PHYS 221 Final Examination

August 17, 2010 Name ______________________________

Time: 180 minutes Student No.______________________________

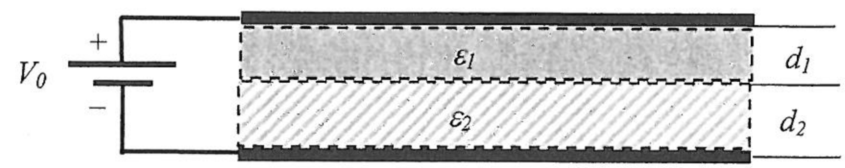
1(10 pts). A DC voltage V_0 is applied across a parallel-plate capacitor. The space between the conducting plates is filled with two different lossless dielectrics of thicknesses d_1 and d_2, permittivity ε_1 and ε_2, respectively. The area of each conducting plate is S.

(a) Determine the capacitance.

(b) Determine the electric field intensity **E** in the dielectrics;

(c) Find the surface free charge density on the plates and at the interface of the dielectrics;

(d) Repeat (b) and (c) for lossy dielectrics, i.e., assuming the conductivity of the two dielectrics is σ_1 and σ_2 respectively.



2(10 pts). A solid sphere with a radius R is uniformly charged. The charge density (charge per unit volume) is ρ. Figure A below depicts its cross section on the x-y plane. The centre of the sphere is at the origin.

(a) Determine the electric field E at point P(2R/3, 0, 0).

(b) A spherical cavity of radius R/2 is created as shown in figure B. The centre of the cavity is located at (R/2, 0, 0). Determine the electric field inside the cavity at point P(2R/3, 0, 0).

(c) If the cavity inside the sphere has a radius w and the centre of the cavity is located at (a, b), find the electric field at a point (x, y) inside the cavity.

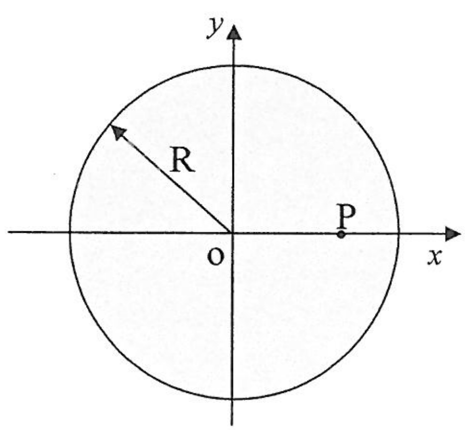


Figure A

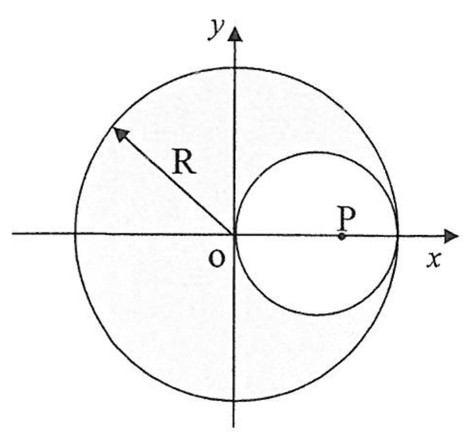


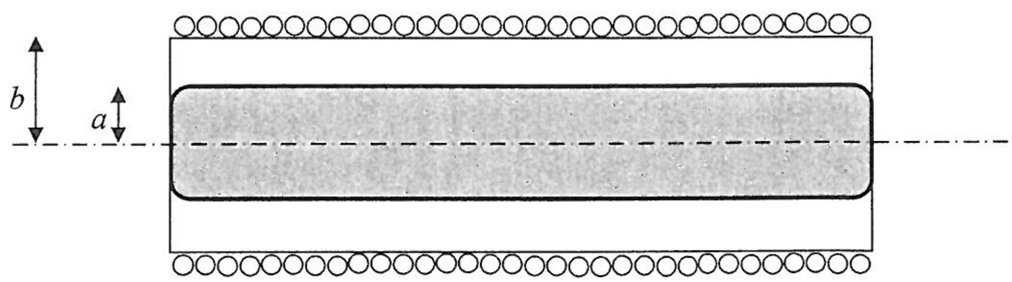
Figure B

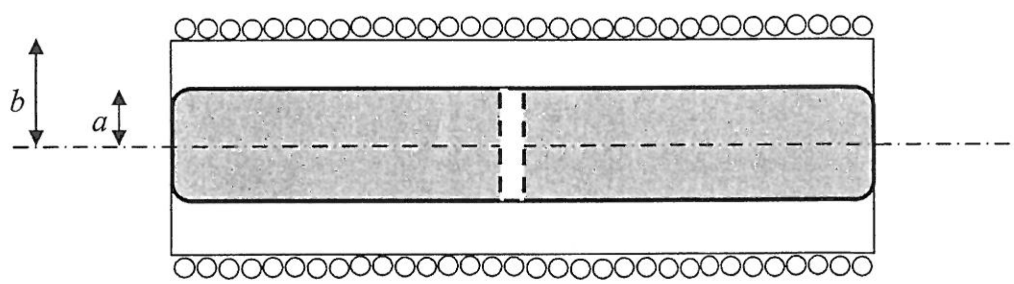
3$_{(10pts)}$. A circular rod of magnetic material with permeability μ is inserted coaxially in a long solenoid, as shown in the first figure below. The radius of the rod, a, is less than the inner radius, b, of the solenoid. The solenoid's winding is n turns per unit length and carries a current I.

(a) Use Ampere's law to determine the magnetic field intensity , ***H***, inside the solenoid for $r<a$ and for $a<r<b$.

(b) Determine ***B*** inside the solenoid for $r<a$ and for $a<r<b$.

(c) If there were a small gap in the middle of the magnetized rod as shown in the second figure below, what would be the magnetic field intensity ***H*** and magnetic flux intensity ***B*** in the gap? Assume the opening of the small gap does not affect the magnetic field inside the rod.



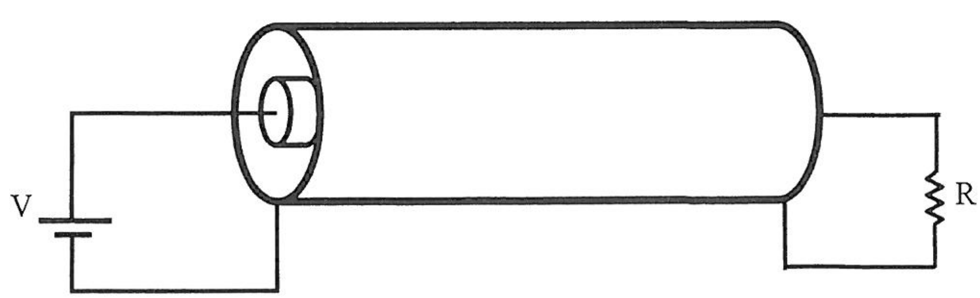


4$_{(10pts)}$. A DC power supply of voltage V is connected to a coaxial cable and then to a load resister R, as shown in the figure below. The length of the coaxial cable is L, the radius of the inner conductor is a and the inner radius of the outer conductor is b. The space between the two conductors is filled with nonmagnetic material whose electric permittivity is ε (assume $\mu=\mu_0$, $\sigma=0$).

(a) Determine the electric field **E** between the two conductors.

(b) Determine the magnetic field **H** between the two conductors.

(c) Determine the magnitude and direction of the Poynting vector **S**.



5$_{(10pts)}$. A coaxial transmission line with an inner conductor diameter of 0.60cm and an outer conductor diameter of 1.0 cm is filled with a dielectric material where $\mu=\mu_0$, $\varepsilon_r=2.5$, and $\sigma=1.0\times10^{-3}$ S/m. The conductors are made of copper with $\mu_c = \mu_0$ and $\sigma_c = 5.8 \times 10^7$ S/m. The operating frequency is 1.0 GHz.

(a) Calculate the line parameters R', L', G', and C';

(b) Calculate α, β, u_p and Z_0.

6$_{(10pts)}$. A circular-loop TV antenna with an area of 0.04 m^2 is in the presence of a uniform-amplitude 300Mhz signal. When oriented for maximum response, the loop develops an emf with a peak value of 60 mV. What is the peak magnitude of **B** of the incident wave?

7$_{(10pts)}$. A robot antenna made of copper ($\mu_c = \mu_0$, $\sigma_c = 5.8\times10^7$ s/m) operates at a frequency of 750 MHz. The length and the radius of the antenna is 10cm and 0.10cm, respectively. Treat the antenna as a monopole on a conducting surface. Under the far-field approximation, determine

a) the radiation efficiency of the antenna;
b) the antenna gain in dB;
c) the antenna current needed to produce 10W of radiation power.

8$_{(10pts)}$. A plane wave is travelling downward in the +z direction in seawater, with the x-y plane denoting the sea surface and z=0 denoting a point just below the surface. The constitutive parameters of seawater are $\varepsilon_r=80$, $\mu_r=1$ and $\sigma=4$ s/m. If the magnetic field at z=0 is given by

$$\vec{H} = \hat{x}50\,cos\left(2\pi\times10^4 t - \frac{\pi}{6}\right) \quad \text{(mA/m)}.$$

(a) Determine the electric field $\vec{E}(z,t)$ and magnetic field $\vec{H}(z,t)$ in the seawater.

(b) Evaluate the average power density at a depth of 20m.

9$_{(10pts)}$. The magnetic field of a wave propagating through a certain nonmagnetic material is given by

$$\boldsymbol{H} = \hat{z}\,4\cos(10^9 t - 5y)$$

Find the following

(a) The direction of wave propagation.
(b) The phase velocity.
(c) The wavelength in the material.
(d) The relative permittivity of the material.
(e) The electric field phasor.

10$_{(10pts)}$. It is commonly known that electromagnetic waves cannot penetrate conductors. For example, an AM radio wave cannot reach the inside of a metal box. On the other hand, seawater is commonly known as a conductor (σ=4 s/m), but it is transparent, i.e., light can penetrate seawater. Explain why.

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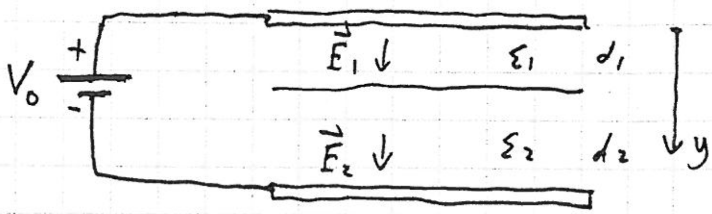
$$\varepsilon_0 = 8.85\times10^{-12} \quad C^2/N\cdot m^2$$

$$\mu_0 = 4\pi\times10^{-7} \ \text{T}\cdot\text{m/A}$$

Phys 221 Final Exam Solutions.

Summer 2010

1. (a).



20/

$$V_0 = E_1 d_1 + E_2 d_2$$

$$D_{1n} = D_{2n}.$$

$$\varepsilon_1 E_1 = \varepsilon_2 E_2, \quad E_2 = \frac{\varepsilon_1}{\varepsilon_2} E_1.$$

$$\therefore \; V_0 = E_1 \left(d_1 + \frac{\varepsilon_1}{\varepsilon_2} d_2 \right)$$

$$D_{1n} = \varepsilon_1 E_1 = \rho_s = \frac{Q}{S} \qquad Q = \varepsilon_1 E_1 S.$$

2

$$C = \frac{Q}{V_0} = \frac{\varepsilon_1 E_1 S}{E_1 \left(d_1 + \frac{\varepsilon_1}{\varepsilon_2} d_2\right)} = \frac{\varepsilon_1 S}{d_1 + \frac{\varepsilon_1}{\varepsilon_2} d_2} = \frac{\varepsilon_1 \varepsilon_2 S}{d_1 \varepsilon_2 + d_2 \varepsilon_1}$$

(b). $$\vec{E}_1 = \hat{y} \frac{V_0}{d_1 + \frac{\varepsilon_1}{\varepsilon_2} d_2} = \hat{y} \frac{\varepsilon_2 V_0}{d_1 \varepsilon_2 + \varepsilon_1 d_2} \quad 1$$

$$\vec{E}_2 = \hat{y} \frac{\varepsilon_1}{\varepsilon_2} E_1 = \hat{y} \frac{\varepsilon_1 V_0}{d_1 \varepsilon_2 + \varepsilon_1 d_2}. \quad 1$$

2

(c). upper plates:

$$\rho_s = \varepsilon_1 E_1 = \frac{\varepsilon_1 \varepsilon_2 V_0}{d_1 \varepsilon_2 + \varepsilon_1 d_2}$$

at interface: $\rho_s = 0$

2

lower plate. $\rho_s = -\varepsilon_2 E_2 = \frac{-\varepsilon_1 \varepsilon_2 V_0}{d_1 \varepsilon_2 + \varepsilon_1 d_2}$

(d). $D_{1n} \neq D_{2n}$, but $J_{1n} = J_{2n} \implies \sigma_1 E_1 = \sigma_2 E_2$

$$E_2 = \frac{\sigma_1}{\sigma_2} E_1.$$

$$V_0 = E_1 \left(d_1 + \frac{\sigma_1}{\sigma_2} d_2 \right).$$

4

$$\vec{E}_1 = \hat{y} \frac{\sigma_2 V_0}{d_1 \sigma_2 + \sigma_1 d_2}, \quad \vec{E}_2 = \hat{y} \frac{\sigma_1 V_0}{d_1 \sigma_2 + \sigma_1 d_2}.$$

1.

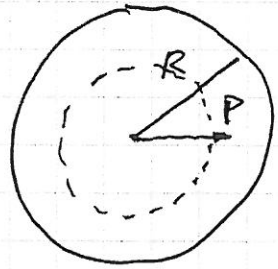
Upper plate: $$\rho_s = E_1 \varepsilon_1 = \frac{\varepsilon_1 \sigma_2 V_0}{d_1 \sigma_2 + \sigma_1 d_2}.$$

lower plate: $$\rho_s = -E_2 \varepsilon_2 = \frac{-\varepsilon_2 \sigma_1 V_0}{d_1 \sigma_2 + \sigma_1 d_2}.$$

interface: $$\rho_s = E_2 \varepsilon_2 - E_1 \varepsilon_1 = \frac{V_0 (\varepsilon_2 \sigma_1 - \sigma_2 \varepsilon_1)}{d_1 \sigma_2 + \sigma_1 d_2}$$

2. $\vec{E} = \hat{r} E$ (use $\vec{r}$ to represent $\vec{R}$)

a). $\oint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\varepsilon_0}$



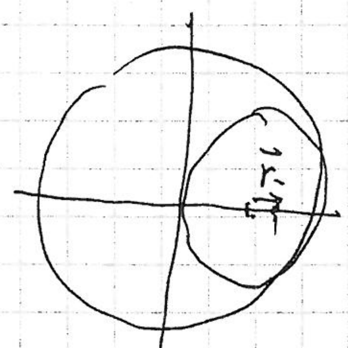
$$E \cdot 4\pi r^2 = \frac{\rho}{\varepsilon_0} \frac{4}{3} \pi r^3$$

$$E = \frac{\rho r}{3\varepsilon_0}, \qquad \vec{E} = \frac{\rho \vec{r}}{3\varepsilon_0}$$

at $\vec{r} = \left(\frac{2R}{3}, 0, 0\right)$. $\quad E = \frac{\rho\, 2R}{3\varepsilon_0 3} = \frac{2\rho R}{9\varepsilon_0}$

$$\vec{E} = \hat{x} \frac{2\rho R}{9\varepsilon_0} \quad \left(\text{or } \hat{r} \frac{2\rho R}{9\varepsilon_0}\right)$$

(b).



$\vec{E} = \vec{E}_L - \vec{E}_S$

$$\vec{E}_L = \hat{r} \frac{2\rho R}{9\varepsilon_0}$$

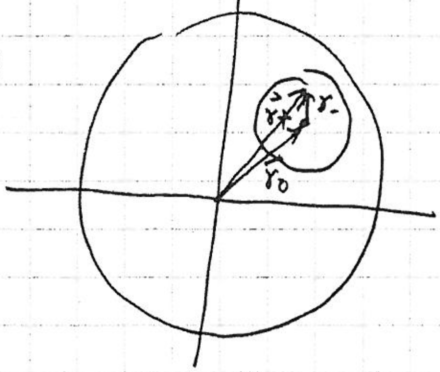
$$\vec{E}_S = \hat{r} \frac{\rho R}{3\varepsilon_0 6} = \hat{r} \frac{\rho R}{18\varepsilon_0}.$$

$$|\vec{r}_-| = \frac{2}{3}R - \frac{1}{2}R = \frac{R}{6}$$

$$\vec{E} = \hat{r} \frac{\rho R}{\varepsilon_0}\left(\frac{2}{9} - \frac{1}{18}\right) = \hat{r} \frac{\rho R}{\varepsilon_0}\left(\frac{3}{18}\right) = \hat{r} \frac{\rho R}{\varepsilon_0 6}.$$

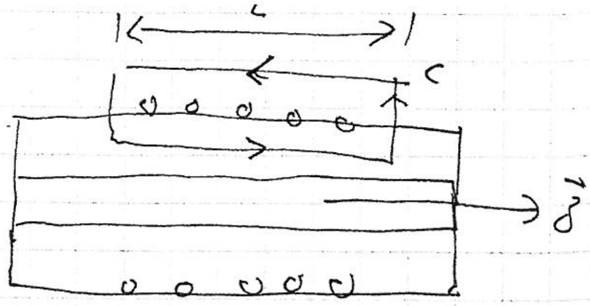
$$\left(\text{or: } \hat{x} \frac{\rho R}{\varepsilon_0 6}\right).$$

(c)



$$\vec{E} = \vec{E}_L - \vec{E}_S = \frac{\rho}{3\varepsilon_0}(\vec{r}_+ - \vec{r}_-) = \frac{\rho \vec{r}_0}{3\varepsilon_0}$$

3. (a)



$$\oint_c \vec{H} \cdot d\vec{l} = nL I$$

$$HL = nLI$$

$$H = nI . \qquad \vec{H} = \hat{z}\, nI . \quad \text{for} \quad 0 < r < b .$$

(b). $B_{1t} = B_{2t}$. for $0 < r < a$. $\vec{B} = \mu \vec{H} = \hat{z}\, \mu n I$.

for $a < r < b$: $\vec{B} = \mu_0 \vec{H} = \hat{z}\, \mu_0 n I$.

(c) . in the gap, and $0 < r < a$.

$$\cancel{B_z =} \quad H_t = 0 . \qquad B_t = 0 . \qquad \vec{B} = \hat{z}\, B_z .$$

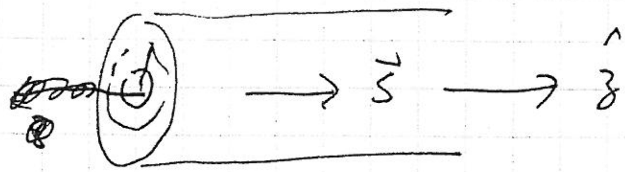
$$B_z = \mu n I \quad \therefore \quad \vec{B} = \hat{z}\, \mu n I .$$

$$H_z = \frac{B_z}{\mu_0} \quad \therefore \quad \vec{H} = \hat{z}\, \frac{\mu}{\mu_0} n I .$$

Gauss' law

4.

(a). $\oint_S \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon}$



$E_r 2\pi r L = \frac{\rho_\ell \cdot L}{\varepsilon}$

$\vec{E} = \hat{r}\cdot\frac{\rho_\ell}{2\pi\varepsilon r}$ $\qquad a < r < b$.

$\int_a^b \vec{E}\cdot d\vec{r} = V$.

$V = \frac{\rho_\ell}{2\pi\varepsilon}\ln\frac{b}{a}$. $\qquad \rho_\ell = \frac{2\pi\varepsilon V}{\ln\frac{b}{a}}$.

$\therefore \vec{E} = \hat{r}\,\frac{2\pi\varepsilon V}{\ln\frac{b}{a}\cdot 2\pi\varepsilon r} = \hat{r}\,\frac{V}{r\ln\frac{b}{a}}$ $\qquad a<r<b$.

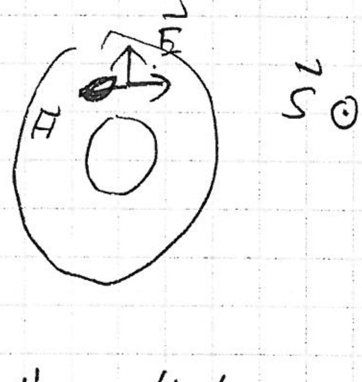
(b). Ampere law : $\vec{H} = \hat{\phi}H_\phi$.

$\oint_C \vec{H}\cdot d\vec{l} = I$. $\qquad H\cdot 2\pi r = I$. $\qquad I = \frac{V}{R}$

$\vec{H} = \frac{I}{2\pi r}\hat{\phi}$ $\qquad a<r<b$.

$= \frac{V}{2\pi r R}\hat{\phi}$.

(c) $\vec{S} = \hat{z}\cdot\frac{V}{r\ln\frac{b}{a}}\cdot\frac{V}{2\pi r R}$.



$= \hat{z}\,\frac{V^2}{r^2 2\pi R\ln\frac{b}{a}}$.

↑ along the axis of the cylinder.

5.

$$a = \frac{0.006}{2} = 0.003\ m = 3\times10^{-3}\ m.$$

$$b = \frac{0.01}{2} = 0.005\ m \quad 6 \quad = 5\times10^{-3}\ m. \quad 6$$

$$\sigma = 1.0\times10^{-3}\ S/m. \qquad \sigma_c = 5.8\times10^{7}\ S/m$$

$$f = 1\ GHz = 10^{9}\ Hz. \qquad \varepsilon_r = 2.5$$

a)

$$R_s = \sqrt{\frac{\pi f \mu_0}{\sigma_c}} = \sqrt{\frac{\pi\times10^{9}\times4\pi\times10^{-7}}{5.8\times10^{7}}} = \pi\times2.626\times10^{-3}$$

$$= 8.25\times10^{-3}\ \Omega$$

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = \frac{8.25\times10^{-3}}{2\pi}\left(\frac{1}{3\times10^{-3}} + \frac{1}{\cancel{5}\,6\times10^{-3}}\right) = 1.313\,(0.533)$$

$$= 0.700\ \Omega/m$$

0.64795

1.386

$$L' = \frac{\mu}{2\pi}\ln(b/a) = \cancel{1.02}\times10^{-7}\ H/m$$

0.90647

$$G' = \frac{2\pi\sigma}{\ln b/a} = \frac{2\pi\times10^{-3}}{\ln(5/3)} = \cancel{1.23}\times10^{-2}\ S/m.$$

$$C' = \frac{2\pi\varepsilon}{\ln(b/a)} = \frac{2\pi\times2.5\times8.85\times10^{-12}}{\ln(5/3)} = \cancel{2.72}\times10^{-10}\ F/m$$

2.00

b)

$$\gamma = \alpha + j\beta = \sqrt{(R'+j\omega L')(G'+j\omega C')}$$

$$R'+j\omega L' = 0.700 + j2\pi\times10^{9}\times1.02\times10^{-7} = 0.700 + j641 = 641\,e^{j89.9374}$$

$$G'+j\omega C' = 1.23\times10^{-2} + j2\pi\times10^{9}\times2.72\times10^{-10} = 0.0123 + j1.71 = 1.71\,e^{j89.587}$$

$$\gamma = \sqrt{641\times1.71}\ e^{j\left(\frac{89.9374^\circ + 89.5879^\circ}{2}\right)} = 33.1\,e^{j89.763^\circ}$$

5

$$\alpha = 33.1 \times \cos(89.763^\circ) = 0.137 \quad m^{-1}$$

$$\beta = 33.1 \times \sin(89.763^\circ) = 33.1 \quad m^{-1}.$$

$$u_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^9}{33.1} = 1.90 \times 10^8 \ m/s.$$

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}}$$

$$= \left(\frac{641 e^{j89.9374}}{1.71 e^{j89.5879}} \right)^{1/2}$$

$$= 19.36 \, e^{j0.175^\circ}$$

$$= 19.36 + j0.0591.$$

6. $A = 0.04\ m^2$ $f = 300\ MHz = 3\times10^8\ Hz.$

$\mathcal{E}_{max} = 60\ mV.$ $B = B_0 \cos(2\pi f t)$

$$\mathcal{E}_{emf} = -\frac{\partial \Phi}{\partial t}. = -A\frac{\partial B}{\partial t} = +A \cdot B_0 . \sin(2\pi f t)\cdot 2\pi f.$$

$$\mathcal{E}_{max} = 60\ mV = A B_0 \cdot 2\pi f.$$

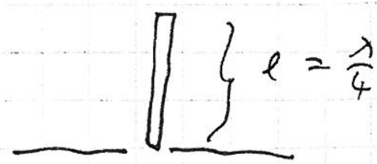
$$B_0 = \frac{\mathcal{E}_{max}}{2\pi f A} = \frac{6\times10^{-2}}{2\pi \times 3\times10^8 \times 0.04} = 7.96\times10^{-10}$$

Tesla.

7. $\sigma_c = 5.8 \times 10^7$ S/m. $f = 750$ MHz $= 7.5 \times 10^8$ Hz.

$\ell = 10$ cm $= 0.1$ m. $a = 0.1$ cm $= 0.001$ m $= 1 \times 10^{-3}$ m

$$\lambda = \frac{v}{f} = \frac{3 \times 10^8}{7.5 \times 10^8} = 0.4 \text{ m}.$$



2. (1) $\ell = \frac{\lambda}{4}$ — quater-λ monopole

(1) (Similar to $\frac{\lambda}{2}$ dipole)

(a)

(1) $$\xi = \frac{P_{rad}}{P_{rad} + P_{loss}} = \frac{R_{rad}}{R_{rad} + R_{loss}}.$$

(1)

$$R_{loss} = \frac{1}{\sigma_c} \cdot \frac{\ell}{2\pi a \delta_s} \qquad \frac{1}{\delta_s} = \alpha = \sqrt{\pi f \mu \sigma}.$$

$$= \frac{\ell \sqrt{\pi f \mu \sigma}}{\sigma 2\pi a} = \frac{\ell \cdot \sqrt{\cancel{\pi} f \cdot \cancel{4\pi} \times \cancel{10^{-7}} \times 5.8 \times \cancel{10^7}}}{5.8 \times 10^7 \cdot 2 \cdot \cancel{\pi} \cdot \times 10^{-3}}$$

$$= \frac{2(0.1)\sqrt{7.5 \times 10^8 \times 5.8}}{5.8 \times 10^7 \times 2 \times 10^{-3}}$$

$$= \frac{6.595 \times 10^3 \times 2}{1.16 \times 10^5}$$

$$= 5.685 \times 10^{-2} \times 2 \simeq 0.057 \ \Omega \times 2$$

$$= 0.114 \Omega.$$

$$\frac{\ell \sqrt{f \times 10^{-7} \sigma}}{\sigma a}$$

$$= \frac{\ell}{a}\sqrt{\frac{f \times 10^{-7}}{\sigma}}$$

$$= \frac{0.1}{0.001}\sqrt{\frac{7.5 \times 10^1}{5.8 \times 10^7}}$$

$$= 100 \times 1.137 \times 10^{-3}$$

0.114Ω.

(1) $$R_{rad} = \frac{1}{2}\left(R_{rad\,\frac{\lambda}{2}}\right) = 36.5 \Omega.$$

(1) $$\xi = \frac{36.5}{36.5 + 0.11} = 0.997 = 99.7\%$$

7 (b)

$G = \xi D$

$= 0.997 \times 1.64 \times 2$

$= 3.27$

(2)

$\beta_G = 10 \log 3.27 = 5.14 \text{ dB}$

c)

$\xi = \frac{P_{rad}}{P_t}$

(2)

$P_t = \frac{P_{rad}}{\xi} = \frac{10W}{0.997} = 10.03 \text{ W} = \frac{1}{2} R I_0^2$

$I_0 = \sqrt{\frac{2P_t}{R}}$ $\qquad R = R_{rad} + R_{loss} = 36.6\ \Omega$

$= \sqrt{\frac{2 \times 10.03}{36.6}}$

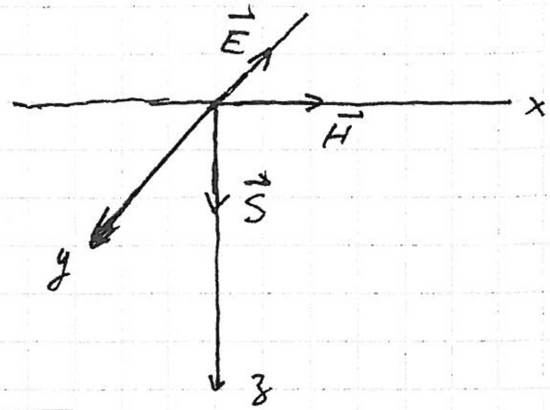
$= 0.74 \text{ A}$ (peak current)

$I_{rms} = 0.52 \text{ A}$

8. $\varepsilon_r = 80,\ \mu_r = 1,\ \sigma = 4\ S/m.$

$z = 0:$

$\vec{H} = \hat{x}\, 50 \cos\left(2\pi \times 10^4 t - \frac{\pi}{6}\right)$



$f = 10^4\ Hz. \qquad \omega = 2\pi \times 10^4.$

(1) $\varepsilon''/\varepsilon' = \dfrac{\sigma}{\omega\varepsilon} = \dfrac{4}{2\pi \times 10^4 \times 80 \times 8.85 \times 10^{-12}}$

$= 8.99 \times 10^4 \quad \gg 1.$

(1) $\therefore$ it's good conductor.

(1) $\alpha = \beta = \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 10^4 \times 4\pi \times 10^{-7} \times 4} = 4\pi\sqrt{10^{-3}}$

$= 0.3974\ m^{-1}.$

(1) (a). $\tilde{\vec{H}} = \hat{x}\, H_0 e^{-\alpha z} \cdot e^{-j\beta z} \cdot e^{-j\frac{\pi}{6}} \qquad H_0 = 50\ mA/m.$

(1) $\tilde{\vec{E}} = -\eta \hat{k} \times \tilde{\vec{H}}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon}} = (1+j)\frac{\alpha}{\sigma}.$

(1) $\tilde{\vec{E}} = -\hat{y}\, H_0 \eta \cdot e^{-\alpha z} e^{-j\beta z} \cdot e^{-j\frac{\pi}{6}}. \qquad \eta = \frac{0.3974}{4}(1+j)$ (1)

$= -\hat{y}\, 50 \times 0.1405\, e^{-\alpha z - j\beta z} \cdot e^{-j\frac{\pi}{6} + j\frac{\pi}{4}}. \qquad = 0.1405\, e^{j\frac{\pi}{4}}$

$= -\hat{y}\, 7.03\, e^{-\alpha z - j\beta z}\, e^{j\frac{\pi}{12}}$

(1) $\therefore \vec{E} = -\hat{y}\, 7.03\, e^{-0.3974 z} \cdot \cos\left(2\pi \times 10^4 t - 0.3974 z + \frac{\pi}{12}\right)$

$\vec{H} = \hat{x}\, 50\, e^{-0.3974 z} \cdot \cos\left(2\pi \times 10^4 t - 0.3974 z - \frac{\pi}{6}\right).$

8. (b) ① $\vec{S}_{av} = \frac{1}{2} Re\left(\vec{\tilde{E}} \times \vec{\tilde{H}}^*\right)$

$$= \frac{1}{2} Re\left[\hat{z}\,(7.03)(150)\, e^{-2(0.3974)z}\, e^{j\frac{\pi}{12}} \cdot e^{j\frac{\pi}{6}}\right]$$

$$= \frac{1}{2} Re\left[\hat{z}\, 351 \cdot e^{-0.7948z}\, e^{j\frac{\pi}{4}}\right]$$

$$= \frac{1}{2} Re\left[\hat{z}\, 351\, e^{-0.7948z} \cdot \left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right)\right]$$

$$= \frac{1}{2}\,\hat{z}\, 351\, e^{-0.7948z}\, \frac{\sqrt{2}}{2}$$

①

$$= \hat{z}\, 124\, e^{-0.7948z}$$

at $z = 20$ m.

$$\vec{S}_{av} = \hat{z}\, 124\, e^{-0.7948\times 20}$$

$$= 124\times 1.25\times 10^{-7}\ \text{W/m}^2.$$

$$= 1.5\times 10^{-5}\ \text{W/m}^2$$

9. $\vec{H} = \hat{z}\, 4 \cos(10^9 t - 5y)$

$= \hat{z}\, H_0 \cos(\omega t - ky)$

$$\begin{cases} \mu = \mu_0 \\ \omega = 10^9 \quad s^{-1} \\ k = 5 = \frac{2\pi}{\lambda} \end{cases}$$

2 a). $+y$ direction. $(\hat{k} = \hat{y})$

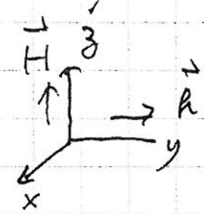
2 b). $u_p : \frac{\omega}{k} = \frac{10^9}{5} = 2.0 \times 10^8$ m/s.

2 c). $\lambda = \frac{2\pi}{k} = \frac{2\pi}{5} = 1.26$ m.

2 d). $\varepsilon_r = \left(\frac{c}{u_p}\right)^2 = \left(\frac{3\times10^8}{2\times10^8}\right)^2 = (1.5)^2 = 2.25$

$\left(u_p = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0}} = \frac{c}{\sqrt{\varepsilon_r}}. \quad \varepsilon_r = \left(\frac{c}{u_p}\right)^2 \right)$

2 e). $\alpha = 0$ lossless.



$\tilde{\vec{H}} = \hat{z}\, 4\, e^{-j5y}$.

$\tilde{\vec{E}} = -\vec{k} \times \tilde{\vec{H}}\, \eta =$

$= -\hat{x}\, 4 \cdot \sqrt{\frac{\mu}{\varepsilon}}\, e^{-j5y}$.

$= -\hat{x}\, \frac{4 \cdot 120\pi}{\sqrt{\varepsilon_r}}\, e^{-j5y}$

$= -\hat{x}\, \frac{480\pi}{1.5}\, e^{-j5y}$

$= -\hat{x}\, 320\pi e^{-j5y}$

$= -\hat{x}\, 1005\, e^{-j5y}$

10. $\dfrac{\varepsilon''}{\varepsilon'} = \dfrac{\sigma}{\omega\varepsilon}$. (2) — depends on frequency. (2)

(3) For radio frequencies: $f = 10^6$ Hz.

$$\omega \sim \cancel{10^7 \text{ Hz}}\ 2\pi f \simeq 6 \times 10^6 \text{ Hz}$$

$$\varepsilon = \varepsilon_r \varepsilon_0 = 80 \times 9 \times 10^{-12} = 7 \times 10^{-10}.$$

$$\frac{\varepsilon''}{\varepsilon'} \sim \frac{4}{6 \times 10^6 \times 7 \times 10^{-10}} \sim 10^3 \gg 1.$$

sea water is a good conductor.

(3) For visible light: $f \sim 10^{14}$ Hz. $\omega \sim 10^{15}$ Hz.

$$\frac{\sigma}{\omega\varepsilon} \sim \frac{4}{10^{15} \times 7 \times 10^{-10}} \sim 10^{-5} \ll 1$$

low loss. (good insulator).

for high frequencies, even if $\sigma > 1$,

$$\frac{\sigma}{\omega\varepsilon} \ll 1.$$