PHYS 221 Midterm examination #1

May 30, 2008 Name: Key

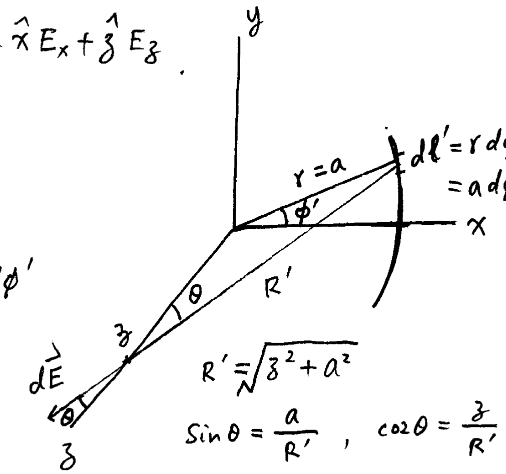
Time: 50 minutes Student No.

Please show complete solutions and explain your reasoning, stating any principles that you have used.

****

1(5/15 marks). Electric charge is uniformly distributed along an arc located in the x-y plane and defined by $r = a$ and $-\frac{\pi}{8} \le \phi \le \frac{\pi}{8}$. The linear charge density is ρ_l. Determine the electric filed $\mathbf{E}$ at (0, 0, z).

By Symmetry, $E_y = 0$. i.e, $\vec{E} = \hat{x} E_x + \hat{z} E_z$.



$R' = \sqrt{z^2 + a^2}$

$\sin\theta = \frac{a}{R'}$, $\cos\theta = \frac{z}{R'}$

$$E_x = \frac{-1}{4\pi\varepsilon_0}\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{\rho_l a\, d\phi' \sin\theta \cdot \cos\phi'}{(z^2+a^2)}$$

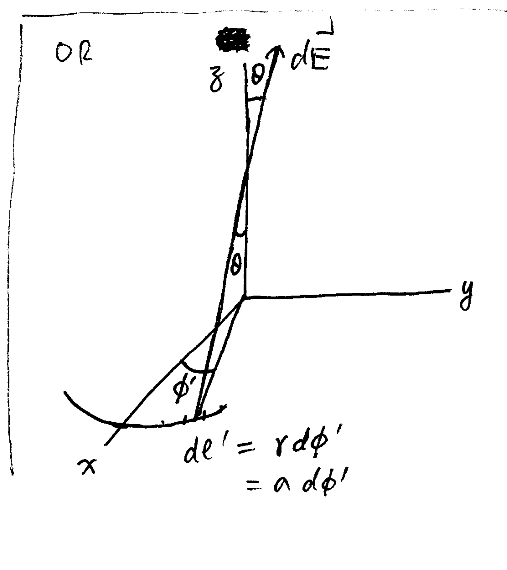
$$= \frac{-\rho_l \cdot a^2}{4\pi\varepsilon_0 (z^2+a^2)^{3/2}} \cdot \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \cos\phi'\, d\phi'$$

$$= \frac{-\rho_l a^2 \sin\frac{\pi}{8}}{2\pi\varepsilon_0 (z^2+a^2)^{3/2}}$$

$$E_z = \frac{1}{4\pi\varepsilon_0}\int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} \frac{\rho_l a\, d\phi' \cdot \cos\theta}{z^2+a^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{\rho_l a z}{(z^2+a^2)^{3/2}} \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} d\phi'$$

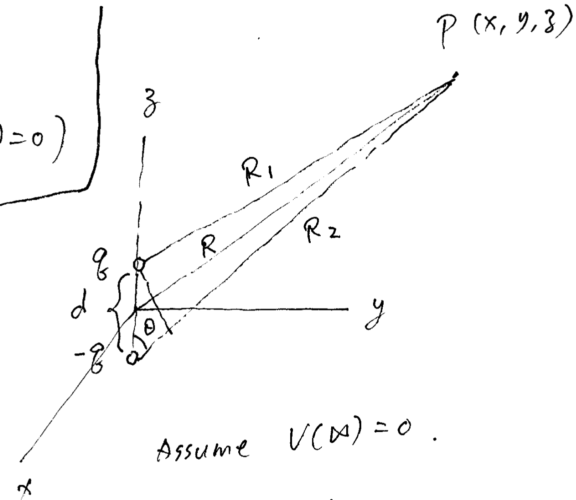
$$= \frac{\rho_l a z}{16\varepsilon_0 (z^2+a^2)^{3/2}}$$



2(5/15 marks). An electric dipole consists of two charges q and –q located at (0, 0, d/2) and (0, 0, -d/2) respectively.
(a) Determine the electric potential V at any point P(x, y, z) in free space, given that P is far away from the dipole.
(b) Determine the electric field **E** (both magnitude and direction) at point P.

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R_1} - \frac{q}{R_2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\,\frac{R_2 - R_1}{R_1 R_2} \qquad (V(\infty)=0)$$



Assume $V(\infty) = 0$.

when $R \gg d$,

$$R_1 R_2 \approx R^2$$

$$R_2 - R_1 \approx d\cos\theta$$

$$\cos\theta = \frac{z}{R} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$R = \sqrt{x^2+y^2+z^2}$$

a). when $R \gg d$.

$$V \approx \frac{q}{4\pi\varepsilon_0}\,\frac{d\cos\theta}{R^2}$$

OR:

$$V = \frac{q\, d\cdot z}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}.$$

b). $\vec{E} = -\nabla V$.

$$= -\left[\hat{R}\frac{\partial V}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial\theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial V}{\partial\phi}\right]$$

$$= -\hat{R}\frac{\partial V}{\partial R} - \hat{\theta}\frac{1}{R}\frac{\partial V}{\partial\theta}.$$

$$E_R = -\frac{\partial V}{\partial R} = \frac{q}{2\pi\varepsilon_0}\,\frac{d\cos\theta}{R^3}$$

$$E_\theta = -\frac{1}{R}\frac{\partial V}{\partial\theta} = \frac{1}{R}\,\frac{q\cdot d\cdot\sin\theta}{4\pi\varepsilon_0 R^2} = \frac{qd\cdot\sin\theta}{4\pi\varepsilon_0 R^3}$$

$$E_\phi = 0$$

$$\therefore\ \vec{E} = \hat{R}\,\frac{qd\cos\theta}{2\pi\varepsilon_0 R^3} + \hat{\theta}\,\frac{qd\sin\theta}{4\pi\varepsilon_0 R^3} \qquad (R \gg d)$$

3(5/15 marks). A solid sphere with a radius R is uniformly charged. The charge density (charge per unit volume) is ρ. Figure A below depicts its cross section on the x-y plane. The centre of the sphere is at the origin.
(a) Determine the electric field E at point P(2R/3, 0, 0).
(b) A spherical cavity of radius R/2 is created as shown in figure B. The centre of the cavity is located at (R/2, 0, 0). Determine the electric field inside the cavity at point P(2R/3, 0, 0).
(c) If the cavity inside the sphere has a radius w and the centre of the cavity is located at (a, b, 0), find the electric field at a point (x, y, 0) inside the cavity.

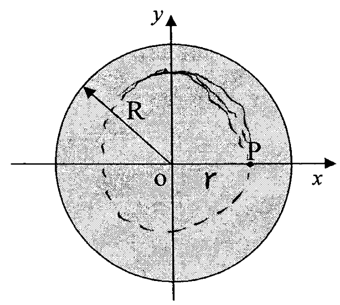
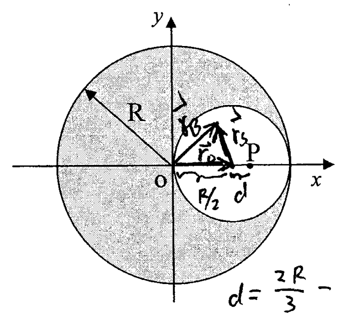


Figure A



$d = \frac{2R}{3} - \frac{R}{2} = R/6$

Figure B

(a). $\vec{E} = \hat{R} E_R$.

spherical Gaussian surface.

$$\oint_s \vec{E} \cdot d\vec{s} = \frac{Q}{\varepsilon_0}$$

$$E_R \cdot 4\pi r^2 = \frac{\frac{4}{3}\pi r^3 \cdot \rho}{\varepsilon_0}$$

$$E_R = \frac{\rho r}{3\varepsilon_0}$$

i.e, $\vec{E} = \hat{R} \frac{\rho r}{3\varepsilon_0} = \frac{\rho \vec{r}}{3\varepsilon_0}$

at point P:

$$\vec{E} = \hat{x} \cdot \frac{\rho}{3\varepsilon_0} \cdot \frac{2R}{3} = \hat{x} \frac{2\rho R}{9\varepsilon_0}$$

(b): $\vec{E} = \hat{x}\left(\frac{2\rho R}{9\varepsilon_0} - \frac{\rho R}{18\varepsilon_0}\right) = \hat{x} \frac{\rho R}{6\varepsilon_0}$

[OR: $E_x = E_x^+ + E_x^- = \frac{\rho}{3\varepsilon_0} \cdot \frac{2R}{3} - \frac{\rho}{3\varepsilon_0} \frac{R}{6} = \frac{\rho R}{6\varepsilon_0}$

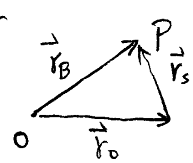
$E_y = E_z = 0$.]

(c) $\vec{E} = \vec{E}_\rho + \vec{E}_{-\rho}$

$$= \frac{\rho}{3\varepsilon_0}(\vec{r}_B - \vec{r}_s) \qquad r_s = W .$$

$$= \frac{\rho}{3\varepsilon_0} \vec{r}_0$$

($\vec{r}_0$ — position of center of cavity)

(

$\vec{r}_B - \vec{r}_s = \vec{r}_0$)