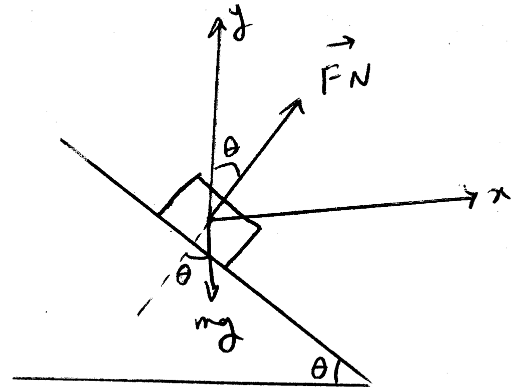
$R = 67\ m$

$v = 95\ km/h = 26.4$

$\mu = 0.30$



(no friction)

The design banking angle, θ, is determined in the absence of friction

Newton's law: $\vec{F}_{net} = m\vec{a}$

x: $F_N \sin\theta = m\frac{v^2}{R}$

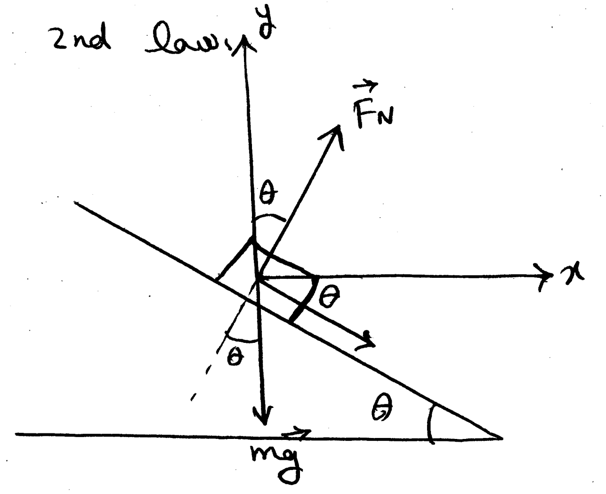
y: $F_N \cos\theta - mg = 0 \Rightarrow F_N = \frac{mg}{\cos\theta}$

$\therefore \frac{mg}{\cos\theta} \cdot \sin\theta = \frac{mv^2}{R} \Rightarrow tg\,\theta = \frac{v^2}{Rg} \Rightarrow \theta = tg^{-1}\left(\frac{v^2}{Rg}\right)$

$\Rightarrow tg^{-1}\left(\frac{26.4^2}{67 \times 9.8}\right) = tg^{-1}(1.061) = 46.7 \simeq 47^\circ$

Case ① fast $\rightarrow$ The x-component of the normal force is not large enough to provide the required centripetal force. Therefore, the car tends to skid upward along the banked road, and the frictional force is downward.

The component form of Newton's 2nd law:



case ①: fast

$$x: \; F_N \sin\theta + F_f \cos\theta = \frac{mv^2}{R}$$

$$y: \; F_N \cos\theta - F_f \sin\theta - mg = 0$$

at maximum static friction : $F_f = \mu_s F_N$

Then:

$$F_N \sin\theta + \mu_s F_N \cos\theta = m\frac{v^2}{R}$$

$$F_N \cos\theta - \mu_s F_N \sin\theta - mg = 0$$

There are two unknowns : F_N and v (m will cancel out : $F_N \propto m$)

We can eliminate F_N and solve for v:

from the second equation : $F_N = \dfrac{mg}{\cos\theta - \mu_s \sin\theta}$

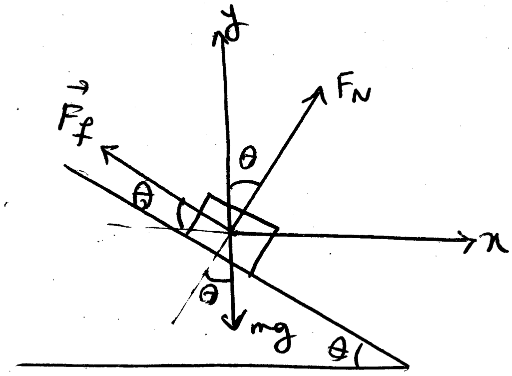
substitute into the first equation:

$$\frac{mg}{\cos\theta - \mu_s \sin\theta}\left(\sin\theta + \mu_s \cos\theta\right) = \frac{mv^2}{R}$$

$$\therefore \; V = \sqrt{\frac{(\sin\theta + \mu_s \cos\theta)\, gR}{\cos\theta - \mu_s \sin\theta}} = 36.2 \text{ m/s}$$

max speed allowed

Case ②: slow – the frictional force acting on the car is upward along the banked road surface



Newton's 2nd law,

x: $F_N \sin\theta - F_f \cos\theta = \frac{mv^2}{R}$

max friction, $F_f = \mu_s F_N$

y: $F_N \cos\theta + F_f \sin\theta - mg = 0$

(A) $F_N \sin\theta - \mu_s F_N \cos\theta = \frac{mv^2}{R}$

(B) $F_N \cos\theta + \mu_s F_N \sin\theta = mg$

$$\frac{(A)}{(B)} \Rightarrow \frac{\sin\theta - \mu_s \cos\theta}{\cos\theta + \mu_s \sin\theta} = \frac{v^2}{Rg}$$

$$\therefore v = \sqrt{\frac{(\sin\theta - \mu_s \cos\theta) Rg}{\cos\theta + \mu_s \sin\theta}} = 19.5 \text{ m/s}$$

min speed allowed

Therefore the range is

$$70.2 \frac{km}{h} = 19.5 \text{ m/s} < v < 36.2 \text{ m/s} = 130 \frac{km}{h}$$