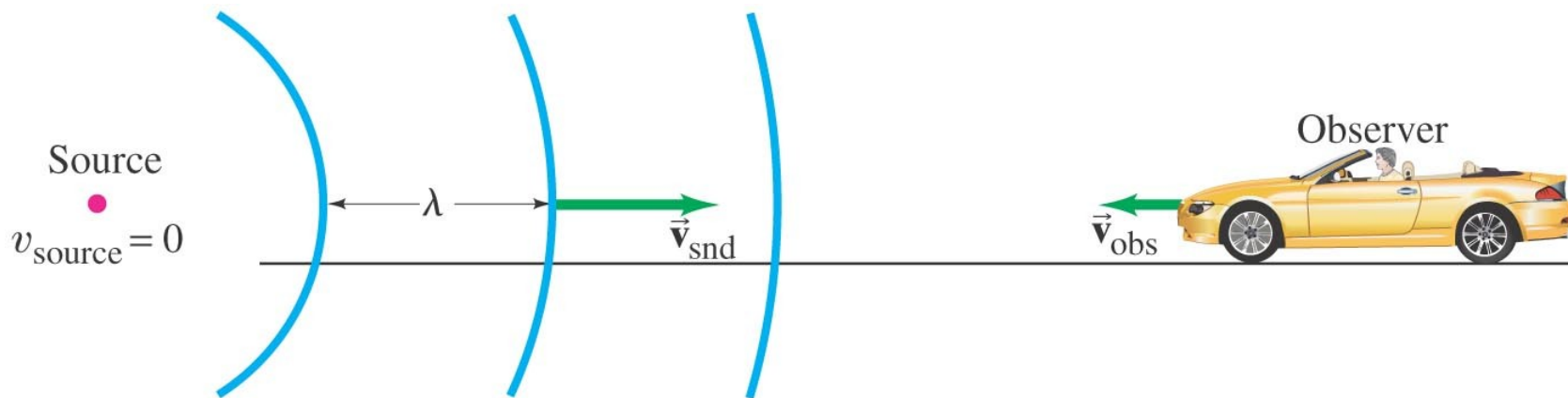## Similarly, if the source is moving **away** from the observer:

$$f' = \frac{f}{\left(1 + \dfrac{v_{source}}{v_{snd}}\right)}.$$

Source receding:
Lower freq.
f'<f

Doppler Effect

$$f = \frac{v}{\lambda}$$

If the observer is moving with respect to the source, things are a bit different. The wavelength remains the same, but the wave speed is different for the observer.

Source

$v_{\text{source}} = 0$

λ

$\vec{v}_{\text{snd}}$

Observer

$\vec{v}_{\text{obs}}$

$$v' = v + v_{obs} \quad \text{when the observer is approaching}$$

$$f' = \frac{v'}{\lambda} = \frac{v + v_{obs}}{\lambda} = \frac{v}{\lambda}\left(1 + \frac{v_{obs}}{v}\right) = f\left(1 + \frac{v_{obs}}{v}\right)$$

For an observer moving toward a stationary source:

Observed sound velocity: $v' = v + v_{obs}$

Observed frequency:

$$f' = \frac{v'}{\lambda} = \frac{v + v_{obs}}{\lambda} = \frac{v}{\lambda}\left(1 + \frac{v_{obs}}{v}\right) = f\left(1 + \frac{v_{obs}}{v}\right)$$

Observer approaching:
Higher freq.
f'>f

And if the observer is moving away:

$$v' = v - v_{obs}$$

$$f' = \frac{v'}{\lambda} = \frac{v - v_{obs}}{\lambda} = \frac{v}{\lambda}\left(1 - \frac{v_{obs}}{v}\right) = f\left(1 - \frac{v_{obs}}{v}\right)$$

Observer receding:
Lower freq.
f'<f

Example: A moving siren.

The siren of a police car at rest emits at a predominant frequency of 1600 Hz. What frequency will you hear if you are at rest and the police car moves at 25.0 m/s (a) toward you, and (b) away from you?

(a) Moving source, approaching: Higher freq.

$$f' = \frac{f}{1 - \dfrac{v_{source}}{v}} = \frac{1600}{1 - \dfrac{25}{343}} = 1726\,Hz \quad (\text{speed of sound in air}: v = 343\,m/s)$$

(b) Moving source, receding: Lower freq.

$$f' = \frac{f}{1 + \dfrac{v_{source}}{v}} = \frac{1600}{1 + \dfrac{25}{343}} = 1491\,Hz$$
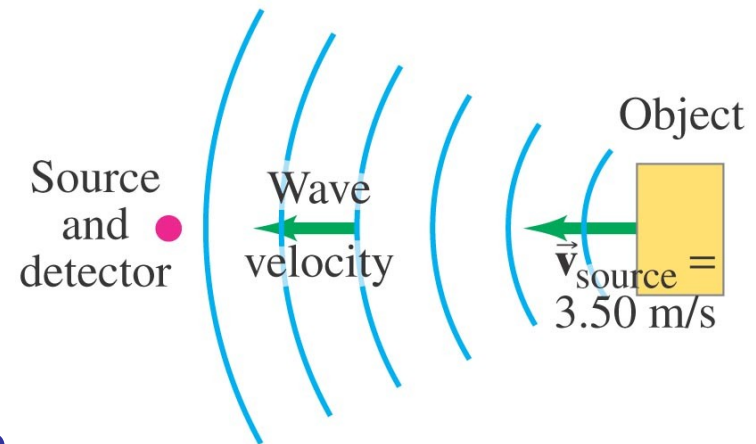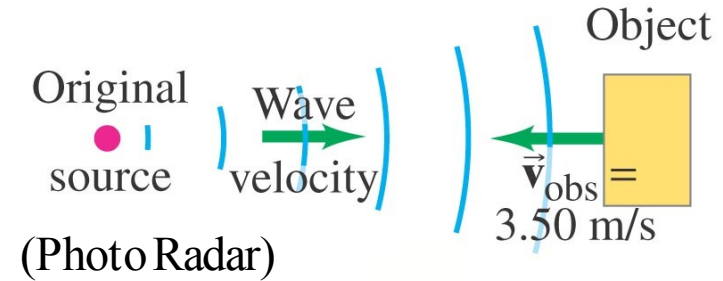
Example 12-15: Two Doppler shifts.

A 5000-Hz sound wave is emitted by a stationary source. This sound wave reflects from an object moving 3.50m/s toward the source. What is the frequency of the wave reflected by the moving object as detected by a detector at rest near the source?

Frequency received by the moving object:

$$f' = f\left(1 + \frac{v_{obs}}{v}\right) = 5000\left(1 + \frac{3.5}{343}\right) = 5051\,Hz$$

Then, the object becomes a moving source when the sound is reflected back to the detector. The frequency received by the detector is:

$$f'' = \frac{f'}{1 - \frac{v_{source}}{v}} = \frac{5051}{1 - \frac{3.5}{343}} = 5103\,Hz$$

Original ● ǀ source Wave velocity (Photo Radar)

Object

$\vec{v}_{obs} = $ 3.50 m/s

Source and detector ● Wave velocity

Object

$\vec{v}_{source} = $ 3.50 m/s

This is known as a double Doppler effect. The moving object first acts as a moving observer and then a moving source.

Doppler Effect

All four equations for the Doppler effect can be combined into one; you just have to keep track of the signs!

$$f' = f\left(\frac{v_{\text{snd}} \pm v_{\text{obs}}}{v_{\text{snd}} \mp v_{\text{source}}}\right).$$

The signs:

Approaching: higher freq. $f' > f$

Receding: lower freq. $f' < f$